A TREATISE

ON

THE STRENGTH OF TIMBER, CAST AND MALLEABLE IRON,

AND OTHER MATERIALS:

WITH RULES FOR APPLICATION IN ARCHITECTURE,
THE CONSTRUCTION OF SUSPENSION BRIDGES, RAILWAYS, &c.;
AND
AN APPENDIX ON THE POWER OF LOCOMOTIVE ENGINES,
AND THE EFFECT OF INCLINED PLANES AND GRADIENTS.

WITH SEVEN PLATES.

BY PETER BARLOW, F.R.S.

MEM. INST. OF FRANCE; OF THE IMP. AND ROYAL ACADEMIES OF PETERSBURG AND BRUSSELS; OF THE AMER. SOC. ARTS; AND HON. MEM. INST. CIVIL ENGINEERS.

A New Edition,

REVISED AND CORRECTED BY I. F. HEATHER, M.A.
OF THE ROYAL MILITARY ACADEMY, WOOLWICH.

TO WHICH IS ADDED

AN ESSAY ON THE EFFECTS PRODUCED BY CAUSING WEIGHTS TO TRAVEL OVER ELASTIC BARS.

BY THE REV. ROBERT WILLIS, M.A., F.R.S.

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WITH NINE ILLUSTRATIONS.

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TO

THE REV. ROBERT WILLIS,

M.A., F.R.S.

JACKSONIAN PROFESSOR IN THE UNIVERSITY OF CAMBRIDGE,

This Edition of

PROFESSOR BARLOW'S WORK

IS RESPECTFULLY DEDICATED

BY HIS OBEDIENT SERVANT,

THE PUBLISHER.

PREFACE TO THE FOURTH EDITION.

The first edition of my 'Essay on the Strength and Stress of Timber' was published in 1817, since which it has gone through three editions: another edition having been called for, I have thought it right to remodel the whole, and to introduce into it a great variety of matter not found in the original work. At the time of the first publication, the construction of suspension bridges was in its infancy, and the application of malleable iron for the purposes of railways unknown. These, and various novel applications of timber, iron, and other materials, to different mechanical works, have rendered it necessary to investigate, experimentally and theoretically, many subjects which were not known when the first edition of this work was published, and which it was difficult to introduce without remodelling the whole.

This has been accordingly done, and it is hoped that the utility of the work has been thereby greatly increased. The arrangement which it has now been thought proper to adopt may be thus stated: the first subject treated of is the strength of direct cohesion of the fibres of timber, with an account of the experiments of Musschenbroeck, Du Hamel, Emerson, and others; and lastly, of those made by the author, with a description of the apparatus by which the results were obtained.

The next division treats of the mechanism of the transverse strain to which timber and other materials are exposed when loaded in any part of their length, and the

mechanical action of the fibres to resist this strain. We then investigate theoretically the laws of deflections under all the varieties of position and fixing to which timber and iron are subjected in architectural and other constructions. Having thus examined theoretically the nature of the several strains and the consequent deflections, we proceed to a detail of various experiments by Buffon, Girard, Beaufoy, &c., on the transverse strength of timber; and lastly, the original experiments of the author, which laid the foundation of the first edition, and on which is founded the Table of Data adopted in the subsequent part of this division of the work. Another section is employed in the detail of experiments on bent timber, as used in ship-building—on the effect of boiling and steaming timber; experiments by Girard on vertical pressure, and a series of illustrative problems and examples. A short chapter follows on the strength of cement and building materials, as stone, brick, &c., and on the subject of revetment walls.

The next division treats on the direct strength of cast iron and its application in the construction of hydrostatic presses; also on the direct strength of copper, brass, yellow metal, &c., from experiments made by Mr. Kingston on the testing machine in Woolwich Dockyard; and others by Messrs. George Rennie, Tredgold, and Duleau.

The following chapter treats on the transverse strength and deflection of cast-iron beams under a great diversity of forms, principally from a highly interesting and valuable Paper by Eaton Hodgkinson, Esq., in volume v. of the 'Manchester Memoirs.' We come now to the subject of malleable iron; and as the experiments on this material were principally made on the testing machine in the Dockyard at Woolwich, it was thought that an

accurate drawing and description of this machine would be acceptable to the reader: two new plates have been therefore introduced, illustrating its entire construction and operation.

A detail of experiments is then given on the strength of direct cohesion of iron bars and bolts, the testing strengths of the different descriptions of iron cables used in the British Navy; Mr. Telford's experiments on iron wires; and lastly, Tables by Davies Gilbert, Esq., for the calculation of the several particulars connected with the construction of suspension bridges.

The next subject of investigation is the application of malleable iron to the purposes of railway bars, being the substance of two Reports by the author, addressed to the Directors of the London and Birmingham Railway Company, with the addition of several subsequent experiments on railway bars of various forms and dimensions, and of miscellaneous experiments on the effect of locomotive engines and trains on the bars of the Liverpool and Manchester line.

These form the subject of the principal matters treated of in the body of the work, but an Appendix is added, on the practical action of locomotive engines, and on the effect of inclined planes and gradients, with a view to the comparison of the mechanical advantages and disadvantages of rival lines of railway.

May 10th, 1837.

PREFACE TO THE PRESENT EDITION.

In the latter part of this present edition of Professor Barlow's valuable treatise much space has been saved by the omission of such remarks as appeared to be of mere temporary or local importance, and by avoiding the diffuseness incidental to the form of Reports made on special occasions, while all the experiments, with the talented author's deductions from them, which have a permanent interest, have been retained. Room has thus been made for the insertion, by Professor Willis, of his treatise on the effects produced by causing weights to travel over elastic bars.

With respect to the corrections in this edition, there is only one which appears to call for any particular remark. The basis of this error occurred at page 80 of the previous edition, in supposing that with the same amount of strain there could be different amounts of deflection, according to the manner in which the beam was fixed. From this cause the values of the constant E, at page 145 of this edition, were given as large again as they should have been. As these errors arose from the application of an erroneous formula only, they were corrected by the corresponding error of the formula in the computation of the deflections of all beams supported in the same manner as those which had been experimented upon to obtain the tabular constants,—that is, of all beams supported at both ends and loaded in the middle; but the results obtained from their application to the computation of beams fixed at one end would be erroneous. This important error has now been corrected, and the whole work having been carefully revised, it is hoped that the tables and formulæ contained in it will be found to preserve that accuracy which is so essential to their practical utility.

I. F. H.

Royal Military Academy,
21st April, 1851.

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A TREATISE

ON

THE STRENGTH OF MATERIALS.

ON THE STRENGTH OF TIMBER.

1. There are four distinct strains to which a beam of timber, a bar of metal, or any other hard body, may be exposed, and in which the mechanical effort to produce the fracture, and the resistance opposed to it by the fibres or particles, are differently exerted; while each of these again is subject to various modifications, according to the manner in which the bodies are supported or fixed, the positions in which they are placed, and the direction of the forces or strains to which they are exposed.

These four distinct cases or strains may be stated as follow:

1st. A body may be torn asunder by a stretching force applied in the direction of its fibres, as in the case of ropes, stretchers, king-posts, tie-beams, &c.

2ndly. It may be broken across by a transverse strain, or by a force acting either perpendicularly or

obliquely to its length, as in the case of levers, joists, &c.

3rdly. A beam or bar may also be destroyed by a pressure exerted in the direction of its length, as in the case of pillars, posts, and truss-beams.

4thly. It may be twisted or wrenched by a force acting in a perpendicular direction, at the extremity of a lever or otherwise, as in the case of the axle of a wheel, the lever of a press, &c.

These several cases will form the subject of inquiry in the following pages.

Experiments on the Strength of Direct Cohesion of the Fibres of different kinds of Wood.

2. It is usual to distinguish by the expression *force of direct cohesion of bodies*, or simply *direct cohesion*, that force by which the fibres or particles of a body resist a separation, and which must ultimately be traced to that unknown cause we are accustomed to speak of under the denomination of *corpuscular attraction*.

This is by far the simplest strain of the four above alluded to with regard to its mechanical action; but the most difficult to submit to experiment, in consequence of the enormous forces that are requisite to produce the rupture even on pieces of small dimensions, and the great difficulty there is in applying those forces in the direct line of the fibres of the body; and if this is not done, the first rupture may

be occasioned by some unequal action of the weight on a part of the fibres only, or by some force of torsion whereby a part of them may be wrenched asunder.

The consequence in either case is, that the force of direct cohesion will be estimated at less than its real value; and it is probably owing to this circumstance that so little agreement is found in the results of such experiments as have been made with a view to this determination. The strength of different woods of the same kind, and of different parts of the same timber, is also very different, as has been shown by the experiments of Musschenbroeck, Robison, Buffon, and others; but, as regards this difference, we still unfortunately meet with strange discrepancies. Musschenbroeck's experiments were made with great care, and he has given a very minute detail of them, particularly those on ash and walnut. In these he states the weights required to tear asunder slips taken from the four sides of the tree, and on each side in a regular succession from the centre to the circumference. His pieces were all formed into slips fitted to his apparatus, and cut down to the form of parallelopipedons of $\frac{1}{5}$th an inch square, and therefore $\frac{1}{25}$th of a square inch section; and the several weights required to produce the rupture when the rods are reduced to a square inch, are as stated in the following Table:

3. *Musschenbroeck's results on the Strength of Direct Cohesion.*

	lbs.		lbs.
Locust-tree . . .	20,100	Pomegranate . . .	9,750
Jugeb	18,500	Lemon	9,250
Beech, Oak . . .	17,300	Tamarind	8,750
Orange	15,500	Fir	8,330
Alder	13,900	Walnut	8,130
Elm	13,200	Pitch Pine	7,650
Mulberry	12,500	Quince	6,750
Willow	12,500	Cypress	6,000
Ash	12,000	Poplar	5,500
Plum	11,800	Cedar [1]	4,880
Elder	10,000		

In these experiments, it was found, that the wood immediately surrounding the pith or heart was the weakest. Dr. Robison also asserts, under the article STRENGTH, 'Encyclopædia Britannica,' from his own observation on *very large* oaks and firs, that the heart was weaker than the exterior parts. He observes also, that the wood next the bark, commonly called the *white*, or *sap*, is again weaker than the rest; and that, generally, the greatest strength is found between the centre and the sap.

With regard to our experiments, they were not particularly directed towards this inquiry; but, in most cases, the heaviest wood was found the strongest; and this was generally the case with those parts that grew nearest the centre of the trunk, and nearest to the root, provided it was so far removed from the latter as not to be very cross-grained. M. Girard [2]

[1] See Musschenbroeck's 'System of Natural Philosophy,' published after his death, by Lulofs, 3 vols. 4to; or the French translation of the same, by Sigaud de la Fond, Paris, 1760.

[2] Traité Analytique de la Résistance des Solides.

is also of the same opinion, stating it as a well-established fact, that the strongest part of a tree is nearest the centre.

4. From this contrariety of results, it is difficult to draw any satisfactory conclusion: the probability is, that much depends upon the age of the timber, and on the soil in which it was grown. While the tree is advancing in its growth, the last-formed wood, that is, the exterior parts, are probably weaker than the heart; but when a tree has attained complete maturity, and approaches, though imperceptibly, towards decay, the circumstances may be reversed; the exterior parts, or last-formed wood, becoming harder and stronger, while the central parts are beginning to experience that dissolution which ultimately pervades the whole. It may be observed, that Dr. Robison states his timbers to be *very large;* and Musschenbroeck's must have likewise been of considerable size, from the number of slips he was able to cut out between the centre and circumference: both which circumstances seem to give a degree of probability to the above suggestions.

Very nearly the same view is taken of this subject by Du Hamel, in his work, 'Sur l'Exploitation des Bois,' where the same ideas are given, not (as those above) merely as conjectures, but as facts, drawn from numerous experiments and observations. The author concludes his chapter on this subject as follows: "Si ce que nous venons d'avancer est vrai, il faut nécessairement que le bois qui est vers le centre

du pied d'un arbre, encore en crûe, soit plus pesant que celui qui est au haut de la tige, et dans toutes les parties de l'arbre; que celui qui est au centre, doit être plus pesant que celui qui est à la circonférence. Au contraire, quand les arbres sont sur leur retour, le bois du centre doit être moins pesant que celui qui est plus près de la superficie, à cause de l'altération qu'il a soufferte. C'est un fait que nous avons vérifié par plusieurs expériences."

The work above referred to by Du Hamel contains many very curious and interesting experiments connected with this subject, as to the chemical analysis and natural decomposition of wood; of the quality of different woods, as depending upon the nature of the soil, &c.

From a great number of experiments and observations on the latter point, the author concludes that the best oaks, elms, and other great trees are the produce of good lands, rather of a dry than of a moist quality: they have a fine and clear bark; the sap is thinner in proportion to the diameter of the trunk; the ligneous layers are less thick, but are more adherent the one to the other, and have a greater uniformity of texture, than trees which grow in moister situations. The grain of these woods is fine and compact; and when they are examined with a good glass, their pores are observed to be filled with a species of varnish, or glutinous matter, strongly adherent, which gives them commonly a pale yellow colour, by which they may be distinguished from trees that are the growth of a different soil.

Also, in consequence of the closeness of their pores, they are more dense and heavy, become extremely hard, and resist the attack of worms.

The specific gravity of a tree growing in such soil as that above described, is to that of a similar tree in a wet marshy situation, frequently as 7 to 5; and the weights which a similar beam will support without breaking, in the two cases, are in about the ratio of 5 to 4.

May not this account for the superior quality of the Sussex oak? which I am informed by Mr. Hookey, timber-master in Deptford Dockyard, he has always found to be the best for strength and durability: that the next in quality is that which grows in the south-west parts of Kent, and the north-east parts of Hampshire.

5. As to the density of the top and bottom of the same tree, and of the centre and external parts, much depends upon the age of the timber when felled; but generally, in a sound tree, the density is found to decrease from the butt upwards, and from the centre to the circumference. On the former point, the following experiments, the result of many years' observation, which have been made with great care by Mr. B. Couch, timber-master in Plymouth Dockyard, are highly valuable; and they are given in preference to those of Du Hamel; not only on account of their containing a greater variety of woods, but because the results are given in weights and measures which are more familiar to English engineers.

TABLE OF EXPERIMENTS,

Instituted in order to ascertain the Weight of a Cubic Foot of different kinds of Wood; the Foreign when first imported, those of the Growth of England when felled: also the Weight of each when fully seasoned; showing, at the same time, the Loss sustained in Dimensions during the Process of Seasoning.

BY MR. BENJAMIN COUCH, OF PLYMOUTH DOCKYARD.

Species (in the language of Commerce).	Country where produced.	What part of the tree the pieces experimented on were cut from.	Dimensions. When first planed for experiment. Length. ft.	in.	Breadth and thickness, or diameter. inches.	When seasoned. Length. ft.	in.	Breadth and thickness, or diameter. inches.	Weight in air of a cubic foot, oz. avoirdupois. When first planed for experiment. ounces.	When seasoned. ounces.
Riga Masts, superior	Russia	Butt[3]	7	6	18 diameter[4]	7	6	$17\frac{3}{4}$ diameter	672	644
		Top	4	5	11 by 11	4	5	$10\frac{13}{16}$ by $10\frac{3}{4}$	546	552[5]
Riga Masts, inferior	Russia	Butt	12	0	12 diameter	12	0	$11\frac{7}{8}$ diameter	577	494
		Top	6	6	$8\frac{3}{4}$ diameter	6	6	$8\frac{1}{2}$ diameter	464	464
Pitch Pine Mast	Baltimore	Butt	2	6	10 by 10	2	6	$9\frac{3}{4}$ by $9\frac{3}{4}$	755	741
	North America	Top	6	0	$7\frac{1}{2}$ by $7\frac{1}{2}$	6	0	$7\frac{3}{8}$ by $7\frac{3}{8}$	518	524
	Virginia	Butt	3	4	$18\frac{3}{8}$ diameter	3	4	$18\frac{1}{4}$ diameter	628	597
	North America	Top	7	6	10 by 8	7	6	$9\frac{15}{16}$ by $7\frac{15}{16}$	540	529
Yellow Pine Mast	Canada	Butt	2	4	18 by 18	2	4	$17\frac{7}{8}$ by $17\frac{7}{8}$	683	461
	North America	Top	5	0	16 by 16	5	0	$15\frac{7}{8}$ by $15\frac{7}{8}$	495	420
White Pine Mast	American States	Butt	3	6	12 by 12	3	6	$11\frac{7}{8}$ by $11\frac{3}{4}$	555	405
		Top	9	6	12 by 12	9	6	$11\frac{7}{8}$ by $11\frac{7}{8}$	633	448
Northern Pine Mast	New York	Butt	3	6	$17\frac{5}{8}$ diameter	3	6	$17\frac{5}{8}$ diameter	658	549
		Top	4	0	[illegible]	4	0	[illegible]	[illegible]	[illegible]

		Butt . .	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
White Pine Mast .	New Brunswick .	Top . .	3	11	8 by 9	3	11	$7\frac{1}{16}$ by $8\frac{1}{16}$	411	368
Red Pine Mast . .	Canada	Butt . .	2	0	12 by 12	2	0	$11\frac{3}{4}$ by $11\frac{3}{4}$	672	569
		Top . .	14	0	11 by 9	14	0	$10\frac{5}{8}$ by $8\frac{3}{4}$	570	503
Spruce Spar . .	Halifax	Butt . .	4	0	$8\frac{1}{2}$ diameter	4	0	$8\frac{3}{8}$ diameter	587	580
		Top . .	4	0	$5\frac{1}{2}$ diameter	4	0	$5\frac{3}{8}$ diameter	541	554
	Canada	Butt . .	4	0	7 diameter	4	0	$6\frac{15}{16}$ diameter	528	524
		Top . .	4	0	$4\frac{1}{2}$ diameter	4	0	$4\frac{3}{8}$ diameter	485	512
Poon	East Indies . .	Butt . .	4	0	17 diameter	4	0	$16\frac{3}{4}$ diameter	651	576
		Top . .	6	0	9 by 9	6	0	9 by $8\frac{7}{8}$	771	695
Teak	Ditto	Butt . .	4	0	12 diameter	4	0	12 diameter	662	657
		Top . .	4	6	$6\frac{1}{2}$ by $6\frac{1}{2}$	4	6	$6\frac{1}{2}$ by $6\frac{1}{2}$	688	675
Yellow Wood . .	Cape of Good Hope	Butt . .	4	0	$5\frac{1}{2}$ diameter	4	0	$5\frac{7}{16}$ diameter	661	657
		Top . .	4	0	5 diameter	4	0	5 diameter	632	630
Stink Wood . . .	Ditto	Uncertain	6	0	9 by 4	6	0	9 by 4	700	681
Letter Wood . .	Surinam	Butt . .	0	7	$4\frac{1}{4}$ by 4	0	7	$4\frac{1}{4}$ by 4	1286	1286
Cedar	Spanish America .	Root . .	3	2	$5\frac{7}{8}$ by 6	3	2	$5\frac{7}{8}$ by 6	722	681
	Ditto	Trunk .	2	0	12 by $5\frac{1}{2}$	2	0	12 by $5\frac{1}{2}$	457	453
	Canada	Uncertain	1	2	14 by 14	1	2	$13\frac{1}{4}$ by $13\frac{1}{4}$	909	753
Oak	England	Butt . .	1	10	12 by $11\frac{3}{4}$	1	10	$11\frac{5}{8}$ by $11\frac{3}{8}$	1113	743
		Top . .	2	0	$6\frac{1}{2}$ by 6	2	0	$6\frac{3}{16}$ by $5\frac{3}{4}$	1071	777
Elm	Ditto	Uncertain	4	0	11 by 11	4	0	$10\frac{3}{4}$ by $10\frac{3}{4}$	940	588
Bog Oak	Ireland	Ditto . .	0	$11\frac{3}{4}$	3 by $1\frac{1}{2}$	0	$11\frac{3}{4}$	3 by $1\frac{1}{2}$	1046	1046

[3] The butts and tops were cut from the same tree.

[4] When diameter is expressed, the pieces are cylindrical; all the others are parallelopipedons.

[5] Should it be asked, Why a cubic foot of some of the pieces increases in weight in *seasoning?* the reason is, that they lost more in dimensions than in weight in undergoing that process.

Columns 5, 6, 7, *&c. will possibly be found of advantage to practical men, as they will enable them to form an idea of the decrease of dimensions in seasoning.*

6. To the same gentleman I am indebted for the following Table relative to the loss of weight sustained by oak in seasoning. The eight pieces on which the experiments were made, were English oak, varying from 3 inches to 10⅝ inches in thickness, and from 24 inches to 40 inches in length; the particulars of which are stated in the three upper lines in the following Table; the dimensions there given being those of the pieces when first taken from the saw-pits in their rough state, viz., without planing; and not being originally cut for the purpose of these experiments, most of the dimensions are found partly fractional.

These several pieces were laid on the beams of a smith's shop, and placed at such a distance from the forges that the fire might only operate sufficiently to keep the air dry. They were converted from trees just received from the forest, and were weighed every month, from February, 1810, to August, 1812; at which latter period, it was observed that the larger pieces lost but little of their weight, and the weighing of them monthly was therefore discontinued, and only performed annually, as shown in the annexed Table: from which it appears that the

Total weight, February, 1810, was	972¼ lbs.
Ditto, August, 1815	630½
Weight lost	341¾

That is, more than one-third of the weight is lost in seasoning.

The specific gravity of No. 1, before seasoning, was 1074, and after that process only 720; and it is probable that the specific gravity of oak is always within these limits; or, at least, that it seldom much exceeds the greatest, or falls below the least of these numbers.

TABLE OF EXPERIMENTS

RELATIVE TO THE LOSS OF WEIGHT IN SEASONING ENGLISH OAK.

BY MR. COUCH.

	No. 1.	No. 2.	No. 3.	No. 4.	No. 5.	No. 6.	No. 7.	No. 8.
	Inches.	Inches.	Inches.	Inches.	Inches.	Inches.	Inches.	Inches.
Length........	24¾	25⅝	30⅝	31⅛	39¾	30⅛	37⅞	38¾
Breadth.......	16⅜	14⅛	16⅛	12⅝	16⅜	12¾	14⅛	14¼
Depth	10⅜	9¾	8⅛	7⅛	6	5⅛	4	3
Periods of Weighing.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.
February, 1810..	163½	133	164	104½	163½	77¼	92	74½
March	154½	123¼	155¼	99	148¾	71½	82¼	65½
April	149½	118	151¼	96	142½	68¾	78	60¾
May	144½	113½	147	92¾	135¾	66¼	75	56¾
June..........	140½	109¼	143½	90¼	130¼	64	71¼	53¼
July	137¾	106¾	141	88¼	127	62	69½	51½
August........	135¼	104½	139¼	87	123¾	61	67¾	50½
September.....	133	102¾	137¼	85¼	121	59¾	66	49¼
October	131¾	101½	136	84½	119¼	58¾	65	48¼
November	130¼	100¼	134¾	84	117¾	58¼	64½	47¾
December	129¾	99½	134¼	83¼	117½	58	63¾	47¾
January, 1811 ..	129	99	133¾	82¾	116¾	57¾	63½	47½
February[6]......	130¼	100¼	135	84	118	57¾	65	47¼
March	127¼	98	132	81¾	115	57	62¾	47
April[7]	127¾	97	132¾	83	116	58½	64	46½
May	125¼	96½	130	80¼	113¼	56¼	61¾	46¼
June..........	124¼	95¼	129¼	79½	112¼	55¾	61	46¼
July[8]	124½	96	129½	80¾	112½	55½	62¼	46
August........	121¾	93¼	127	78½	109¾	54½	60¼	45½
September.....	122	92¼	127½	79¼	109	56	59½	45½
October	119¾	92	125½	77½	108¼	54	59½	45½
November	121	93¾	126½	77	110	56	59¼	45¾
December	119¾	91¾	125	77¼	108½	54	59½	45¾
January, 1812 ..	119	91¼	124½	76½	107¾	53¾	59	45¾
February	118¾	91¼	124	76¾	107½	53½	59	45¾
March	118½	91	124	76¾	107¾	53½	59¼	45¾
April	117½	90¾	123	76¼	106¾	53¼	58¾	45½
May	117¼	90½	122¾	75¾	106½	53	58¼	45½
June..........	116¾	89¼	122	75½	106	52¾	58	45½
July	116	89¼	121½	75¼	105¾	52¾	58¼	45½
August, 1812...	115½	89	121	74¾	105¾	52½	58½	45¼
August, 1813...	111¼	85¾	116¾	72½	103¼	51½	57¼	45
August, 1814...	108½	85	114¼	71¼	103	51	57½	45½
August, 1815...	106¾	84½	112½	70½	102¾	51¼	57¼	45

[6] Very much rain since last weighed.

[7] Rained several days previous to weighing.

[8] Constant rain for two days previous to weighing.

The loss of weight in the preceding experiments was more rapid than in the similar experiments of Du Hamel: but much depends upon the nature of the soil in which the trees grow, as the timber of moist land loses more of its weight in seasoning than that which is the produce of a drier and better soil.

7. The process of seasoning may be facilitated by boiling, steaming, &c., as appears from the following experiments of Mr. Hookey. The three pieces marked Nos. 1, 2, and 3, were English oak, each four feet long and three inches square; all cut from the same timber. No. 1 was placed in the steam kiln for twelve hours; No. 2 was boiled for the same time in fresh water; and No. 3 was left in its natural state. The weights of the three pieces, previous to the experiment, and at the end of each month for half a year afterwards, were as stated below.

Times of Weighing.	No. 1. Steamed.	No. 2. Boiled.	No. 3. Natural State.
	Weight. lbs. oz.	Weight. lbs. oz.	Weight. lbs. oz.
Previous to the experiment	16 12¼	16 15	16 14
After ditto	16 6	16 14	16 14
June	15 1	15 10	16 5
July	14 2	14 12	15 14
August	13 13	14 0	15 5
September	12 10	13 6	15 0
October	12 5	12 10	14 12
November	11 10	12 5	14 8

All the pieces were placed in the same place, in the open air, and in the same position, (i. e. ver-

tically,) after the experiment, and were continued so during the six months that their weights were taken.

From the above it appears that the process of seasoning went on more rapidly in the piece that was steamed that in that which was boiled; but that in the latter, the process was carried on much quicker than in the piece which was left in its natural state:

The first had its specific gravity reduced from 1050 to 744.
The second .... from 1084 to 788.
And the third .... from 1080 to 928.

We must look to the philosopher for a satisfactory solution of the problem presented in these results. Mr. Hookey[9] accounts for the facts by supposing, that the process of boiling or steaming dissolves the *pithy* substance contained in the air tubes, by which means the latter fluid circulates more freely, and that the seasoning thereby proceeds with greater rapidity.

8. From the several experiments above given, and from others found in Du Hamel's work above referred to, it appears,

1. That the density of the same species of timber, and in the same climate, but on different soils, will vary as much as in the ratio of seven to five; and

[9] To this gentleman is due the ingenious idea of bending large ship timbers.—See 'Transactions of the Society of Arts,' vol. xxxii.

that the strength of the same will be, both before and after seasoning, in nearly the ratio of five to four.

2. In healthy trees, or those which have not already passed their prime, the density of the butt is in some cases to that of the top in about the ratio of four to three, and that of the centre to the circumference as seven to five.

3. The contrary occurs when the tree is left standing after it has acquired full maturity; viz. the butt will in this case be specifically lighter than the top, and the centre than the outward part of the trunk within the bark.

4. That oak, in seasoning, loses at least one-third of its original weight; and this process is much facilitated by steaming or boiling.

On these subjects, as well as a variety of others, relative to the quality of timber, &c., which do not properly fall within the plan of this work, the reader is referred to the Treatise of Du Hamel above mentioned, where he will find much useful and important information.

Experiments made for determining the Strength of Direct Cohesion of different Woods.

9. It has been before remarked, that notwithstanding the mechanical operation in this kind of fracture is by far the most simple of the four alluded to, yet

it is the most difficult to submit to actual experiment in wood; and it was not till after some consideration and one or two failures, that we were led to adopt the apparatus exhibited in Plate I.

Here A B, fig. 1, represents one of the pieces whose strength is to be determined, its whole length being 12 inches; the length of each square end $3\frac{1}{2}$ inches, and the side of the square end $1\frac{1}{2}$ inch: the intermediate part of 5 inches was turned in an excellent instrument by a very correct workman,[10] and brought down in the centre to $\frac{1}{3}$rd or $\frac{1}{4}$th of an inch in diameter;[11] but the other cylindrical parts were made each $\frac{3}{4}$ inch in diameter. C C, D D, fig. 2, represent two strong iron bars, brought to the form shown in the Plate; G G are two screws which are passed through the holes H H, in the bar D D, and are there screwed fast by the nuts I, I; E, E are two semicircular collars, riveted one to each bar, which, when the two are fixed together, form a circular plate, as represented in fig. 4. The circular hollow parts *e*, *e*, are $\frac{3}{4}$ inch in diameter, so as to fit exactly the larger part of the cylinder shown in fig. 1. These bars, after being screwed together, were rested on their supports, as in fig. 4, and, as the workmen

[10] Mr. Short, modeller to the Royal Military Academy.

[11] As it was difficult to measure very exactly the diameter of the small cylinder, it was found by winding a fine thread of silk ten times about it, and then dividing its length by the number of volutions, in order to get the mean circumference, and hence the diameter.

express it, *brought out of winding*, and accurately adjusted to a horizontal position by a spirit level.

The two iron boxes M N O, M′ N′ O′, fig. 3, were made exactly to fit the square head B, of fig. 1, having also two semicircular holes at top, correctly fitted to the larger part of the cylinder: these were shut by passing the bolts through the holes N, M, and were thus secured by the two sheers shown in fig. 4.

Having thus described the separate parts of the apparatus, the reader will perceive at once the manner in which they were employed in the experiment: viz. the head A, of fig. 1, was placed above the collar E E, fig. 2, the upper larger cylindrical part of fig. 1 being placed in the hollow parts *e*, *e*, of fig. 2, when the two parts were securely fixed together by the nuts and screws I, G; I, G. In the same manner the lower end B, of fig. 1, was enclosed in the two iron boxes MNO, M′N′O′, fig. 3, and fastened in that position by means of the bolts, seen in fig. 4, and the sheers above described. The whole was then rested on the props, fig. 4; and the hook of the scale being inserted in the circular hole formed by O, O′, fig. 3, the whole was ready for the experiment, as shown at large in the former figure.

Every thing being thus prepared, the wedges shown in the Plate were introduced under the scale, to keep it steady, while the larger weights were put in; the former were then removed, and smaller

weights added in succession till the fracture took place.

The weights were 10-inch, 8-inch, and 5½-inch shells, loaded each with as many musket-balls as brought them respectively to 100 lbs., 50 lbs., and 15 lbs. A few common weights of 7 lbs., 4 lbs., 2 lbs., &c. were also employed toward the conclusion of an experiment, where it was necessary to increase the weight by small degrees.

It should also be observed, that as a slight vibration of the scale might cause a fracture in the small cylinder submitted to the operation of the weight, four small braces were made use of, one at each corner of the scale, to prevent any such motion. These were attached to the four inward legs of the stand, which are omitted in the Plate, to avoid a complication of parts.

The results of these experiments are exhibited in the following Table.

TABLE I.

10. *Experiments on the Direct Cohesion of different Woods.*

No. of experiments.	Names of the woods.	Specific gravity.	Circumference.	Weight in lbs.	Weight reduced to a square inch.	Mean value of direct cohesion on a squ. inch.
1	Fir	600	1·05	1140	12993	12857
2	do.	600	1·10	1260	13073	
3	do.	600	1·10	1191	12037	
4	do.	600	1·05	1160	13220	
5	do.	600	1·11	1213	12371	
6	do.	600	1·05	1180	13448	
7	do.	581	1·10	1059	11000	11549
8	do.	564	1·10	1201	12472	
9	do.	601	1·10	1094	11360	
10	do.	611	1·10	1130	11736	
11	do.	532	1·10	1076	11180	
12	do.	590	1·10	1112	11548	

The first six experiments were made upon the fragments of the 4-foot pieces (Art. 88), which were the same also as the triangular pieces, Nos. 3, 4, 7, and 8 (Art. 93), were cut from.

These pieces were all cut from a plank remarkably free from knots and irregularities, which throughout gave more uniform results than any other specimen.

No. 7, broke by a part of the fibres drawing out of the head of the piece: it was probably first broken by an accidental motion of the scale.

No. 9, broke by the whole of the middle cylinder drawing out of the head, to the length of about 2 inches, where there was a knot, which might break off the continuation of the fibres. The others were all complete fractures.

TABLE I.

11. *Experiments on the Direct Cohesion of different Woods.*

No. of experiments.	Names of the woods.	Specific gravity.	Circumference.	Weight in lbs.	Weight reduced to a square inch.	Mean value of direct cohesion.
13	Ash	594	·8800	1100	17850	
14	do.	611	·9000	1096	17003	17207
15	do.	611	·8750	1024	16770	
16	do.	600	·8375	881	15784	
17	do.	600	·8625	1025	17315	16947
18	do.	600	·8750	1081	17742	
19	Beech	712	·880	716	11626	
20	do.	694	·890	721	11437	11467
21	do.	700	·900	731	11338	
22	Oak	770	1·10	856	8889	
23	do.	770	1·10	887	9211	9198
24	do.	770	1·10	908	9494	
25	do.	920	·8800	740	12008	
26	do.	920	·8750	712	11660	11580
27	do.	920	·8900	698	11072	

Nothing remarkable happened in the course of these experiments, except that No. 4 of the ash, viz. No. 16 above, was observed to twist, during the action of the weight, about 7½°, but the fracture took place in the small part of the cylinder: as this piece, however, bore less weight than any other of the ash, it is probably to be attributed to the above circumstance: a similar effect was observed in the specimens of mahogany, as stated in the following page.

It is proper to observe, that Nos. 13, 14, and 15 were made from the fragments of the 2-inch square ash pieces, Art. 98; those of the beech from the fragments of the similar pieces, Art. 99.

The first three oak pieces were off the same plank as the several battens, Art. 96. It was a very fine piece of English oak, which had been a considerable time in store, and was perfectly dry: the other specimen, viz. Nos. 25, 26, and 27, appears, from its specific gravity, to have been more recently felled: it was also of a closer texture.

TABLE I.

12. *Experiments on the Direct Cohesion of different Woods.*

No. of experiments.	Names of the woods.	Specific gravity.	Circumference.	Weight in lbs.	Weight reduced to a square inch.	Mean value of direct cohesion.
28	Teak	860	·8625	868	14662	
29	do.	860	·8625	900	15203	15090
30	do.	860	·8625	912	15405	
31	Box	960	·8625	1168	19730	
32	do.	960	·8625	1160	19595	19891
33	do.	1024	·8625	1200	20348	
34	Pear	646	·8625	683	11537	
35	do.	646	·8500	523	9096	9822
36	do.	646	·8625	523	8834	
37	Mahogany	637	1·1125	783	7950	
38	do.	637	1·1125	783	7950	8041
39	do.	637	1·1125	810	8224	

Nos. 28, 29, and 30 were from a piece of teak which had been taken from an old ship. Some other specimens were tried, but the results were so irregular, that it would be useless to give them; and exactly the same occurred in the first experiments on the transverse strain of this wood.

In the first two experiments on box, the small part of the cylinder drew out of the head, which was $5\frac{1}{4}$ inches in length, but not so perfectly as in the fir piece already mentioned; the part that drew out being very tapering, so that we could barely see through the hole thus formed. It is therefore obvious that, although the mean strength amounts to nearly 20,000 lbs. upon a square inch, this is still short of the absolute strength of direct cohesion of this wood.

The same may be observed with regard to the mahogany, but it proceeded from a different cause; viz. the twisting of the pieces, which, in all the experiments, wrenched the fibres asunder, instead of drawing them apart. The effect seems to have been exactly the same as would happen to a weight suspended to a rope, which would have a tendency to untwist; and it is highly probable that the fibres of the tree had acquired, in their growth, a situation with regard to each other similar to that of the component fibres of the rope, but of course in a much smaller degree.

13. *Experiments on the Lateral Adhesion of Fir.*

It is stated in a few of the preceding experiments, that the fibres, instead of breaking, as was intended, in some instances drew out, either wholly or in part, from the head of the pieces, notwithstanding these were, in one instance, more than 5 inches in length. This circumstance suggested the following experiments, in which the head of the piece was bored down very accurately to the distances stated in the third column, viz. to the insertion of the smaller cylinder into the greater part; the several pieces were then suspended, as in the foregoing experiments, and the weights put on as usual, till the separation took place; that is, till the small part was drawn out, or broken.

TABLE II.

No. of experiments.	Names of the woods.	Length drawn out.	Circumference.	Weight in lbs.	Weight reduced to one inch surface.	Mean value of lateral cohesion on one inch surface.
1	Fir.	1·625	1·1	996	556	592
2	do.	1·625	1·15	1187	621	
3	do.	1·625	1·15	1117	584	
4	do.	1·375	1·15	1066	634	
5	do.	1·500	1·15	1000	578	
6	do.	1·500	1·15	1000	578	

Nos. 1, 3, and 5 were drawn out very completely; the part which came out being nearly as perfect a cylinder as that which was turned: the other three were more or less irregular.

Nos. 2 and 4 twisted at least 10° before the separation took place.

It appears from the above, that the lateral adhesion is not more than one-twentieth of the direct cohesion in fir. With the other woods we did not attempt any experiments.

14. From a mean derived from the preceding experiments, and employing only the nearest whole numbers, it appears that the strength of direct cohesion on a square inch of

	lbs.
Box, is about	20,000
Ash	17,000
Teak	15,000
Fir	12,000
Beech	11,500
Oak	10,000
Pear	9,800
Mahogany	8,000

Also, that the strength of the lateral adhesion of

the fibres in fir is about equal to 600 lbs. on a square inch.

Some of these numbers differ considerably from those given by Musschenbroeck, as is stated in Art. 3; on which head it will be sufficient to observe, that the preceding experiments, from which the above results are drawn, were made with every possible care that the delicacy of the operation required.

15. *Practical Rule.*—Since the strength of direct cohesion must necessarily be proportional to the number of fibres, or to the area of the section, it follows, that the strength of any rod will be found by multiplying the number of square inches in its section by the corresponding tabular number, as given above.

This, however, gives the absolute strength, or rather the weight that would destroy the bar; and practical men assert, that not more than one-fourth of this ought to be employed. I have, however, left more than three-fourths of the whole weight hanging for twenty-four or forty-eight hours, without perceiving the least change in the state of the fibres, or any diminution of their ultimate strength.

On the Transverse Strength of Timber.

16. By the *transverse strength of timber* is to be understood the resistance which this material

opposes to a force or weight acting upon it transversely to its length, either perpendicularly or obliquely; and it naturally divides itself into three distinct considerations, viz.:

1st. The mechanical strain which a given force acting in a given direction produces on the section of fracture.

2dly. The nature of the mechanical action of the fibres to resist this strain.

3dly. The actual strength of the fibres when thus excited; which of course varies considerably in woods of different kinds.

The two former are merely questions relating to theoretical mechanics and geometry, while the latter is wholly experimental.

Mechanism of the Transverse Strain.

17. A beam of timber A C I F, fig. 1, Plate II., fixed with one end in a wall, and loaded with a weight W at the other, will be deflected from its first horizontal position A H, into an oblique direction A F, fig. 2, supposing it for the present inflexible in every point, except in the section of fracture A C. And this deflection, as we shall see, takes place about a line denoted by *n* in the figure (called the *neutral axis*) within the centre of fracture, which it is very important to determine, when we are considering the nature of the resisting forces of the fibres; but at present our object is merely to estimate the

exciting or straining force, which is obviously the product of the weight into the effective length of the lever n F; that is, analytically denoting the strain by f;

$$f = n\,\mathrm{F}.\cos n\,\mathrm{F\,B}\,.\,\mathrm{W}, \text{ or}$$
$$f = l \cos \Delta\ \mathrm{W},$$

denoting n F by l, the weight by W, and the angle n F B of deflection by Δ.

It will be observed that n F is not the length of the beam, but the distance of the neutral axis from the point on which the weight is suspended; nor is the angle n F B actually the angle of deflection of the beam; but as the depth of beams is generally small in comparison of their length, and the depth of the neutral axis still smaller, we shall in what follows, except the contrary be expressed, consider l as the length of the beam, and Δ as the angle of deflection, as it will simplify the investigation, and can produce no sensible error.

When a beam, instead of being fixed at one end into a wall, is merely rested on a support at its middle point, and loaded at each end, the tension of the upper fibre is still the same as in the former case; the length of the beam in the latter instance being supposed double what it is in the former; that is, supposing the beam F F′, fig. 3, to be double A F, fig. 2, then the three weights being equal, the tension of the fibre A b, in both cases, will be the same; excepting only so much of it as depends upon the cosine of the angle of deflection,

which in fig. 3 will be only half that in fig. 2: the same general expression, however, will apply in both cases, by merely changing l in the former into $\frac{1}{2}l$ in the latter; so that we shall have in this case

$$f = \tfrac{1}{2} l \cos \Delta \text{ W}.$$

18. Now, a beam resting on a fulcrum C, in the middle of its length, as in fig. 3, and acted upon by two weights W, W′, has commonly been considered in the same state, with regard to the strain upon it, as the equal beam F F′, fig. 4, which is rested on the two props F F′, and loaded with a double weight P, at its centre: and this is sufficiently correct in all common cases, although not strictly so when the deflection of the beam is considerable, as may be demonstrated as follows.

In the first place, it is obvious that the resistance of the props is not made in a direction parallel to that of the vertical weight P, but perpendicular to the arms of the lever F *n*, F′ *n*; and therefore, that the beam is, with regard to its strain, kept in equilibrio by the action of the three forces, F O, F′ O, and O R; the former, F O, F′ O, being supposed perpendicular to F *n*, F′ *n*.

The reaction of the fulcrums F F′ will therefore be to the weight P, as F O to half O R, or O C; or as radius to the cosine of the angle F O *n* or *n* F′ C; that is, as radius to the cosine of the angle of deflection.

Hence, when a beam is rested upon two fixed props, and loaded at its middle point by any weight

P, the strain upon that middle point, arising from the reaction of the props, will be found by the following proportion, as

$$OC : OF :: \tfrac{1}{2}P : \frac{FO \cdot P}{2OC}, \text{ or}$$

$$\cos \Delta : \text{rad} :: \tfrac{1}{2}P : \frac{P \text{ rad}}{2 \cos \Delta}, \text{ or } \frac{P}{2 \cos \Delta},$$

taking radius equal to unity; or if we call $\frac{1}{2}P = W$, then, according to our former notation,

$$f = \frac{\frac{1}{2}l \cdot W}{\cos \Delta} = \frac{l \cdot P}{4 \cos \Delta}.$$

This supposes the arms of the lever F n, F′ n, to remain of the same length; but it is obvious that this is also an erroneous hypothesis; for the props, or fulcrums, being fixed, these arms, either by the stretching of the fibres, or by the piece of wood slipping between the points of support, are more and more lengthened as the piece descends; viz. the length of the lever is to half the distance of the props, as rad to cos Δ; and, consequently, the strain on this account is again increased in the ratio of $\frac{\text{rad}}{\cos \Delta}$ or $\frac{1}{\cos \Delta}$ to radius 1; whence, by introducing this consideration, our former expression becomes

$$f = \frac{lP}{4 \cos^2 \Delta} = \frac{lP \sec^2 \Delta}{4}.$$

19. In all practical cases the angle of deflection Δ is so small that the secant may be considered as unity; but in extreme cases of experimental fracture

it is considerable; and as attending to this circumstance may serve to explain what has hitherto been considered as an anomaly in the experiments of Buffon and others, it may not be amiss to examine the question a little more particularly, especially as it seems to have escaped the attention of other authors.

Let, then, A C B, Plate III. fig. 1, represent a beam of timber, or simply a lever, which, in the first place, we will suppose to be kept in equilibrio by the two equal weights W, W′, and the resistance of the fulcrum C, or by a weight P, acting in an opposite direction C Q; then it is obvious that the weight P must be exactly equal to the two weights W, W′, or $P = 2\,W$, the lever being supposed void of gravity. But the effect of the weights W, W′, on the two levers A C, B C, as they relate to any strain at C, may be produced by two less weights w, w', acting perpendicularly to these levers; and these less weights, from the nature of the composition and resolution of forces, are to the two given weights W, W′, in the ratio of O B, or O A, to O C.

If, therefore, the lever A B be kept in equilibrio by the weights w, w', in the directions A O, B O, the reaction of the fulcrum, that is, the weight P, must be reduced in the ratio of $OC^2 : OB^2$; for the weights themselves are less in the simple ratio of these lines, and their vertical action is also less in the same proportion; and, consequently, the resistance at the fulcrum, or the weight P, will be decreased in the

duplicate ratio of O C to O B, or as $OC^2 : OB^2$. And, on the other hand, if the weight P remain the same in both cases, then the equilibrium will require the weights w, w', to be increased in the ratio of $OB^2 : OC^2$; and, consequently, the effect of these on the two levers A C, B C, to produce a fracture or strain at C, will have the same increased energy.

The reader will perceive immediately that these two cases of equilibrium are similar to those of the two beams in fig. 3 and fig. 4, Plate II., and that they agree with the former deductions;

$$\text{the first being } f = \tfrac{1}{2}\, l\, W \cos \Delta = \tfrac{1}{4}\, l\, P \cos \Delta,$$

$$\text{and the second, } f = \frac{\frac{1}{2}\, l\, W}{\cos \Delta} = \frac{l\ .\ P}{4 \cos \Delta},$$

where these two forces, or strains, are obviously to each other in the ratio of $\text{rad}^2 : \cos^2 \Delta$, or as the square of radius to the square of the cosine of deflection.

In this case, however, the length of the lever is not changed, because the weights are supposed to act at a fixed point; whereas in the former case, that is, when the beam is rested on two props, there is an actual lengthening of the arms of the lever; and in the latter instance, therefore, as before shown, the strain must be increased by multiplying the second formula by $\frac{1}{\cos \Delta}$, or the strain

$$\text{in the first case} = \tfrac{1}{4}\, l\, P \cos \Delta,$$

$$\text{and in the second} = \tfrac{1}{4}\, l\, P\ .\ \frac{1}{\cos^2 \Delta};$$

that is, they are to each other as $\cos^3 \Delta$ to rad^3;

whereas all writers that I am acquainted with on this subject consider them equal to each other.

Some mathematical readers may probably think I have been much more minute and explicit in the preceding investigation than was necessary; but those who are not so conversant with the resolution of forces, may not disapprove of the pains taken to render the deductions clear and satisfactory.

It may not, however, be improper again to remark, that although the $\cos^2$ of the angle of deflection being introduced into the general formulæ, may serve to explain some anomalies in the final results of different sets of experiments, it is a quantity which may always be dispensed with when our object is only to obtain the proper dimensions of beams for building, or other practical applications; because in these cases the deflection is always very inconsiderable, and its cosine little less than radius: in all cases, therefore, except when it is in contemplation to compare the ultimate results of different experiments, we shall omit the introduction of the $\cos \Delta$, and consider the straining forces under the more simple form

$$f = l\text{W}, \text{ or } f = \tfrac{1}{4} l\text{W},$$

according as the beam is fixed at one end, or supported at both; writing in the latter expression W, for what has been before denoted by P, viz. the suspended weight.

20. Let us now endeavour to ascertain the strain

upon the centre of a beam which is loaded at that point, having each of its ends fixed in a wall or other immoveable mass.

If the beam, instead of being fixed at each end, were merely rested on two props, and extended beyond them, on each side, a distance equal to half their distance; and if weights *w*, *w′*, fig. 12, Plate III., were suspended from these latter points, each equal to one-fourth the weight W, then this would be double of that necessary to produce the fracture in the common case; for, dividing the weight W into four equal parts, we may conceive two of these parts employed in producing the strain or fracture at E, and one of each of the other parts as acting in opposition to *w* and *w′*, and by these means tending to produce the fractures at F and F′.

This is the case which has been by most authors erroneously confounded with the former, but the distinction between them is sufficiently obvious; because here the tension of the fibres, in the places where the strains are excited, are all equal; whereas in the former case the strains at the fixed points are manifestly less from the compressibility and consequent yielding of the material in which they are fixed. In fact, in every experiment that I made, after the complete fracture in the middle, the two fragments had been so little strained at the points of fixing, that they soon after recovered their correct rectilinear form.

Parent and Belidor, in their experiments, and

indeed all experimentalists except Musschenbroeck, make the strength of their beams, when fixed at the ends, to the same when merely supported, in the ratio 3 to 2; but theorists have always made the ratio that of 4 to 2, as above stated, which is obviously erroneous.

The formula, therefore, in this case, will be $f = \frac{1}{6} l W$, or more accurately, $f = \frac{1}{6} l W \sec^2 \Delta$.

21. At present we have considered the load as being placed upon the middle of the beam; let us now endeavour to ascertain what strain will be excited in it when the weight is placed in any other part than the centre, as at C, fig. 2, Plate III.

Here, since the tension of the fibre A B is the same, whether it be estimated towards F or F′, we may suppose the weight W to be divided into two weights which shall have to each other the ratio of I C to I′ C; that is,

$$\text{as } \mathrm{I\,I'} : \mathrm{I\,C} :: \mathrm{W} : \frac{\mathrm{I\,C\,.\,W}}{\mathrm{I\,I'}},$$

$$\mathrm{I\,I'} : \mathrm{I'\,C} :: \mathrm{W} : \frac{\mathrm{I'\,C\,.\,W}}{\mathrm{I\,I'}}.$$

Then it is obvious, that whether we consider the first of these weights as acting at the point C of the lever C I′, or the latter as acting at the point C of the lever C I, or both of them as acting at the point C of the beam, or compound lever, I I′, the strain or tension of the fibre A B will be the same, and will

be expressed by

$$f = \frac{I'C \cdot W}{I I'} \times I C = \frac{I C \cdot I'C \cdot W}{I I'}; \text{ or}$$

$$f = \frac{I C \cdot W}{I I'} \times I' C = \frac{I'C \cdot I C \cdot W}{I I'}.$$

Hence, if l be taken to denote the length of the beam I I′, and m and n, the two distances I C, I′ C, then

$$f = \frac{m\,n}{m+n} W = \frac{m\,n}{l} W.$$

That is, the strain varies as the rectangle of the two parts into which the beam is divided by the point of suspension: and hence it follows, that the strain will be the greatest when this rectangle is the greatest; that is, when the weight acts at the centre.

22. Let us now take the case of two weights suspended from any two points of a beam, to determine the strain upon the beam at any given point.

Conceive F I I′ F′, Plate III. fig. 3, to be a beam resting on the two props F F′, and having two weights, equal or unequal, suspended from the two points D, E; then, from the preceding formula, it appears that the strain at D, arising from the weight at D, is

$$f = \frac{I D \cdot D I'}{I I'} \cdot W;$$

and the strain at E, arising from the weight E, is

$$f = \frac{I E \cdot E I'}{I I'} \cdot W'.$$

Now, in order to find the strain at any point C, we have only to make the following proportions, viz.

$\mathrm{D\,I'} : \mathrm{C\,I'} :: \frac{\mathrm{ID.DI'}}{\mathrm{I\,I'}}\mathrm{W} : \frac{\mathrm{ID\,.\,CI'}}{\mathrm{I\,I'}}\mathrm{W} =$ the strain at C, as arising from that at D; and again,

$\mathrm{E\,I} : \mathrm{C\,I} :: \frac{\mathrm{IE'\,.\,I'E}}{\mathrm{I\,I'}}\mathrm{W'} : \frac{\mathrm{I'E\,.\,CI}}{\mathrm{I\,I'}}\mathrm{W'} =$ the strain at C, as arising from that at E.

Consequently, the whole strain at C, arising from both weights, will be expressed by

$$f = \frac{\mathrm{ID\,.\,CI'\,.\,W + I'E.CI\,.\,W'}}{\mathrm{I\,I'}}.$$

23. From this general formula may readily be deduced that for any particular case: for example,

1st. Suppose the beams uniformly loaded throughout, and the stress at any point C required.

In this case, D and E will be the centres of gravity of the two parts I C and C I′; consequently,

$$\mathrm{I\,D} = \tfrac{1}{2}\mathrm{I\,C} \text{ and } \mathrm{I'E} = \tfrac{1}{2}\mathrm{C\,I'};$$

whence the expression becomes

$$f = \frac{(\tfrac{1}{2}\mathrm{\,IC\,.\,I'C\,.\,W}) + (\tfrac{1}{2}\mathrm{\,I'C\,.\,IC\,.\,W})}{\mathrm{I\,I'}}; \text{ or}$$

$$f = \frac{\mathrm{I,C\,.\,I'C\,.\,(W+W')}}{2\mathrm{\,I\,I'}}.$$

Where (W + W′) and I I′ being constant, it follows that f varies as the rectangle I C . I′ C; that is, in this case, the strain at any point C varies as the

rectangle of the two parts into which the beam is divided by that point.

2ndly. Suppose, as another example, that the weights W, W′, are equal to each other, and that C is the centre of the beam; then, since

$$I'C = IC = \tfrac{1}{2} II', \text{ and } W = W',$$

the general expression becomes, in this particular case,

$$f = \frac{(ID + I'E) \,.\, I'C \,.\, W}{II'} = \frac{ID + I'E}{2} \times W.$$

And if we further suppose $ID = I'E$, then it becomes simply

$$f = ID \,.\, W.$$

Now, if both weights acted at the centre, it appears, from the preceding investigation, that

$$f = \tfrac{1}{4} II' \,.\, (2W) = \tfrac{1}{2} II' \,.\, W = IC \,.\, W.$$

Whence the strain in the two cases will be to each other as ID to IC; and hence the following practical deduction, viz.

24. When a beam is loaded with a weight, and that weight is appended to an inflexible bar or bearing, as D E, fig. 4, Plate III., the strain upon the beam will vary as the distance I D, or as the difference between the length of the beam and the length of the bearing; for the bearing D E being inflexible, the strains will be exerted in the points D and E, exactly in the same manner as if the bearing was removed, and half the weight hung on

at each of these points. This remark may be worth the consideration of practical men in various architectural constructions.

25. In the same manner as in Art. 23, it may be shown, that if a beam be loaded with many weights, W, W′, W″, W‴, &c., as in fig. 5, Plate III., all equal to each other, and every two of which are equally distant from the centre, the strain excited on the middle point C will be expressed by

$$f = (\mathrm{ID} + \mathrm{ID}' + \mathrm{ID}'' + \&c.) \,.\, \mathrm{W}.$$

Hence, if the length of the beam be l, and the number of equal weights m, and the sum of all the weights W, then the above becomes

$$f = \left(o + \frac{l}{m} + \frac{2l}{m} + \frac{3l}{m} + \&c. \frac{\frac{1}{2}ml}{m}\right) \times \frac{\mathrm{W}}{m}; \text{ or,}$$

$$f = \frac{l\mathrm{W}}{m^2} \times (1 + 2 + 3 + 4, \&c. \tfrac{1}{2}m); \text{ or,}$$

$$f = \frac{l\mathrm{W}}{m^2} \times \frac{(\frac{1}{2}m + 1)\frac{1}{2}m}{2} = \frac{\frac{1}{4}l\mathrm{W}m^2 + \frac{1}{2}l\mathrm{W}m}{2m^2} = \tfrac{1}{8}l\mathrm{W} + \frac{l\mathrm{W}}{4m}.$$

Hence, when the weight is uniformly distributed through the whole length, the number of points of suspension, m, becoming infinite, the last term of the preceding expression, $\frac{l\mathrm{W}}{4m}$, vanishes; and there results

$$f = \tfrac{1}{8}l\mathrm{W},$$

for the strain on the centre of a beam, when the weight W is uniformly distributed throughout its

length; which is half what it would be if it were all suspended from its middle point.

26. At present the weight has been supposed to act in a direction perpendicular to the fibres; that is, the different deflections to which the beam may be exposed in consequence of the different positions of the weight have not been taken into consideration; and it has been before explained, that it is not necessary to introduce the latter datum while we are merely contemplating the comparative strengths and strains of beams for architectural and mechanical constructions, in which the deflections are always inconsiderable, but that they are essentially necessary in the comparison of experiments on the ultimate strength; and, therefore, when we treat of those comparisons, it may be necessary to modify some of the preceding results. I shall not, however, pursue the subject further in this place, except so far as relates to the strain on beams when the direction of the fibres and the exciting forces are placed obliquely to each other.

27. When a beam A C F I, or A′ C′ F′ I′, fig. 6, Plate III., is placed obliquely in a wall, whether it be descending, as in the former, or ascending, as in the latter, the strain excited by the equal weights W, W′, on the equal arms I C, I′ C′, will be the same, being in both cases expressed by

$$f = l\,W \cos I,$$

where l is the length, W the weight, and I the angle of inclination.

For, let I W, in both cases, be taken to represent the perpendicular force of the weight W, and let this be resolved into two other forces; the one, I K, perpendicular to the lever C I, and the other, K W, parallel to it; then it is obvious that K I will represent the only effective force to turn the lever about the point C; that is, the exciting force will be to the weight W as K I : I W, or as radius : cosine of KIW; but the angle KIW = CIL = the angle of inclination = I; therefore,

$$1 : \cos I :: W : W \cos I = IK,$$

which, combined with the lever $CI = l$, gives for the strain at C,

$$f = l W \cos I.^{12}$$

Therefore, while we omit the consideration of the

[12] It has been assumed by some writers on this subject, and strangely adopted by others, that not only is the exciting force diminished in the ratio of rad to cos, but also that the power of resistance is increased in the ratio, viz. of cos to rad, because they say the area of fracture C A is increased in the latter proportion; whence they conclude, that the weight necessary to break a beam in an inclined position is to the weight when it is horizontal, as $\text{rad}^2 : \cos^2$.

Nothing, however, can be more obviously false than to suppose the power of resistance to be increased; for if the force or weight W, or W′, fig. 6, which is denoted by I W, be resolved into the two, I K, K W, it is evident that the force I K will have the same effect upon this beam (and no other), as if the beam were placed horizontally, and loaded with a vertical weight, which should be to W as I K to I W.

There might be some plausibility for the above hypothesis in

quantity of deflection, the strain on the two beams (their lengths, weights, and inclinations being the same) will be exactly equal to each other: and this is true, as has been before observed, while we are merely considering the application of timber to architectural purposes, but fails entirely in determining the ultimate strengths.

For the deflection of the beam I C brings it nearer and nearer to a horizontal position, where the effect of the weight is the greatest; while the deflection of the descending beam I C brings it more and more towards a vertical, where the effect of the weight is the least.

Conformably to this, I have always found, of three equal and similar beams, of which the one inclined upwards at a certain angle, another downwards at the same angle, and the third horizontal, that which had its inclination upwards was the weakest; the one which declined, the strongest; and the strength of the horizontal one, about a mean between both. —(See "Experiments," Art. 95.) It is obvious, indeed, that the ultimate strength of a beam does not depend upon its original position, but upon that which it has attained immediately before the fracture takes place.

It may be proper to observe, that in the preceding expression, $f = l\,W \cos I$, that force only is included

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crystallized bodies, but it will certainly not apply to fibrous ones, the number of fibres on which the resistance depends being still the same.

which has a tendency to turn the beam about the point C: there is, however, also another exciting force, but which does not act at any mechanical advantage, that is, the force represented by K W, which in the declining position of the beam A F C I acts by tension, and in the ascending position of A′F′C′I′ by pressure: the entire expression, therefore, for the exciting force, is

$$f = l\,\mathrm{W}\cos\mathrm{I} + k\,\mathrm{W}\sin\mathrm{I},$$

the value of k depending upon the proportion between the areas of compression and of tension.

But in most practical cases this latter force is very inconsiderable; first, because it does not act at any mechanical advantage through the intervention of the lever; and, secondly, because it acts equally upon the compressed and extended fibres; and, consequently, while it increases the one of these forces, it diminishes the other, and, therefore, in a certain degree, neutralizes its effect on both, on which account it may in most cases be omitted: and we must necessarily omit it in this place, because its real effect depends upon the proportionality between the area of compression and that of tension, the determination of which will form the subject of experiment in a following section. It will, therefore, in this place, be sufficient to observe, that in the cases where the beam is vertical, and consequently $\cos\mathrm{I} = 0$, and $\sin\mathrm{I} = 1$, the former part of the expression disappears, and we have simply $\mathrm{F} = \mathrm{W}$; where, in the declining position,

W must be equal to the force of direct cohesion in the area of fracture, and in the ascending position it will represent the weight necessary to crush the beam with a vertical pressure.

28. At present we have only considered the strain a beam is exposed to by being charged at any point with a given weight, without making any reference to the resistance to which it is opposed. Now, this resistance obviously depends upon the figure and area of the section of the beam at the breaking point, and experiments make this resistance vary in rectangular beams as the breadth and square of the depth. That the strength or resistance is as the breadth, is obvious; because, whatever resistance any given beam offers to fracture, two, three, or more such beams will offer two, three, or more times that resistance: and this is in fact the same as a beam of two, three, &c. times the breadth. And with regard to the depth, the resistance will be, in the first place, as the number of fibres; that is, as the depth: and, secondly, it varies as the length of the lever by which those fibres act; that is, as the distance of the several fibres from the centre about which the beam turns, wherever that point may be, which is also obviously as the depth; and hence, by combining the two causes, it will vary as the square of the depth when the breadth is the same: and therefore, generally, the resistance opposed to fracture by rectangular

beams is as the product of the breadth and square of the depth.

If we represent the breadth of a beam of any given wood by a, its depth by d, its length by l, all in inches, its angle of deflection by Δ, and the weight necessary to break it in lbs. by W; also, the resistance of a rod an inch square by S: then $a\,d^2\,S$ will be the resistance of the beam whose breadth is a and depth d. Now, in the instant before breaking, there must be an equilibrium between the strain and the resistance; and hence we obtain the following equations, viz.

1. *When the beam is fixed at one end, and loaded at the other,*

$$l\,W\cos\Delta = a\,d^2\,S, \text{ or } \frac{l\,W\cos\Delta}{a\,d^2} = S, \textit{ a constant quantity.}$$

2. *When the beam is supported at each end, and loaded in the middle,*

$$\tfrac{1}{4}\,l\,W\sec^2\Delta = a\,d^2\,S, \text{ or } \frac{l\,W\sec^2\Delta}{4\,a\,d^2} = S, \textit{ constant.}$$

3. *When the beam is fixed at each end, and loaded in the middle,*

$$\tfrac{1}{6}\,l\,W\sec^2\Delta = a\,d^2\,S, \text{ or } \frac{l\,W\sec^2\Delta}{6\,a\,d^2} = S, \textit{ constant.}$$

4. *When the beam fixed as in either of the last two cases is loaded at any other point than the centre,*

We shall have in the former case, by denoting the two unequal lengths by m and n,

$$\frac{m\,n\,W}{l}\sec^2\Delta = a\,d^2\,S, \text{ or } \frac{m\,n\,W\sec^2\Delta}{l\,a\,d^2} = S;$$

and in the second,

$$\frac{2mnW}{3l}\sec^2\Delta = ad^2 S, \text{ or } \frac{2mnW\sec^2\Delta}{3lad^2} = S,$$

still the same constant quantity.

The first formula will also apply to a beam fixed at any given angle of inclination; observing only, that the angle Δ, in this case, will represent the angle of the beam's inclination, increased or diminished by the angle of its deflection, according as its first position is ascending or descending; or rather, it will denote the angle of the beam's inclination at the moment of fracture.

In all these cases, as has been before stated, when it is only intended to apply the results to the common application of timber to architectural and other purposes, the angle of deflection may be omitted, and the equations then become simply,

$$1.\ \frac{lW}{ad^2} = S, \qquad 2.\ \frac{lW}{4ad^2} = S,$$

$$3.\ \frac{lW}{6ad^2} = S, \qquad 4.\ \frac{mnW}{lad^2} = S,$$

$$5.\ \frac{2mnW}{3lad^2} = S.$$

But in the comparison of the ultimate strength, under different circumstances, the angle of deflection must be retained; and it remains to show how far the introduction of this datum will explain what has hitherto been considered as paradoxical in the best conducted experiments.

29. One of the most remarkable discrepancies

between theory and experiment is that already explained (Art. 20); viz. that the strength of a beam fixed at the ends is to that of a like beam merely supported, in the ratio of 3 to 2.

The next anomaly, or what has hitherto been considered as such, is that in which the strength has been observed to decrease in a higher ratio than that of the inverse of the lengths; or, which is more correct, that the strain increases in a higher ratio than the direct ratio of the lengths. Now, it appears from the preceding formulæ, that this is what ought to be the case; for the strain being denoted by

$$f = \tfrac{1}{4}\, l\, W \sec \Delta^2;$$

and as the ultimate deflection, *in quantity*, varies as the square of the length, (see Art. 54,) the *angle* Δ will vary as the length; and consequently, if the length of one beam be supposed l, and the other any number of times the same length, as $m\, l$, then the strain in the two cases will be as

$$\tfrac{1}{4}\, l\, W \sec^2 \Delta, \text{ to } \tfrac{1}{4}\, m\, l\, W \sec^2 m\, \Delta;$$

and therefore, where the resistance to be overcome is the same, W' will be to W as $\sec^2 \Delta : m \sec^2 m\, \Delta$, instead of being in the simple ratio of $1 : m$, as stated by most writers on this subject. This defalcation of strength was observed by Buffon in his experiments, and has been considered as an inexplicable paradox. Some of the reasons assigned by Dr. Robison may probably have their effect; but it is singular that the above explanation escaped so keen

a mathematician: it may not, perhaps, account for the whole discrepance observed in the results, but it will certainly tend considerably towards reconciling them with each other. The case in which a beam is fixed at one end and loaded at the other presents a deviation from the commonly established ratio of an opposite kind; for it has been seen (Art. 28) that the strain in this case is $l\text{W} \cos \Delta$; and since the angle Δ varies as the length, the strain upon a beam of m times the length will be $m\, l\, \text{W} \cos m\, \Delta$; and hence, when the resistances are the same, we shall have

$$\text{W} : \text{W}' :: m \cos m\, \Delta : \cos \Delta,$$

instead of the simple ratio of $m : 1$; and, consequently, the strength will not decrease so rapidly as in the inverse ratio of the lengths.

The only experiments that I am aware of, bearing on this point, are those of M. Parent, the results of which are published in the 'Academy of Sciences' for 1707 and 1708, from which the author concludes that the weight necessary to break a beam fixed at one end and loaded at the other, and that of a beam of double the length supported at each end and loaded in the middle, and another equal to the latter, but fixed at each end, were as the Nos. 4, 6, and 10, and the preceding deductions (Art. 28) give the values of those weights

$$\frac{f}{l \cos \Delta}, \qquad \frac{4f}{2l \sec^2 \Delta}, \qquad \frac{6f}{2l \sec^2 \Delta},$$

observing, that $2\,l$, in the two latter expressions, is substituted for l in the formulæ referred to, because the beams are of double length: these ratios are the same as

$$3.\ \frac{1}{\cos\Delta},\qquad 6.\ \frac{1}{\sec^2\Delta},\qquad \text{and } 9.\ \frac{1}{\sec^2\Delta},$$

which, if the angle be considerable, will approximate towards the above numbers; but in the references I have seen to these experiments, neither the dimensions of the beams nor the amount of their deflection are stated.

Of the Mechanical Action of the Fibres to resist Fracture.

30. This is a subject which has engaged the attention of several very able mathematicians, whose results have differed very considerably from each other; and although the subject is now properly understood, and all writers adopt the same general view of the theory, yet it will not be uninteresting to take a rapid sketch of the doctrines which have been advanced in support of different hypotheses, by the writers alluded to.

31. Galileo, to whom the physical sciences are so much indebted, was the first who connected this subject with geometry, and endeavoured to compute the strength of different beams upon pure mathematical principles, by tracing the proportional strengths

which different bodies possessed, as depending upon their length, breadth, depth, form, and position.

It appears that this philosopher was led to these investigations in consequence of a visit which he made to the arsenal and dockyards of Venice, and that they were first published in his 'Dialogues' in 1633. He considered solid bodies as being made up of numerous small fibres applied parallel to each other; and sought, or assumed, at first, the force with which they resisted the action of a power to separate them when applied parallel to their length; and thence readily deduced, that their resistance in this direction was directly as the area of the transverse perpendicular section; that is, as the number of fibres of which the body is composed.

He next considered in what manner the same fibres would oppose a force applied perpendicularly to their length, and ultimately came to the following conclusion: "that when a beam is fixed solidly in a horizontal position in a wall, or other immoveable mass, the resistance of the integrant fibres is proportional to their sum, multiplied into the distance of the centre of gravity of the area of fracture from the lowest point."

32. In order to illustrate this theory a little more explicitly, let R S T V, fig. 1, Plate II., represent a solid wall, or other immoveable mass, into which the beam C G is inserted, and let W be a weight suspended from its other extremity: then supposing

the beam to be insuperably strong in every part except in the vertical section A B C D, the fracture must necessarily take place in that section only; and, *according to the hypothesis of this author*, it will turn about the line C D, whereby the fracture will commence in the line A B, and terminate in the former, C D. Galileo also further supposes that the fibres forming the several horizontal plates, or laminæ, from C D to A B, act with equal force in resisting the fracture, and therefore differ in their energy only as they act at a greater or less distance from the supposed quiescent line, or *centre of motion*, C D.

Now, from the known property of the lever, it is obvious that the equal forces acting at the several distances, $o1$, $o2$, $o3$, $o4$, &c. of the lever oe, will oppose resistances proportional to their respective distances; and therefore that their sum, that is, the constant force of each particle into its respective distance, is the force which must be overcome by the weight W, acting as on a lever, at the distance oK.

33. This will perhaps be better understood from the illustration given by M. Girard, in his '*Traité Analytique de la Résistance des Solides*,' which is as follows:

Let A C I F, fig. 7, Plate II., represent a longitudinal section of the beam C G, and w', w'', w''', &c. so many small equal weights passing over pins or pulleys, at r', r'', r''', r'''', &c., acting at the several

distances, Cm', Cm'', Cm''', &c., each weight being supposed equal to the cohesion of its respective lamina; then, denoting each of these weights by the constant quantity f, the sum of all their energies, or resistances, will be expressed by the formula

$$C m'.f + C m''.f + C m'''.f + C m''''.f + \&c. =$$
$$f \times (C m' + C m'' + C m''' + C m'''' + \&c.)$$

This, however, supposes the section to be rectangular, or that the number of fibres in each horizontal lamina is the same. When the beam is triangular, cylindrical, or has any other than a rectangular section, the several small weights must be made proportional to the breadth of the section at the point where each is supposed to act: the illustration, however, is equally obvious.

Since, then, the whole resistance to fracture is made up of the sum of the resistance of every particle or fibre, acting at different distances on the lever C A, which is supposed to turn upon C as a fulcrum, there must necessarily be some point in that lever, in which, if all the several forces were united, their resistance to the weight W would be exactly the same as in the actual operation; *and this point is the centre of gravity of the section represented by* A C.

For let A B C, fig. 10, represent the section of any formed beam whatever, F H, any variable absciss, $= x$, and D E, the corresponding double ordinate, $= y$; then, by what is stated above, the energy or

force of all the particles in the line D E will be as D E . H F, or as xy; and consequently, the differential of that force will be $y\,x\,d\,x$, and the sum of all these forces will, therefore, be denoted by $\int y\,x\,d\,x$. Now the area of the section may be expressed by $\int y\,d\,x$; and, assuming G as the centre of energy sought, we shall have

$$\text{F G} \,.\int y\,d\,x = \int y\,x\,d\,x.$$

$$\text{Whence F G} = \frac{\int y\,x\,d\,x}{\int y\,d\,x},$$

which is the well-known expression for the centre of gravity.

34. From these considerations, or at least from others tantamount to them, Galileo deduces his general theorem for the resistances of solids; which, from what is above stated, is obviously as follows: viz.

When a beam is solidly fixed with one end in a wall, or other immoveable mass, the weight necessary to produce the fracture, is to the force of direct cohesion of all its fibres, as the distance of the centre of gravity of the section of fracture, from the lowest point of that section, to the length of the beam, or the distance at which the weight acts from the same point.

From other investigations, which it is unnecessary to exhibit in this place, the author endeavours to show, that whatever weight is sufficient to break a beam, fixed as above, double that weight will be

necessary to break a beam of equal breadth and depth, and of twice the length, when supported at each end on two props; and four times the same weight, when the latter is fixed with each end solidly in a wall, &c., &c.

35. Nothing can be desired more simple than the results obtained by this theory; but, unfortunately, it is founded on hypotheses which have nothing equivalent to them in nature. In the first place, it assumes the beam to be inflexible, and insuperably strong, except at the section of fracture: secondly, that the fibres are inextensible and incompressible: and, thirdly, that the beam turns about its lowest point when fixed at one end, or its upper when supported at both, and therefore, that every fibre in the section is exerting its force in resisting extension: and, lastly, if this be not implied in the former objection, that every fibre acts with equal energy, whatever may be the tension to which it is exposed.

With regard to the first of these suppositions, it is obvious that no beam of timber, or any other body with which we are acquainted, is perfectly inflexible; nor any (and more particularly timber) whose fibres are not both extensible and compressible; and consequently, a beam of such matter will not turn about its lowest point, as a fulcrum: and, lastly, the supposition of every fibre exerting a constant resistance is now known to be decidedly erroneous.

The theory of Galileo having these radical defects, it necessarily happened, as soon as it was attempted to compare its results with experiments, (which the author himself had never done,) that it was found defective. The first person, we believe, who did this, was Mariotte, a member of the French Academy, who, having soon discovered its inaccuracy, proposed to substitute another theory in its place, which was published in 1680, in his '*Traité du Mouvement des Eaux*;' and here we find the first notice of extensible and compressible parts of the section of fracture, the neutral axis, &c. This attracted the attention of Leibnitz, who, after examining the theory of Galileo and the experiments of Mariotte, published his own thoughts on the subject in a Memoir which appeared in the 'Leipsic Acts,' in 1684.

36. He stated that every body, before breaking, was subject to a certain degree of deflection, which could not have place if the fibres were, as Galileo had supposed, inextensible; and thence, assuming the principle first suggested by Dr. Hooke, viz. *ut tensio sic vis*, or that the tension varies as the force, he concluded that every fibre, instead of acting with an equal force, exerted a power of resistance proportional to its quantity of extension; or, which is the same, proportional to its distance from the line about which the beam was supposed to turn: but he still considered the fibres to be incompressible, or

at least, what amounts to the same, that the beam turned about its lowest or highest point, accordingly as it was fixed at one end or supported at both.

Thus, to use a similar illustration in this case to that we have done in the former, instead of the fracture being opposed by the action of the equal forces or weights w', w'', w''', &c., fig. 7, the resistance is supposed to be equal to the decreasing weights w', w'', w''', &c., fig. 8, these being to each other in the proportion of their respective distances from the axis of rotation.

The only alteration which this hypothesis introduced into the final results, was the removal of the centre of energy G, to another point I, fig. 10, nearer or further from the centre of motion, according to the figure of the transverse section of the beam: and this new point is found to be distant from that axis, *by a quantity equal to the product of the distances of the centres of gravity and oscillation from the axis of motion, divided by the depth of the section.*

For, let A B C, fig. 10, represent, as before, the area of fracture of any beam, F H $= x$ any variable absciss, and D E $= y$, the corresponding double ordinate. Also, make C F $= d$, and let f represent the absolute and ultimate force of a fibre at C, at the instant of rupture: then, since the resistance opposed by each fibre is supposed to vary as its tension, or as its distance from F, we have $d : x :: f : \frac{fx}{d} =$ the

force of a fibre at H; and the number of fibres acting at this distance being y, we shall have $\frac{f x y}{d}$ for the sum of the resistances of all the fibres or particles in the line D E: but this force, acting upon the lever at the distance H F, its resistance will be expressed by $\frac{f x^2 y}{d}$; and hence the sum of all the resistances of every fibre in the section will be $= \int \frac{f y\, x^2\, d x}{d}$.

Now this is to be equal to the direct cohesion of all the fibres acting at some required distance F I; that is,

$$\mathrm{F\,I} . \int y\, d x . f = \frac{f}{d} \times \int y\, x^2\, d x, \text{ or}$$

$$\mathrm{F\,I} = \frac{1}{d} . \frac{\int y\, x^2\, d x}{\int y\, d x}.$$

The variable part of this expression is exactly equivalent to the general formula for the centre of oscillation of a surface, multiplied by the former expression for the centre of gravity, that is, using $\frac{1}{d}$ as a coefficient,

$$\mathrm{F\,I} = \frac{1}{d} . \frac{\int y\, x^2\, d x}{\int y\, d x} = \frac{\int y\, x\, d x}{\int y\, d x} \times \frac{\int y\, x^2\, d x}{\int y\, x\, d x} . \frac{1}{d},$$

as is obvious. And since these centres are generally known in most of the figures which fall under consideration in the present inquiry, we may avail ourselves of them, independently of calculation, in determining what may properly be termed the *centre of energy*, or *centre of tension*: but, in other

cases, recourse must be had to the general differential expression

$$\mathrm{F\,I} = \frac{1}{d} \times \frac{\int y\,x^2\,dx}{\int y\,dx}.$$

37. Referring again to the formula previously found for the centre of energy on the Galilean hypothesis, and denoting the absolute strength of cohesion on a square inch by f; also writing d for the depth of the beam in inches, a the area of fracture, and l the length likewise in inches; then the general expression for the ultimate strength of any beam, fixed with one end in a wall, would be, on the hypothesis of

$$\text{Galileo,}\quad S = \frac{\int y\,x\,dx}{\int y\,dx} \cdot \frac{af}{l},$$

$$\text{Leibnitz,}\quad S = \frac{\int y\,x^2\,dx}{\int y\,dx} \cdot \frac{af}{dl}.$$

When the beam is *supported* at both ends, these must be each multiplied by four; and when *fixed* at both ends, by eight.

This being the case, both theories give the same results, so far as relates to the comparison of similar-formed beams, but of different dimensions: thus, for example, it appears from both, that when the breadth and depth are the same, the strength varies inversely as the length; when the length and depth are the same, the strength varies directly as the breadth; and when the length and breadth are the same, the strength varies as the square of the depth:

deductions which have been found to agree very nearly with experiment.

38. There are other conditions, however, resulting from the same formulæ, in which the two theories are totally irreconcilable with each other, and in which neither will agree with actual experiment.

In the first place, although the proportions are the same, the absolute strength in the one case is to that in the other as two to three in rectangular beams; and in triangular ones the disagreement is still more striking. Again, according to Galileo, the strength of a triangular beam with its edge upwards, when fixed by one end in a wall, or with its base upwards, when supported at both ends, is to the strength of the same beam in the reversed position, as one to two; and, according to Leibnitz, as one to three: whereas experiment shows it to be neither the one nor the other.

So also, square beams fixed, in one instance with the side vertical, and in the other with the diagonal vertical, have their strengths, according to Galileo, in the ratio of 1 to $\sqrt{2}$; and, according to Leibnitz, in that of $\frac{4}{3} : \sqrt{2}$; whereas experiments show the beam to *be stronger in the former position than in the latter.*

In both theories, also, the strength of hollow cylinders, not bored through the axis, but nearer one side than the other, varies according as the boring is nearer the upper or lower surface, and is

greatest of all when the cylinder is infinitely thin on that side about which it is supposed to turn; whereas experiment shows the very reverse of this, and that the beam is absolutely weakest, when, according to both these writers, it ought to be the strongest.

39. The subject was afterwards taken up by James Bernouilli, who observed, that the instant before a body is broken across with a transverse strain, such as we have been considering, a part of the fibres only are in a state of tension, and a part in a state of compression,—a circumstance that had not before been introduced into the conditions of this problem (except perhaps by Mariotte); and he moreover doubted of the justness of the principle, *ut tensio sic vis*, employed by Leibnitz, and made some experiments, whereby he proved that, at least, this is not a universal law of nature. But he unfortunately stopped at this point, contenting himself with showing the inadequacy of the theory he had been examining; but without substituting any new one in its place, except so far as his theory of the *elastic curve* (a problem which arose out of the present question) may be considered as applicable to this subject. Had he pursued the idea he seems first to have promulgated, of a part of the fibres being stretched, and a part compressed—and, consequently, that the line about which the beam turns is somewhere within the area of the section of fracture—we might have expected, from his extra-

ordinary talents, a complete solution of this interesting problem: instead of which, he contented himself with stating a few general observations, and with pointing out the difficulty of determining the neutral axis, or of that line which suffers neither compression nor extension; which is the principal desideratum for establishing a correct theory.

40. The next important step in this inquiry was made by Dr. Robison, under the article STRENGTH, in the 'Encyclopædia Britannica;' and here for the first time the position of the neutral axis, or that line in a beam which suffers neither extension nor compression, is introduced as a necessary datum. The position of this line was not, however, determined by Dr. Robison, nor had it been attempted to be found, to the best of my knowledge, when I made the experiments on which I founded my 'Essay on the Strength of Timber.' In these, I found its position in two or three different kinds of wood experimentally, and thence endeavoured to determine the law of action of the fibres at different distances from the neutral axis, and arrived at a conclusion, "that, however difficult it might be to account for the fact, the theory of resistance assumed by Galileo was nearer the truth than the generally admitted law, *ut tensio sic vis.*" But in this investigation, I had fallen into an error, by assuming the momenta of the forces on each side the neutral line to be equal to each other, instead

of the forces themselves,—an error which was first pointed out by Mr. Eaton Hodgkinson, in a very able Paper on this subject, in vol. iv. of the 'Manchester Memoirs,' new series. This correction being made, the agreement is greatly in favour of the latter hypothesis, the truth of which is not now, I believe, doubted by any one,—making, of course, great allowance for the very variable force of the fibres in different kinds of wood, and even of the fibres in the same section, when the latter is of considerable area.

Fortunately it is seldom that the strength of timber is of great importance, except in the form of rectangular or square beams; and its strength in these forms is deducible from experiments on similar-formed beams, without any reference to the exact position of the neutral axis; but still, as a point of theory, and wherever the question relates to beams of other figures, it is essential that we should have reference to it. Without, therefore, pursuing our historical sketch to a greater length, we shall proceed at once to illustrate the nature of the neutral axis, and the consequent mechanical action of the fibres in resisting fracture.

41. It has been before remarked, that when a beam is submitted to a transverse strain, being either supported at its two extremities and loaded in the middle, or fixed at one end in a wall and loaded at the other, it will not, as was formerly assumed by

Galileo and Leibnitz, turn about its upper or lower surface, but about a line within the area of fracture; which line is what is denominated the *neutral line*, or *neutral axis of rotation*.

If the fibres of a beam (referring, for instance, to fig. 1, Plate II.) were wholly incompressible, there is no doubt that the beam, when loaded at the end I, would turn about the line C D; and every fibre of it, from C to A, would be in a state of tension.

And, on the contrary, if the fibres were wholly inextensible, then, if the beam turned at all, it must be about the line A B, and every fibre from A to C would be in a state of compression.

But we know of no bodies in nature that are either inextensible or incompressible; and, therefore, the rotation of the beam will neither take place about A nor C, but on an intermediate point or line, *n*; and all the fibres above that line will be in a state of tension, and those below it in a state of compression; while those which are situated so as exactly to coincide with its plane, will be neither extended nor compressed, but be in a state perfectly neutral with regard to both.

42. It is obvious, that the fibres submitted to tension are more and more extended as they are situated further from the point *n*, and at A their extension is the greatest. The same has also place with the fibres submitted to compression, this being greatest at C; and, whatever may be the law of the

forces necessary for producing these several degrees of tension and compression, or whatever may be the law of the resistances which they offer after they are produced, we may conceive some point situated between A and *n*, into which, if all the resistances to tension were united, and some point between *n* and C, into which, if all the resistances to compression were condensed, the reaction arising from these two aggregate forces would be the same as in the actual operation; and these points are what are designated the *centres of tension and compression.*

43. With regard to the situation of the neutral axis, we have nothing to guide us in the determination but experiments; and these seem to indicate, that in rectangular fir beams it is at about $\frac{5}{8}$ths of the depth of the section of fracture when the beam is broken on two supports; or, at $\frac{3}{8}$ths of the same when it is broken by having one end fixed in a wall, and loaded at the other;—that is, in both cases the number of fibres exposed to compression are to those submitted to tension in about the ratio of 5 to 3.

This was pointed out very unequivocally in several of the experiments stated in the following pages; the beams in most cases showing very distinctly, after the fracture, what part of the section had been compressed, and what had experienced tension; the compressed fibres always breaking very short, having been first crippled by the pressure to

which they had been exposed, while the lower part was drawn out in long fibres, frequently 5 or 6 inches in length.

Another criterion was found in the external appearance of the side of the beam exposed to pressure before the fracture took place: this always exhibited itself in a wedge-like form, the lower point of which, when the beam was broken on two props, was commonly found to divide the depth in about the ratio above stated.

It should be observed, however, that Mr. Hodgkinson, in the experiments he has described in the article above referred to, finds the ratio to be nearly 4 to 4, instead of 3 to 5; and unquestionably there must be considerable irregularities in the position of this line in different specimens of timber, even of the same kind, and much more in woods of different kinds. Without, therefore, attempting to determine this point, we may at all events assume, from what has been above stated, that there is necessarily such a point in the area of fracture in all beams; and this is sufficient for our present purpose, as it is intended in the first instance to speak here only of rectangular beams.

44. Referring to fig. 2, Plate II., let *n* denote the neutral axis of the rectangular beam A C I F, *b* A *n* representing the part suffering extension, and *n* C *d* that submitted to compression. Let also *t* denote the amount of tension of the extreme fibre *b* A, and C

the compression of the extreme fibre C d. Then, assuming that the resistance to tension of a fibre is proportional to the quantity of tension, or to its distance from the neutral axis, if we call the whole depth of extension A $n = d'$, and denote any variable distance from n by x, we have $d' : t :: x : \frac{t\,x}{d'}$, the tension of a fibre of that part. Consequently, the sum of all the tensions will be expressed by

$$\int \frac{t\,x\,d\,x}{d'} = \tfrac{1}{2}\,t\,d' \text{ (when } x = d'\text{)};$$

and in the same way, assuming the same law of compression, the sum of all the compressions will be expressed by

$$\int \frac{c\,x\,d\,x}{d''} = \tfrac{1}{2}\,c\,d'' \text{ (when } x = d''\text{)};$$

d'' denoting the depth of compression; which two forces are equal to each other; for it is this equality which determines the motion to take place about the line n: therefore $\frac{1}{2}\,t\,d' = \frac{1}{2}\,c\,d''$, or $t\,d' = c\,d''$.

45. It may be proper to observe, that c here is not intended to represent the force requisite to compress a fibre the same quantity that the force t extends it, but simply the force of compression at C, corresponding to that of the tension at A.

46. Now, to estimate the effect of those forces, it will be seen that the tension of any fibre at the variable distance x being $\frac{t\,x}{d'}$, and this acting at the

distance x, the effect will be $\frac{t\,x^2}{d'}$, and the sum of all the effects

$$\int \frac{t\,x^2\,dx}{d'} = \tfrac{1}{3}\,d'^2 t \text{ (when } x = d');$$

and in the same way the sum of the compressing forces will be

$$\int \frac{c\,x^2\,dx}{d''} = \tfrac{1}{3}\,d''^2 c \text{ (when } x = d'');$$

and therefore the whole sum of both species of resistances will be

$$\tfrac{1}{3}\,d''^2 c + \tfrac{1}{3}\,d'^2 t:$$

and since $d''\,c = d'\,t$, this sum becomes

$$\tfrac{1}{3}\,(d'' + d')\,d'\,t;$$

or, taking $d'' + d' = d$, the whole depth, it becomes

$$\tfrac{1}{3}\,d\,d'\,t.$$

That is, in rectangular beams the resistance is equal to the product of one-third of the whole depth into the depth of tension, and into the force of tension on the extreme fibre.

If, therefore, we knew in all cases the depth of tension, or the relative depth of tension and compression, and the force of direct cohesion, we might compute the transverse strength of rectangular beams, independently of any other data; but these being both very precarious, the best method of determining the strength of beams of wood is by comparative experiments on other beams; for, since the resistance is expressed by $\frac{1}{3}\,d\,d'\,t$, and d' is always proportional to d in the same material, it follows that

the whole resistance is as the square of the depth, as is stated Art. 28; and the resistance being also necessarily as the breadth, it follows that in all rectangular beams the resistance is as the breadth and square of the depth; and we have seen that the strain is as the length into the weight: consequently, calling the breadth b, the depth d, the length l, and the breaking weight w, we ought to have $\frac{lw}{bd^2} = S$, a constant quantity for materials of the same kind, when fixed or supported in the same manner; and when they are fixed or supported in different ways, the formulæ investigated Art. 16 et seq. will enable us still to make the requisite reductions.

The principal data, therefore, that a practical man requires for determining the requisite dimensions of beams, rafters, &c. are such as give this constant quantity S, for all variety of woods; and such will be found in a subsequent part of this Treatise.

On the Deflection of Beams.

47. Hitherto we have considered a beam of timber as inflexible in every part except at its point of fracture, which served to simplify the investigations and the conception of the subject, without in any way affecting the accuracy of the result, the strain at the point of fracture being the same in both cases; but it is frequently very important to know the amount

of deflection a given weight will produce, and the law of action which obtains in these cases.

48. In order to this investigation, let A B C D, fig. 5, Plate II., represent a beam fixed into a solid wall, and in its natural horizontal position, its weight being supposed nothing, or inconsiderable with regard to that with which it is loaded: and let us suppose it to be made up of the several parts A B ab, ab a' b', a' b' a'' b'', &c., each of which is considered to be subject to compression and extension: then, when the beam is loaded with a weight W, it will be brought into the curvilinear form shown in the second position in the figure. Draw the several tangents A m, a n, $a'o'$, $a''p$, &c.; and admitting that the quantity of extension and compression is proportional to the extending and compressing forces, we shall have the several angles m A n, n a' o, o $a''p$, p a'''D, (see fig. 6,) proportional to the distances C F, Cf, Cf', Cf'', &c., these being the effective lengths of the levers, by means of which the force or weight W is exerted at those several points: and the same will have place if we suppose the number of laminæ to be indefinitely great, and therefore the thickness of each indefinitely small: and hence we see the fundamental property of the curve which a beam thus fixed and loaded will assume; viz. "that the curvature at every point is as the distance of that point from the line of direction of the weight," which is, in fact, the *elastic curve*, first proposed by Galileo,

but the correct investigation of which we owe to James Bernouilli, who published it in the 'Memoirs of the Academy of Sciences' for 1703. Other investigations of it have since been given by John Bernouilli, 'Opera Omnia,' tom. iv. p. 242; as also in his Essay on the 'Theory and Manœuvres of Ships;' and particularly by Euler, in the Appendix to his celebrated work, 'Methodus inveniendi Lineas Curvas.'

49. It is to be observed, however, that the supposition of the extension and compression being exactly proportional to the exciting forces, is only a particular and very limited case of the elastic curve; for if that extension were as any function of those forces, it would still not wholly change, although it would modify, the fundamental property of it: but its investigation under this general character would carry us far beyond our present purpose, and, at the same time, would be of no use in our future investigation; for it appears from experiment, that the quantity of extension, in consequence of the imperfect elasticity of the fibres, is very irregular, and that after a certain deflection has been obtained, it seems subject to no determinate law; a circumstance which we have endeavoured to illustrate in a subsequent article: but during the early part of the experiment, that is, while the weight is considerably less than that which is required to produce the ultimate fracture,

the law of the deflections is nearly uniform, and proportional to the exciting force; it will, therefore, be sufficient to consider the elastic curve under this particular case, being the only one that is applicable to the present inquiry.

50. Let, then, A B, fig. ~~10~~ 9, Plate III., represent a thin elastic lamina, without weight, and in its first natural horizontal position; A C, the position of it after being loaded with any given weight W: at any point in the curve R, draw the tangent RT, and conceive the curve to be divided into an indefinite number of equal small parts, A *a*, R *r*; and since, by the hypothesis, the extension of each fibre is proportional to the force by which it is excited, if *r s* and *b a* be drawn perpendicular to the curve at *a* and *r*, the former may be taken to denote the extension of the particle A *a*, and the latter that of the particle R *r*; and we shall have $rs : ab :: $ force in R : force in A, or $:: CL \times W : CG \times W$. Let A F and R X be the radii of curvature at the points A and R, then the triangles A *a b* and A *a* F, as also R *s r* and R *r* X, are similar; and therefore, since $Aa = Rr$, we have

$$rs : Rr :: Rr : RX$$
$$ab : Aa :: Aa : AF;$$
$$\text{therefore } rs : ab :: AF : RX;$$
$$\text{but } rs : ab :: CL : CG,$$
$$\text{and consequently, } CL : CG :: AF : RX;$$

whence again,

$$CL . RX = CG . AF, \text{ a } \textit{constant quantity} = A.$$

In order now to trace the property of the curve, let $CL = x$, $RL = y$, and $RC = z$; then, as is shown by writers on the differential calculus, the radius of curvature

$$RX = \frac{dz^3}{-dx \, . \, d^2y} = \frac{(dx^2 + dy^2)^{\frac{3}{2}}}{-dx \, . \, d^2y};$$

and consequently

$$\frac{x\,dz^3}{-dx\,d^2y} = A, \text{ or } \frac{x\,(dx^2 + dy^2)^{\frac{3}{2}}}{-dx \, . \, d^2y} = A.$$

In its present form this equation is not integrable, but we may accommodate it to our purpose, without any sensible error, while the deflections are small, by supposing $dx = dz$, in which case it becomes

$$\frac{x\,dx^2}{-d^2y} = A, \text{ or } x\,dx = A\frac{d^2y}{-dx}.$$

Or assuming dx as constant, and taking the integral

$$\tfrac{1}{2}x^2 + C = \frac{-A\,dy}{dx}, \text{ C being the correction.}$$

Now, when $x = l$, $\frac{1}{2}l^2 + C = 0$, $\frac{dy}{dx}$ being in that case $= 0$, therefore the correct integral is

$$\tfrac{1}{2}(x^2 - l^2) = -A\frac{dy}{dx}.$$

Multiplying now by dx, we have

$$\tfrac{1}{2}dx\,(x^2 - l^2) = -A\,dy,$$

and taking the integral

$$\tfrac{1}{2}l^2x - \tfrac{1}{6}x^3 = Ay,$$

which requires no correction.

By means of this equation, the curve may be constructed while the deflections are small with regard

to the length of the laminæ; but it will obviously apply to no other case, because it is obtained on a supposition of $d x$ being equal to $d z$, which is in no case strictly true; although the difference, while the deflections are small, is inconsiderable, and may be admitted without any sensible error.

Writing l for x and b for y, the above becomes

$$\frac{l^3}{3b} = A, \text{ or } \frac{l^3}{3b} = CG \, . \, AF;$$

or, since in the case here supposed $CG = l$, *very nearly*, this equation may be still further reduced to

$$\frac{l^2}{3b} = AF;$$

and hence it follows, that while A F remains constant, or the curvature at A is the same, that is, while the strain upon the beam at that point is constant, the deflection b must vary as the square of the length.

But the strain (the weight remaining the same) is as l; or A F is reciprocally as l; and therefore, while the weight is the same,

$$\frac{l^3}{3b} = l \, . \, AF = \text{constant quantity}:$$

consequently, while the weight remains the same, the deflection b is as the cube of the length: but we have seen that, *cæteris paribus*, the deflection is as the weight; therefore, generally,

$$\frac{W l^3}{3b} = E, \text{ a constant quantity};$$

that is, the deflection is as the weight into the cube of the length.

51. This deduction being contrary to the experimental results of M. Girard, ought to be examined with caution: we propose, therefore, investigating the nature of the curve on different principles, and on such as will probably be more intelligible to many readers.

It has been shown above, that an approximation to the actual state of the curve is all that can be obtained; and this approximation may be obtained perhaps more satisfactorily as follows.

Let A B C D, figs. 5, 6, Plate II., represent the deflected beam, and let it be divided as above supposed (Art. 48) into any number of equal inflexible parts, A B *ab*, *ab* $a'b'$, &c., and let *ad*, $a'd'$, $a''d''$, &c., drawn perpendicular to the respective tangents at A, *a*, a', &c., represent the deflections at those points, which, from what has been above shown, will be proportional to CF, C*f*, Cf', &c.; and as the investigation is only intended to apply to small deflections, let us consider these several lines, *a d*, $a' d'$, &c., instead of being perpendicular each to its respective tangent, to be all parallel to each other, and perpendicular to A *m*; let us also denote the first of these *a d* by *d*, which may be denominated the *element of deflection*, and let the number of parts or laminæ into which the beam is divided be denoted by *m*, then we shall have

$$\text{------------} \frac{m}{m} d = a\,d$$

$$m : m - 1 :: d : \frac{m-1}{m} d = a'\,d'$$

$$m : m - 2 :: d : \frac{m-2}{m} d = d'' d''$$

$$m : m - 3 :: d : \frac{m-3}{m} d = d''' d'''$$

&c. &c. &c.

Also, according to our supposition,

$$n m = m \times a d = \frac{m^2}{m} d$$

$$n o = (m - 1) d' d' = \frac{(m-1)^2}{m} d$$

$$o p = (m - 2) d'' d'' = \frac{(m-2)^2}{m} d$$

&c. &c. &c.

Whence the whole deflection $m\,\mathrm{D}$ will be expressed by the series

$$m\,\mathrm{D} = \frac{d}{m} \left\{ m^2 + (m-1)^2 + (m-2)^2 + \text{\&c. } 1^2 \right\} \quad . \; . \; (1)$$

or by the summation of the series,

$$m\,\mathrm{D} = \frac{d}{m} \left\{ \frac{m^3}{3} + \frac{m^2}{2} + \frac{m}{6} \right\}, \text{ or}$$

$$m\,\mathrm{D} = d \left\{ \frac{m^2}{3} + \frac{m}{2} + \frac{1}{6} \right\}.$$

That is, while the number of parts m are supposed finite, $m\,\mathrm{D}$ varies as $\left(\frac{m^2}{3} + \frac{m}{2} + \frac{1}{6}\right) d$; but when m is infinite, then the two latter terms vanish, as being inconsiderable with regard to the first; and we have $m\,\mathrm{D} = \frac{m^2 d}{3}$.

In the same manner, if l' were the length of any other beam of which the number of parts were m', but the parts individually in length equal to the

former, and the element of deflection d', we should have

$$m' D' = \frac{m'^2 d'}{3};$$

whence

$$m D : m' D' :: m^2 d : m'^2 d'; \text{ but } m : m' :: l : l';$$

therefore

$$m D \text{ varies as } \frac{l^2 d^*}{3};$$

that is, the deflection varies as the square of the length, and the element of deflection; but the element d obviously varies as the strain; that is, as l W: therefore, again, the deflection varies as $\frac{l^3 W}{3}$; or, denoting the deflection m D by b, we have $\frac{l^3 W}{3 b} = E$, a constant quantity, the same result as before.

52. The same may be otherwise demonstrated as follows:

In the above investigation it is shown that D m, which is supposed to represent the deflection, is expressed by the equation

$$m D = d \left\{ \frac{m^2}{3} + \frac{m}{2} + \frac{1}{6} \right\},$$

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* We have used the above process for the convenience of those who may not be acquainted with the fluxional or differential calculus: those who are will see immediately that the summation, expressed in equation (1), is equal to $\frac{d}{m}$ times the integral of $x^2 d x$; that is,

$$\frac{d}{m} \int x^2 d x = \frac{d x^3}{3 m} = \frac{d m^2}{3} \text{ when } d x = m.$$

and that in any other beam of which the number of parts are m', the deflection is also

$$m'D' = d' \left\{ \frac{m'^2}{3} + \frac{m'}{3} + \frac{1}{6} \right\},$$

from which we conclude, that when m is infinite, the deflections are as

$$d\,m^2 : d'm'^2 ; \text{ or as } d\,l^2 : d'\,l'^2 ;$$

where l and l' denote the two lengths. If this should not appear to involve all that precision and accuracy that may be desired, it may be considered under a point of view somewhat different to the former, and will probably carry more conviction with it to some of our readers:

Supposing, therefore, the equation

$$mD = d \left\{ \frac{m^2}{3} + \frac{m}{2} + \frac{1}{6} \right\}$$

to be established; and calling l the length of the beam, and λ the length of each of the equal sides of the polygon, we shall have $\frac{l}{\lambda} = m$; and substituting this for m in the preceding equation, we obtain

$$mD = d \left\{ \frac{l^2}{3\lambda^2} + \frac{l}{2\lambda} + \frac{1}{6} \right\}, \text{ or}$$

$$mD = d \left\{ \frac{2l^2 + 3l\lambda + \lambda^2}{6\lambda^2} \right\};$$

and in the same manner, if the length of another beam is l', and $m'D'$ denotes its deflection, we find

$$m'D' = d' \left\{ \frac{2l'^2 + 3l'\lambda + \lambda^2}{6\lambda^2} \right\};$$

λ, or the length of each side of the polygon, being,

by the supposition, the same in both cases: we shall have, therefore,

$$mD : m'D' :: d\,\{2l^2 + 3l\lambda + \lambda^2\} : d'\,\{2l'^2 + 3l'\lambda + \lambda^2\}.$$

This result is wholly independent of any particular value of λ, and therefore is true, when λ becomes indefinitely small; that is, in the case of a continued curve. But here, as λ is indefinitely small, the last two terms of each of the third and fourth members of the above ratio vanish, and that ratio then becomes simply

$$mD : m'D' :: dl^2 : d'l'^2;$$

that is, the deflection varies as the element of deflection into the square of the length; or, as the element of deflection into the square of the length divided by 3, as we have found it in the article in question.

53. In a similar manner we may investigate the law of deflection when the weight, instead of being all applied at the extremity of the beam, is equally distributed throughout its whole length, or when it is divided into equal portions, and suspended at equal distances, as at the points a', a'', a''', &c., fig. 5, Plate II.

For, calling d', as before, the element of deflection $= a\,d$, it is obvious that the successive deflections, instead of decreasing as before, in the simple ratio of the length, will now decrease as the square of the length, because both the weight and the length of

lever decrease in the same manner. Our successive deflections therefore, in this case, will be

$$\frac{m^2}{m^2} d' = a d$$

$$m^2 : (m-1)^2 : : d' : \frac{(m-1)^2}{m^2} d' = a' d'$$

$$m^2 : (m-2)^2 : : d' : \frac{(m-2)^2}{m^2} d' = a'' d''$$

$$m^2 : (m-3)^2 : : d' : \frac{(m-3)^2}{m^2} d' = a''' d'''$$

&c. &c. &c.

Also, according to the same supposition as that above adopted, we shall have

$$n m = m \cdot a d = \frac{m^3}{m^2} d'$$

$$n o = (m-1)\ a' d' = \frac{(m-1)^3}{m^2} d'$$

$$o p = (m-2)\ a'' d'' = \frac{(m-2)^3}{m^2} d'$$

&c. &c. &c.

Whence the whole deflection $m\,D$ will now be expressed by the series

$$m\,D = \frac{d'}{m^2}\left\{m^3 + (m-1)^3 + (m-2)^3 + \&c.\ 1^3\right\};$$

or by summation,

$$m\,D = \frac{d'}{m^2}\left\{\frac{m^4}{4} + \frac{m^3}{2} + \frac{m^2}{2}\right\};\ \text{or}$$

$$m\,D = d'\left\{\frac{m^2}{4} + \frac{m}{2} + \frac{1}{4}\right\}:$$

which expression is analogous to that in Article 51, and shows that in this case also, when m is infinite, that is, when the weight is uniformly distributed,

the deflection is as the weight and cube of the length, or as the square of the length and element of deflection, because the expression then becomes

$$m\,\mathrm{D} = \frac{m^2}{4}\,d'.$$

But in order to compare the real quantity of deflection in this case with that of the former, it must be observed, that the weight being the same, the strain on the beam will, in the first instance, be double what it is in the second; and the element d in the former will be double d' in the latter, or $d' = \frac{1}{2}\,d$. Substituting, therefore, $\frac{1}{2}\,d$ for d', our expression

$$m\,\mathrm{D} = \frac{m^2}{4}\,d', \text{ becomes } \frac{m^2}{8}\,d;$$

whereas in the former case it is $\frac{m^2}{3}\,d$; therefore the beams being of the same length, the deflection, when the weight is all collected at the extremity, is to that of the beam equally loaded throughout its length with the same weight, as

$$\frac{m^2}{3}\,d : \frac{m^2}{8}\,d, \text{ or as 8 to 3.}$$

The expression for the elasticity in this case will therefore be $\frac{l^3\,\mathrm{W}}{8\,b} = \mathrm{E}$, the same constant quantity as before.

The principles of investigation given in Art. 52 are equally applicable in this case.

54. In the preceding investigations the deflections have only been considered with reference to beams

fixed at one end: let us now endeavour to investigate the same, on a supposition of their being supported at both ends. In order to which, it may be observed, in the first place, that whatever weight is just sufficient to break a beam fixed by one end in a wall, the same weight may be borne at the other end of it, (the arms or levers being supposed of equal length,) if the wall were removed, and the beam merely supported on a fulcrum, or prop, in its middle point, as in fig. 3, Plate II., the tension in both cases being the same; just as a line passing over a pulley, and loaded at each end with an equal weight, has the same tension as a single fixed line loaded with only one of those weights; and what is here stated of the ultimate degree of tension, is obviously true of any quantity of it: that is, whatever tension the fibres may have in the former case, they will have precisely the same in the latter.

Again, the beam F I F′ I′, fig. 3, is similarly situated, at least as far as our present question is concerned, with regard to the strain upon it, and therefore to its deflections, as the equal beam FIF′I′, fig. 4; whether we consider the latter to rest against a fulcrum at C, and to be strained by the two weights W, W′ passing over the pulleys Q, Q′; or, as being supported on two fulcrums, F, F′, and loaded in the middle with the weight P, equal to the two weights W, W′.

Hence, then, we conclude, that the deflection of a beam fixed at one end in a wall, and loaded at the

other, is equal to that of a beam of twice the length, supported at both ends, and loaded in the middle with a double weight; that is, the strain being the same in both cases: consequently, when the weights are the same, the deflection in the first instance is to that in the second as 2 : 1.

And when the length and weight are both the same, the deflections will be to each other as 1 : 16. For the strain will be four times greater on the beam fixed at one end than on that supported at both; and therefore, all other things being the same, the element of deflection will also be four times greater: also, the entire deflection is as the element of deflection into the square of the length; and, according to our supposition, the length is double; whence, upon the whole, it appears that the deflection in the one case is to that in the other as 1 : 4 × 4, or as 1 to 16.

The same formula will, therefore, apply in this case as in Art. 50; viz. $\frac{l^3 W}{3 b} = E$, a constant quantity; observing only, that the value of E is here sixteen times greater than in the former.

55. When the weight is distributed throughout the length of the beam, instead of being all collected in the middle, it is a known mechanical principle, that the strain on the centre will be the same as it would be with half the entire weight collected in that point; and consequently, the element of de-

flection in the same place will also be one-half of what it would be if the whole weight was collected there.

But now, in order to compare the strain and consequent deflection at any other point, D, fig. 9, Plate II., we must first observe, that the resistance of the fulcrum at B is constant; and therefore, that the strain at D, as arising from that resistance, will be found as follows; viz. $CB : DB : d' : \frac{DB\,d'}{CB}$ = the element of deflection at D, as arising from the resistance at B; d' denoting the deflection at C.

But the point D has a further strain to sustain, and consequently a further deflection, arising from the weight of the part between C and D. Now this weight will be to the whole weight W as C D to A B, or 2 C B; that is,

$$2CB : CD :: W : \frac{CD \,.\, W}{2CB}.$$

Consequently, the deflection arising from this strain, as referred towards B, will be

$$CB^2 : CD \times BD :: d' : \frac{CD \,.\, BD}{CB^2}\, d'.$$

Whence the entire deflection from the tangent of the curve at the point D will be

$$\frac{DB}{BC}\, d' + \frac{CD \,.\, DB}{BC^2}\, d' = \frac{(CB + CD)\, DB}{BC^2}\, d'.$$

Which deflection, referred to the perpendicular B F, will be

$$\frac{(CB + CD)\, DB^2}{BC^2}\, d'.$$

If, now, we denote C B by m, and D B by n, in which case $CD = m - n$, the above will become

$$\frac{(2m - n)\,n^2}{m^2}\,d' = \frac{2\,m\,n^2 - n^3}{m^2}\,d'.$$

And, by giving to n the successive values, 1, 2, 3, &c., as in our preceding investigations, and summing the resulting series, or by finding the value of

$$\int \frac{2\,m\,x^2 - x^3}{m^2}\,d'\,d\,x,$$

when $x = m$, we shall have for the entire deflection,

$$BC = \frac{5\,m^2}{12}\,d'.$$

But it has been shown, that in the former case, where the weight is all collected in the middle, the deflection is $\frac{m^2}{3}\,d$; and, therefore, since $d' = \frac{1}{2}\,d$, the deflections in the two cases will be as $\frac{1}{3} : \frac{5}{24}$, or 8 to 5.

Now it has been seen, that when a beam or rod is fixed only at one end, the deflection, when the weight is uniformly distributed, is to the same when that weight is collected at the extremity, as 3 to 8: whereas we have found above, that when the beam is supported at its ends, the deflections in the like cases are to each other as 5 to 8.

Whence, if a long rod or plank is, in the first instance, supported in the middle, and the ends be deflected; and, in the second, the ends are supported, and the middle left to descend, the deflection in the latter case is to that in the former as 5 to 3.

Of the Deflection as depending on the Breadth and Depth.

56. In the preceding investigations we have supposed the beams, although of different lengths, to be all of the same breadth and depth; or, as opposing equal resistance: when these dimensions are not the same, the resistance is as the breadth and square of the depth, Art 46; and, therefore, when the weight is increased in that proportion, the quantity of extension will, by hypothesis, be the same, the length being here supposed constant; but, by a reference to fig. 2, Plate II., it will appear, that the extension of the fibre *b* A being supposed constant, the angle *bn* A, or H A F, (which is equivalent to what we have denominated the element of deflection,) will be inversely as *n* A, or C A, the depth of the beam.

Hence with the same weight the deflection will be inversely as the breadth and square of the depth into the element of deflection, which is itself inversely as the depth. Hence, every thing else being the same, the deflection will vary inversely as the breadth and cube of the depth; but we have seen that when the breadth and depth are constant the deflections are as the weight and cube of the length; therefore generally, if l denote the length of a beam, b its breadth, and d its depth, also W the weight with which it is loaded, the deflection will vary as $\frac{l^3 . W}{b . d^3}$; and if, therefore, we denote the deflection by δ,

$$\frac{l^3 W}{b d^3 \delta} = E, \textit{ a constant quantity.}$$

57. This is a conclusion which necessarily arises out of the above investigation, but being at variance with the experiments of M. Girard, which are very numerous, I was a little surprised at the result thus obtained, and re-examined my investigations, under an impression that some error had crept in, and escaped my observation. At length, not being able to discover any, I referred to the experimental results, the greater part of which were in favour of my own theoretical deductions: still, however, as these were different beams, and many of the deflections considerable, while the investigation was supposed to apply only to those cases in which it was very small, I was still doubtful, and therefore procured three pieces of fir, each 6 feet 6 inches in length, and 2 inches in depth, by $1\frac{1}{2}$ inch in breadth, and of very uniform texture: these pieces were rested on two props, as represented in Plate IV.; first at the distance of 3 feet, and then at 6 feet.

If, therefore, the deflections varied as the square of the length, according to the results of M. Girard, the deflections ought to be, in the second case, four times what they were in the first; but if the deflections were as the cubes of the lengths, as they should be according to my deduction, then the deflection would be eight times as much. I accordingly made the experiments with great care; and the following are the results that were obtained.

No. 1.

Feet long.	Inches deep.	Breadth.	Weight, lbs.	Deflection.
3	2	1½	120	·09
3	2	1½	180	·12
6	2	1½	120	·68
6	2	1½	180	1·00

The same piece.

3	1½	2	120	·19
3	1½	2	180	·28
6	1½	2	120	1·38
6	1½	2	180	1·91

No. 2.

3	2	1½	120	·10
3	2	1½	180	·15
6	2	1½	120	·72
6	2	1½	180	1·05

The same piece.

3	1½	2	120	·18
3	1½	2	180	·28
6	1½	2	120	1·30
6	1½	2	180	2·00

No. 3.

3	2	1½	120	·07
3	2	1½	180	·11
6	2	1½	120	·65
6	2	1½	180	·96

The same piece.

3	1½	2	120	·16
3	1½	2	180	·24
6	1½	2	120	1·25
6	1½	2	180	1·85

58. It was impossible, after these experiments, any longer to doubt the correctness of the preceding investigations; the deflection of the 6-feet beams

answering so very nearly to the cube, or to eight times that of the same at 3 feet. With regard to the deflection being inversely as the cube of the depth into the breadth, that is, inversely as $b\,d^3 : b^3 d$, or as $b^2 : d^2$, in the above experiments, this also is confirmed as far as the comparison can be made, but the difference in these two dimensions is not so great as in the lengths, and therefore the results, perhaps, not so conclusive.

M. Girard makes the deflections inversely as $b\,d^2 : b^2 d$; that is, in the above cases, as $b : d$, which by no means agrees with the above results: the discrepance will, however, be best seen by computing the deflections; first of the long beam from that of the short one being given, and comparing them with those determined from experiment; and then computing the deflections of the beams in the direction of their least depth, from those given for their greater.

	Feet.	lbs.	Deflection computed according to M. Girard.	Deflection from preceding formulæ.	Deflection from experiments.
No. 1.	6	120	·36	·72	·68
	6	180	·48	·96	1·00
No. 1.	6	120	·76	1·52	1·38
	6	180	1·12	2·34	1·91
No. 2.	6	120	·40	·80	·72
	6	180	·60	1·20	1·05
No. 2.	6	120	·72	1·44	1·30
	6	180	1·12	2·24	2·00
No. 3.	6	120	·28	·56	·65
	6	180	·44	·88	·96
No. 3.	6	120	·64	1·28	1·25
	6	180	·94	1·92	1·81

It only requires a comparison to be made between the last column and the other two, to decide which of the two formulæ best agrees with the actual state of the beam's deflection.

59. The above are obtained from a comparison of the lengths of the beam: let us now make a similar comparison, as depending upon their depth and breadth.

	Feet.	lbs.	Deflection according to M. Girard. Defl. $\propto \frac{1}{b d^2}$	Deflection from the formulæ. Defl. $\propto \frac{1}{b d^3}$	Deflection from experiment.
No. 1.	3	120	·12	·16	·19
	3	180	·16	·21	·28
No. 1.	6	120	·91	1·21	1·38
	6	180	1·33	1·77	1·91
No. 2.	3	120	·13	·18	·18
	3	180	·20	·27	·28
No. 2.	6	120	·96	1·28	1·30
	6	180	1·40	1·87	2·00
No. 3.	3	120	·09	·12	·16
	3	180	·15	·20	·24
No. 3.	6	120	·87	1·16	1·25
	6	180	1·28	1·71	1·85

Here, again, the agreement between the last column and the preceding one is so near, in comparison with that computed according to M. Girard's principle, as to leave no doubt concerning the legitimacy of our formulæ.

Still, however, I was desirous of further proof, and therefore procured three pieces of very clean fir, free from knots, 10 feet 6 inches long, 3 inches deep, and 1½ inch in thickness, and an ivory scale very

accurately graduated into 40ths of an inch, which was now fixed to the batten, instead of the scale of 10ths of inches hitherto employed; by which means the deflections could be accurately observed to within about $\frac{1}{80}$th of an inch.

One of the beams was laid on with the props 9 feet apart, and the weights gradually added till the deflection was 27 of the equal parts on the scale: I then unloaded it, and set the props 6 feet asunder, and applied again the same weights, and the deflection was exactly eight divisions.

Now, in case of the deflections being as the square of the length, we ought to have had

$$9^2 : 6^2 :: 27 : 12$$

for the deflection at 6 feet. But if the deflections were as the cubes,

$$9^3 : 6^3 :: 27 : 8,$$

precisely the same as it was found to be by the experiment.

The props were then brought to the distance of 3 feet, and the same weights being used, the deflection was exactly $\frac{1}{40}$th of an inch, or one division: whereas it ought, according to M. Girard, to have been $\frac{3}{40}$ths, or three divisions.

The second batten was now laid on at 9 feet, and brought to a deflection of $40\frac{1}{2}$ divisions; the same weights brought it at 6 feet to $12\frac{1}{2}$ divisions, and at 3 feet to $1\frac{1}{2}$; whereas if the deflections had been as the squares, they ought to have been 18 and $4\frac{1}{2}$ respectively.

60. The third beam was deflected to 54 divisions at 9 feet, and the same weights brought it to 16½ at 6 feet, and to 2 divisions at 3 feet, instead of 24 and 6, as required by the law which M. Girard had deduced from his experiments.

I next tried each of the pieces again at the distance of 6 feet, laid in the contrary way, viz. with their least thickness vertical; and placing on each the same weights as had been before employed, the deflections were, for

No. 1	32 divisions.
No. 2	48 ditto.
No. 3	64 ditto.

Which show that the deflections were also as the cubes of the depth into the breadth, and not as the squares; for had that law obtained, these deflections would have been 16, 24, and 32.

61. After the preceding experiments were gone through, I made the following series on the same battens, and have computed, in every case, the value of the constant quantity, which we may call the elasticity, E, from the formula $\frac{l^3 W}{b d^3 \delta} = E$, the reduced mean of which is $E = 5317610$, whence we have $\frac{l^3 W}{b d^3 \delta} = 5317610$, from which any one of these five quantities may be found when the other four are given.

62. *Table of the Deflections of Fir Battens.*

Weight in lbs.	Deflections at			Computed value of $E = \frac{l^3 W}{b d^3 \delta}$			Mean values of E.
No. 1.—3 inches deep, 1½ inch thick. Sp. gr. 584.							
	10 ft.	8 ft.	6 ft.	10 ft.	8 ft.	6 ft.	
70	0·65	0·31	0·16	4594960	4918816	4838512	
120	1·05	0·51	0·26	4876144	5139848	5104296	
135	1·18	0·57	0·29	4881184	5227256	5148144	5047599
150	1·30	0·64	0·31	4922960	5195848	5333328	
165	1·44	0·71	0·35	4888888	5076736	5213624	
180	1·57	0·77	0·37½	4893288	5094816	5308144	
No. 2.—1½ inch deep, 3 inches thick. Sp. gr. 558.							
	8 ft.	6 ft.	4 ft.	8 ft.	6 ft.	4 ft.	
35	·625	·275	·075	4893300	4691800	5097200	
50	·825	·400	·112	5295800	4608000	4876200	4877950
65	1·12	·525	·150	5071300	4564100	4723400	
80	1·36	·625	·180	5140100	4718600	4855600	
No. 3.—3 inches deep, 1½ inch thick. Sp. gr. 640.							
	10 ft.	8 ft.	6 ft.	10 ft.	8 ft.	6 ft.	
70	·501	·275	·114	5961400	5560600	5658900	
120	·875	·467	·195	5851400	5613400	5671400	
135	1·000	·525	·220	5760000	5617400	5655300	5693427
150	1·125	·687	·242	5688900	5582300	5712400	
165	1·237	·640	·265	5691300	5619100	5751300	
180	1·350	·700	·287	5688900	5617400	5780100	
No. 3.—viz. the same beam, 1½ inch by 3 inches.							
	8 ft.	6 ft.	4 ft.	8 ft.	6 ft.	4 ft.	
35	·55	·237	·70	5560600	5444100	5461300	
50	·775	·327	1·00	5637500	5707100	5461300	5651466
65	1·02	·425	1·25	5632900	5708600	5679800	
80	1·50	·512	1·50	5867000	5832000	5825400	
Mean . . . . E = 5317610.							

63. As a further confirmation of the preceding deductions, the following, from M. Dupin's experiments, may be added, which I had not seen when the above was written. The pieces on which M. Dupin's experiments were made, were 2 metres in length, and of various lateral dimensions, viz. 1, 2, and 3, &c. centimetres, to a decimetre in the squareage; they were performed with care, and conducted with great ability.

64. The following are some of the principal theorems which this author has drawn from his experiments and investigations, as connected with this part of our inquiry; viz.

1. The deflections of the same beam resting on props at each end, and loaded in the middle with small weights, are as those weights.

2. When the same piece is rested on props at the same distance, and loaded at its middle point with different small weights, these weights are reciprocally proportional to the radius of curvature at that point; and the curvature itself is consequently proportional to the weights.

3. The deflection is, *cæteris paribus,* inversely as the cube of the depth; also the depth being the same, the deflection is inversely as the breadth.

4. The deflection is, therefore, *cæteris paribus,* directly as the cube of the length.

From which it necessarily follows, agreeably to

the preceding deductions, that $\frac{l^3 W}{b d^3 \delta} =$ a constant quantity.

5. M. Dupin also demonstrates, experimentally, the ratio which has been stated between the deflection of beams supported at each end and loaded in the middle, and the deflection of the same when the weight is uniformly spread; at least his experiments give results approximating towards that ratio; viz. experimentally he has found it to be as 19 : 30, while the theory required the ratio of 5 to 8; or reducing both to the same antecedent, the first is as 95 to 150, and the second as 95 to 152, which is as nearly correct as it is possible to expect, considering, in the first place, that it is impossible practically to distribute the weights so as to have them perfectly uniform; and in the second, that the investigation belongs only to infinitely small deflections; while experimentally they are rendered sufficiently obvious to be submitted to actual measurement. The same author has found various other interesting results; but we cannot allow any further abstracts in this place.

65. It is important to observe, before concluding this chapter, that all the foregoing investigations have been made exclusively with reference to rectangular beams, and that they must only be considered as being applicable to that form; for, notwithstanding we have throughout made our deduc-

tions from a comparison of the depths, breadths, &c., it is obviously not the depth of the whole beam, but that of its neutral axis, on which the deflection depends; but as the latter, in rectangular beams, is always as the whole depth, we may use the one for the other indifferently, and we made choice of the latter for the sake of simplicity.

Practical Deductions.

66. The following practical deductions flow immediately from the preceding investigations, and with them we shall conclude this chapter.

1. It has been shown, that the successive deflections are directly as the weight and cube of the length, and reciprocally, as the breadth and cube of the depth, or that when the beam is fixed at one end, and loaded at the other,

$$\frac{l^3 W}{b d^3 \delta} = E,^* \text{ is a constant quantity.}$$

When fixed at one end uniformly loaded, (see Art. 50,)

$$\frac{3 l^3 W}{8 b d^3 \delta} = E, \text{ the same constant.}$$

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* It may be proper to observe, that the original expression is $\frac{l^3 W}{3 b d^3 \delta} = E$, a constant; of course $\frac{l^3 W}{b d^3 \delta} = E$, is constant also; and we prefer the latter expression, for the sake of simplicity.

When supported at both ends, and loaded in the middle,

$$\frac{l^3 W}{16\, b\, d^3\, \delta} = E, \text{ the same constant.}$$

2. And hence it follows, that in order to preserve the same stiffness in beams, the depth must be increased in the same proportion as the length, the breadth remaining constant.

3. In square beams of different lengths, the stiffness will be the same, when $s^{\frac{4}{3}}$ is as l, s being the side of the square, and l the length.

4. If the depth is given, the stiffness will be the same when b is as l^3, or when $b^{\frac{1}{3}}$ is as l.

5. The deflection of different beams arising from their own weight, having their several dimensions proportional, will be as the square of either of their like lineal dimensions. For it has been seen that in all these cases $\frac{l^3 W}{b\, d^3\, \delta} = E$, a constant quantity: and if, therefore, we suppose each of these dimensions to be increased m times; then the weight W will be increased m^3 times, and we shall, therefore, have

$$\frac{l^3 m^3 W m^3}{b\, m\, d^3 m^3 \delta'} = E, \text{ or } \frac{l^3 W m^2}{b\, d^3 \delta'} = E;$$

consequently, since E is the same in both, δ' must have varied as m^2.

The same will apply to beams loaded throughout proportional to the dimensions; and it is a fact which ought to be kept constantly in view in the

construction of models, on a small scale, of works intended to be executed on a large one.

6. With regard to the ultimate deflection of beams before their rupture, the same relations do not obtain; for it is obvious, from what has been already stated, (Art. 54,) that the depth being the same, the element of deflection will, in the breaking state of the beam, be constant; and, consequently, the ultimate deflection will in this case be as the square of the length, and it will be inversely as the depth when the length is the same; and if both these dimensions remain constant, the last deflections will be constant also, whatever may be the breadth of the beam.

The formula, therefore, applicable to this case, is $\frac{l^2}{d\,\Delta} = U$, a constant quantity, where Δ is the last deflection, l the length, and d the depth of the beam.

But little dependence, however, can be placed on this last deduction, because the law of deflections becomes very uncertain, after the elasticity has ceased to be perfect; which is some time before the rupture takes place.

Experiments on the Transverse Strength of Timber.

67. Reference has already been made to the experiments of Galileo, Mariotte, Musschenbroeck, and others, which had been made in the earlier stages

of this inquiry; but unfortunately, from one cause or other, the results obtained from them are little accordant with each other; and we think it useless to embarrass the reader by giving a long detail of labours on which no dependence can be placed: passing over, therefore, many early experiments, we come to those of M. Buffon, by far the most valuable, both as respects the number of them, and the size of the pieces of timber on which they were made; many of them having been from 20 to 28 feet in length, and from 4 to 8 inches square. This philosopher was furnished by the French Government with ample funds, and every necessary means for carrying on his experiments on a grand scale; and he discharged the duty thus imposed upon him in a manner highly creditable to himself, and to the satisfaction of the Academy; but he did not, perhaps, possess the mathematical knowledge necessary for making the best use of his results. His experiments, however, are not the less valuable; as they are, no doubt, faithfully related, and furnish a sound foundation for the establishment of a correct theory.

He commenced his operations, with Du Hamel, on pieces of small dimensions; and tried them in succession from the heart to the bark of the tree, and from the root upwards. From these experiments it was found that the heart was the densest, that the density decreased from hence to the circumference, and that the strength decreased also in nearly the same proportion.

He also made trial of the proportional strength of battens, accordingly as they were laid, with the annual layers, vertical or horizontal, and found a difference in the strength, in these two cases, nearly in the ratio of 8 to 7; the difference, no doubt, arising from the cohesion of the layers with each other being considerably less than that between the fibres themselves. Some experiments have been referred to, in Art. 13, to show the quantity of this lateral cohesion, although it must be allowed to be rather a subject of curiosity than utility; for large beams, whose strength it is the most important to be acquainted with, commonly occupy the whole, or nearly the whole, section of the tree.

M. Buffon found likewise, that oak timber lost much of its strength in the course of drying, or seasoning; and therefore, in order to secure uniformity, his trees were all felled in the same season of the year, were squared the day after, and experimented on the third day. Trying them in this green state, gave him an opportunity of observing a very curious phenomenon; namely, that when the weights were laid briskly on, nearly sufficient to break the log, a very sensible smoke was observed to issue from the two ends, with a sharp hissing noise, which continued all the time the tree was bending or cracking.

This philosopher, as above stated, drew no important conclusions from his experiments: he seems to have had in view no favourite theory, either of

his own or of any other writer, and was therefore free from any bias, or any desire to accommodate his experiments to a particular hypothesis: besides, his beams were too large for him to deceive himself in this respect, as there is reason to believe has been the case with some authors. Upon the whole, these were certainly the most valuable experiments that had yet been made upon the transverse strength and strain of oak timber, whether they be considered as the means of furnishing practical precedent or theoretical data: the following Table of this author's results will therefore, it is presumed, be acceptable to the English engineer, for whose convenience the several results are reduced to English weights and measures.

68. *Table of the Results of Buffon's Experiments on the Transverse Strength of Square Oak Beams.*

No.	Side of square.		Length.		Weight of the pieces.		Weights which broke the pieces.		Deflection before cracking.	
	In inches.	In metres.	In feet and inches.	In metres.	In lbs.	In kilogr.	In lbs.	In kilogr.	In inches.	In metres.
1			7 6	2·2732	64·56	29·34	5756	2616	3·47	·0946
2			7 6	2·2732	60·25	27·39	5676	2580	4·82	·1217
3			8 6·8	2·5979	73·17	33·26	4950	2250	4·01	·1013
4			8 6·8	2·5979	67·79	30·81	4842	2201	5·00	·1262
5	4·28	·1082	9 7·7	2·9227	83·85	37·66	4401	2005	5·17	·1307
6			9 7·7	2·9227	76·39	34·73	4250	1932	5·89	·1488
7			10 8·5	3·2473	90·37	41·09	3900	1773	6·25	·1578
8			10 8·5	3·2473	88·23	40·11	3884	1760	6·96	·1758
9			12 10·3	3·8969	107·60	48·91	3281	1491	7·50	·1894
10			12 10·3	3·8969	105·45	47·93	3174	1430	7·50	·1894
11			7 6	2·2732	101·14	45·98	12670	5759	2·67	·0676
12			7 6	2·2732	95·32	43·33	12133	5515	2·67	·0676
13			8 6·8	2·5979	111·91	50·87	10653	4842	2·85	·0721
14			8 6·8	2·5979	109·76	49·89	10411	4732	3·12	·0789
15			9 7·7	2·9227	126·97	57·72	9038	4108	3·21	·0811
16			9 7·7	2·9227	124·82	56·74	8958	4072	3·51	·0878
17			9 7·7	2·9227	173·75	56·25	8822	4011	3·75	·0946
18			10 8·5	3·2473	142·04	64·69	7774	3534	3·39	·0856
19			10 8·5	3·2473	139·91	63·60	7586	3448	3·74	·0946
20			10 8·5	3·2473	138·28	62·85	7639	3472	4·28	·1082
21			12 10·3	3·8969	167·87	76·30	6510	2959	5·89	·1488
22	5·35	·1353	12 10·3	3·8969	165·71	75·32	6563	2983	6·16	·1556
23			15 0	4·5464	191·54	87·06	5811	2641	8·57	·2164
24			15 0	4·5464	189·38	86·09	5595	2543	8·83	·2231
25			17 1·7	5·1959	224·90	102·23	4861	2164	8·65	·2186
26			17 1·7	5·1959	220·59	100·27	4600	2091	8·74	·2209
27			19 3·4	5·8453	249·65	113·48	4034	1834	8·57	·2164
28			19 3·4	5·8453	248·57	113·00	3248	1799	8·74	·2209
29			21 5·1	6·4947	284·12	129·64	3523	1601	9·46	·2389
30			21 5·1	6·4947	278·71	126·69	3416	1553	10·71	·2706
31			25 8·6	7·7939	333·59	151·63	2367	1076	11·88	·2976
32			25 8·6	7·7939	330·36	150·16	2286	1039	12·05	·3652
33			30 0	9·0928	391·69	178·04	1936	880	19·78	·4870
34			30 0	9·0928	365·40	166·09	1882	855	23·57	·5952

TABLE—(CONTINUED.)

No.	Side of square.		Length.		Weight of the pieces.		Weights which broke the pieces.		Deflection before cracking.	
	In inches.	In metres.	In feet and inches.	In metres.	In lbs.	In kilogr.	In lbs.	In kilogr.	In inches.	In metres.
35			7 6	2·2732	138·27	62·85	20715	9416		
36			7 6	2·2732	129·66	58·93	20069	9122		
37			8 6·8	2·5979	160·33	72·88	16894	7679	2·50	·0631
38			8 6·8	2·5979	157·11	71·41	16517	7508	2·58	·0653
39			9 7·7	2·9227	178·63	81·19	14473	6579	2·67	·0676
40			9 7·7	2·9227	177·02	80·46	13607	6285	2·83	·0766
41			10 8·5	3·2473	202·30	91·96	12347	5612	3·21	·0811
42			10 8·5	3·2473	200·18	91·00	11863	5392	3·74	·0946
43	6·43	·1624	12 10·3	3·8969	241·04	109·56	9900	4500	4·28	·1082
44			12 10·3	3·8969	237·82	108·10	9684	4402	4·38	·1107
45			15 0	4·5464	274·40	124·73	8016	3644	4·82	·1217
46			15 0	4·5464	273·33	124·24	8070	3668	4·38	·1107
47			17 1·7	5·1959	316·37	143·80	6725	3057	5·89	·1488
48			17 1·7	5·1959	315·29	143·32	6967	3167	6·25	·1578
49			19 3·4	5·8453	359·42	163·37	6052	2751	7·94	·2006
50			19 3·4	5·8453	156·18	161·90	5918	2690	9·10	·2299
51			21 5·1	6·4946	405·69	184·40	5406	2457	10·17	·2570
52			21 5·1	6·4946	403·53	183·43	5246	2384	9·46	·2389
53			8 6·8	2·5979	219·52	100·00	28140	12791	2·94	·0743
54			8 6·8	2·5979	219·52	100·00	27926	12693	2·68	·0676
55			9 7·7	2·9227	244·28	111·03	24535	11152	3·31	·0836
56			9 7·7	2·9227	242·12	110·05	23562	10712	3·12	·0789
57			10 8·5	3·2473	273·33	124·24	21145	9611	2·73	·0689
58			10 8·5	3·2473	271·15	123·25	20769	9440	3·21	·0811
59			12 10·3	3·8969	324·98	147·72	18078	8217	3·12	·0789
60			12 10·3	3·8969	323·90	147·23	16733	7606	3·56	·0900
61	7·5	·1894	15 0	4·5464	382·01	173·64	14634	6652	4·46	·1127
62			15 0	4·5464	377·72	171·69	13828	6285	4·01	·1013
63			17 1·7	5·1959	436·90	198·59	11944	5429	5·17	·1307
64			17 1·7	5·1959	433·67	197·12	11729	5331	5·62	·1420
65			19 3·4	5·8453	488·55	222·07	10163	4622	5·89	·1488
66			19 3·4	5·8453	488·55	222·07	10113	4597	6·25	·1578
67			21 5·1	6·4946	653·45	247·01	9200	4182	8·39	·2119
68			21 5·1	6·4946	654·55	247·50	8608	3913	9·10	·2299
69			10 8·5	3·2473	356·19	161·90	29916	13598	3·21	·0811
70			10 8·5	3·2473	356·19	161·90	28709	13049	2·41	·0608
71			12 10·3	3·8969	427·22	194·19	24619	11190	3·21	·0811
72			12 10·3	3·8969	425·60	193·45	23654	10750	3·12	·0789
73			15 0	4·5464	496·08	225·49	21575	9807	4·10	·1036
74	8·57	·2165	15 0	4·5464	493·93	224·51	20894	9538	3·39	·0856
75			17 1·7	5·1959	564·96	266·80	18078	8217	6·53	·1398
76			17 1·7	5·1959	563·88	266·31	17163	7801	4·01	·1013
77			19 3·4	5·8453	639·21	290·55	14526	6603	4·82	·1217
78			19 3·4	5·8453	638·53	290·06	13801	6309	4·37	·1104
79			21 5·1	6·4946	712·34	323·79	12670	5759	6·96	·1758
80			21 5·1	6·4946	710·23	322·83	13128	5967	6·43	·1623

It has been observed, that the preceding Table may be considered as furnishing the most useful results, relative to the transverse strength of oak beams, of any hitherto made public; both as they regard practical precedent and theoretical data; but, with reference to the former, the engineer must bear well in mind the green state of the wood when the experiments were performed, which adds much to its strength, on account of the fibres in that state offering a much greater resistance to compression than when the timber has been well dried and seasoned.

We come now to more recent experiments.

69. A knowledge of the strength and elasticity of timber being subjects of the highest importance in the construction of ships, &c., the Surveyors of His Majesty's Navy have, at different times, ordered experiments to be made, directed to this object; and they have in the most handsome manner supplied me with every information they were in possession of, relative to those inquiries; a favour for which I am equally indebted to the liberal views of those gentlemen and to the friendly interference and recommendation of John Knowles, Esq., Secretary to that Board, through whom it was solicited.

The following Table contains the results of experiments carried on in the dockyard at Deptford, by Colonel Beaufoy, on English and Dantzic oak, Riga fir, and pitch pine. The several pieces

were each 5 feet long and 2 inches square, fixed at one end in a mortise to the length of 1 foot, so that the part projecting was 4 feet; and the weight was hung on at that distance from the fulcrum. The twenty-five pieces of Dantzic oak were cut from the same tree, of which the mean specific gravity was 854. The several pieces of Riga fir were also all from one tree, of which the mean specific gravity was 537; as were those of pitch pine, but the specific gravity is not stated. Of the English oak, the first six pieces were from one tree, of which the specific gravity was 922, and the other thirteen from another; the latter very irregular and cross-grained, but its weight is not given: nor do I find any indication of the particular weight of each piece, nor the situation it occupied with regard to its distance from the heart or centre. It is simply stated, that the last piece of oak was the heart of the tree, and that it was the weakest.

The deflections were measured in degrees and minutes, on a graduated arc of the same radius as the beam, viz. 4 feet, and were taken as every 14 lbs. were put on: we have given, however, only the mean, the last weights, and the corresponding deflections. It appears from all these experiments, that the deflections are very nearly in the ratio of the weights, till about one-half, or a little less than one-half the weight, is laid on, after which they become more rapid, and very irregular.

70. *Table of Experiments carried on in the Dockyard, Deptford, on Beams of different woods, fixed at one end: by Col. Beaufoy.*

No. of experiment.	Dantzic oak, 25 pieces, 4 ft. long, 2 in. square.	Deflection in degrees, &c.	Riga fir, 25 pieces, 4 ft. long, 2 inches square.	Deflection in degrees, &c.	Pitch pine, 24 pieces, 4 ft. long, 2 inches square.	Deflection in degrees, &c.	English oak, 19 pieces, 4 ft. long, 2 inches square.	Deflection in degrees, &c.
	lbs.	° ′	lbs.	° ′	lbs.	° ′	lbs.	° ′
1	98	2 3	98	1 24	98	1 15	98	1 15
	106	6 12	182		287	7 0	266	
2	98	2 6	98	1 21	98	1 18	98	1 14
	193	7 0	175	5 6	266	6 24	273	6 24
3	98	2 21	98	1 14	98	1 20	98	1 12
	175		182	4 42	280	5 36	224	
4	98	2 36	98	1 12	98	1 6	98	1 10
	168	7 12	182	5 48	257	4 50	284	7 0
5	98	2 48	98	1 23	98	1 8	98	1 19
	151	5 54	238	5 0	270	5 30	231	5 0
6	98	2 32	98	1 26	98	1 6	98	1 14
	161	6 12	168	3 0	274	6 0	273	6 50
7	98	2 24	98	1 36	98	1 0	98	1 17
	175	7 12	182	4 6	294	6 0	245	
8	98	2 9	98	1 25	98	1 6	98	1 16
	184	6 15	203	5 30	266	6 30	238	
9	98	1 54	98	1 20	98	1 20	98	1 20
	192	6 30	238	6 0	245	7 30	238	
10	98	1 46	98	1 16	98	1 24	98	1 32
	193	5 25	259	7 12	203	5 30	238	
11	98	1 58	98	1 26	98	1 10	98	1 47
	183		217	5 30	274	6 0	224	
12	98	1 51	98	1 39	98	1 8	98	1 26
	202	9 0	168	3 50	274	7 42	231	
13	98	2 24	98	1 15	98	1 0	98	1 32
	154	6 0	154	4 0	326	7 0	189	
14	98	2 4	98	1 26	98	1 15	98	1 20
	175		182	4 30	280	5 30	245	
15	98	2 23	98	1 21	98	1 10	98	1 28
	140	3 54	210	4 30	287	6 30	231	
16	98	2 37	98	1 14	98	1 15	98	1 44
	112	2 54	252	6 12	256	7 30	231	
17	98	2 1	98	1 20	98	2 12	98	1 34
	174	6 30	189	3 36	182	6 30	238	
18	98	1 57	98	1 30	98	1 12	98	1 30
	179	6 12	161	4 0	277	8 30	210	
19	98	1 40	98	1 38	98	0 54	98	1 46
	214		154	4 36	308	5 30	182	
20	98	1 54	98	1 31	98	1 2		
	133		238	5 0	301	7 30		
21	98	2 19	98	1 20	98	1 8		
	161	7 0	224	6 12	301	7 30	*1st Six,*	
22	98	1 57	98	1 18	98	1 12	Sum 1551	
	161	5 24	238	4 48	224	5 0	Mean 258 lbs.	
23	98	2 36	98	1 16	98	1 12		
	147	5 30	175	5 48	252	5 30	13 *following,*	
24	98	2 30	98	1 24	98	0 54	Sum 2740	
	133	4 24	245	3 0	336	7 0	Mean 211 lbs.	
25	98	3 18	98	1 16				
	130	5 48	231					

These experiments furnish the absolute and comparative strength of the four following woods, viz.:

Length 4 feet, 2 inches square.	Dantzic oak	167 lbs.	Sp. gr. 854
	Riga fir [13]	202 lbs.	Sp. gr. 537
	Pitch pine	272 lbs.	Sp. gr.
	English oak	258 lbs.	Sp. gr. 922
	Ditto	211 lbs.	

Other experiments were made by the same gentleman on battens of $2\frac{1}{4}$, $2\frac{1}{2}$, $2\frac{3}{4}$, and 3 inches square, and of the same length. The particulars are not stated; but it appeared from them, that the ratio of the strengths a little exceeded the ratio of the cubes of the sides.

71. Other experiments were also made upon pieces of the same dimensions, spliced and fixed in different ways: the *scarph* in all of them was 12 inches long and 13 inches from the end, viz. about an inch from the fulcrum. The results were as follow:

Scarph up and down	No. 1, broke in the splice	112 lbs.
	No. 2, ditto	109 lbs.
Scarph flatwise, large end uppermost, and towards the fulcrum	No. 1, nails drew through the small end of the scarph	104 lbs.
	No. 2, ditto	98 lbs.
Scarph flatwise, small end towards the fulcrum	No. 1, broke in the thick part of the scarph	84 lbs.
	No. 2, ditto	90 lbs.

[13] It may be proper to observe, that No. 13, in Col. Beaufoy's Report of the Riga fir, was very irregular, having been broken

From these experiments it is inferred, that the two former positions of spliced pieces are preferable to the last.

72. The following experiments were made under the same authority, by Messrs. Peake and Barrallier.

It is necessary, in order that the reader may properly understand the results contained in the fourth, fifth, and sixth columns of the following Table, to explain the nature of the apparatus by which these several pieces were submitted to experiment. An oak pillar, 12 inches square, had a hole of 2 inches square cut in it, for the purpose of receiving the end of the batten, the pillar itself being securely fixed, between the principal floor-joist and the tie-beam, in the mould-loft in Woolwich dockyard; and a semicircular piece of oak, of 6 inches radius, was well fixed to the principal pillar, to prevent the batten from crippling at its lower side. This semicircle was divided into inches and parts, and as the weights were successively applied, the batten was deflected, and in some measure bent over this arc; and the numbers in the columns above mentioned show to what extent this bending took place.

As to the numbers in the other columns, they will be readily understood, from the description given at

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with only 98 lbs.: this experiment is therefore rejected, and its place is supplied with experiment No. 26. It may also be further stated, that the above mean weights are obtained by dividing the sum of all the breaking weights by the number of them.

their heads in the Table;—the first showing the number of the experiments; the second, the number of years the pieces had been in store; the third, the specific gravity; the fourth and fifth, the part of the arc which came in contact with the batten, with 56 lbs. and 112 lbs. respectively; the sixth, the contact which remained after removing the last weight: the seventh column shows the whole curvature; the eighth, the weight under which the piece crippled; the ninth, the weight under which it broke; and the tenth contains sundry remarks.

73. *Table of Experiments on Riga Fir Battens, 2 inches square, fixed at one end, and the weight acting at 5 feet from the fulcrum.*

Note.—These pieces were all kept dry.

No. of experiment.	Years in store.	Specific gravity.	Arc received by the battens under the weight of 56 lbs.	Arc received by the battens under the weight of 112 lbs.	Arc remaining after the last weight was removed.	Total curvature.	Weight under which the beam crippled.	Weight under which it broke.	Remarks.
			[illegible]	inches.	inches.	in.	lbs.	lbs.	
1	[illegible]	474		7½	1	12	112	144	Part of a jib-boom.
2	6	[illegible]		6½	[illegible]	16	202	220	
3	13	474		7½	1	[illegible]	112	144	Same as No. 1.
4	13	513		5¾	0¼	13	167	[illegible]	Ditto.
5	6	768		12½	2½		112	112	Sound batten.
6	6	804		7	0¾		126	129	Ditto.
7	6	756		7	0½		[illegible]	127	Ditto.
8	6	696		7½	0½		133	141	Ditto.
9	6	720		7½	0¾		126	126	Broke short.
10	6	726		7¾	0½		137	137	Ditto.
11	6	756					77	77	Very shaky.
12	6	726	3¾	8½	0½		126	[illegible]	Sound.
13	6	720	2¾	5¾	0½		127	[illegible]	Ditto.
14	6	[illegible]	[illegible]	5¾	0¾		147	147	Ditto.
15	6	708	2½	6	0¾		147	147	Ditto.
16	6	522	3¼	7¾	0¾		133	135	Very dry.
17	[illegible]	558	3¾	7¾	0¾		133	133	Broke short.
18	[illegible]	[illegible]	3½	6¾	0½		[illegible]	149	Ditto.
19	10	522	3	5¾	0½		140	140	Fine texture.
20	8	546	3¼	6¾			133	[illegible]	Ditto.
21	8	558	2¼	5¾	0¾	13½	146	147	Broke short.
22	3	828	3¼	6¾	0¾		161	161	Ditto.
23	6	[illegible]	3¾	6½	1	[illegible]	202	[illegible]	Same as No. 2.
24	6	705	[illegible]	6¾	0½	13¾	168	182	Broke suddenly.
25	[illegible]	[illegible]	[illegible]	7½	1	12	112	[illegible]	
26	10	[illegible]	3½	5½	0½	13¾	167	194	
27	8	546	3¾	5¾	0	15	[illegible]	202	
[illegible]	8	561	2½	[illegible]	0¾	9¾	[illegible]	[illegible]	
		27)17856 Mean, 633	Sum, rejecting No. 11.					27)4132 Mean, 153	

The preceding Table, by Colonel Beaufoy, reckoning the strength to be inversely as the length, gives 5 : 4 : : 202 : 161 lbs. for the mean; which is in defect only 1 lb.; the mean of the former being 162 lbs. at 4 feet.

TABLE—(CONTINUED).

Experiments in every respect similar to the last, except that the several pieces were kept wet.

No. of experiment.	Years in store.	Specific gravity.	Are received by the battens under the weight of 56 lbs.	112 lbs.	Are remaining after the last weight was removed.	Total curvature.	Weight under which the beam crippled.	Weight under which it broke.	REMARKS.
			inches.	inches.	inches.	inches.	lbs.	lbs.	
1	29	639	3	5	0	12⅛	193	207	
2	6	615	2¾	4⅛	0⅜	13⅛	248	261	
3	13	534	3¼	6	0¼	14	126	158	
4	13	555	2¾	5	0	15	153	208	
5	29	639	3	5	0	12⅛	193	207	
6	6	876	2¾	5⅛	1⅛		136	136	Very shaky.
7	6	666	2⅛	4⅛	0⅛		140	199	Sound.
8	6	666	2⅛	4⅛	0⅛		158	190	Ditto.
9	6	696	2⅛	4¼	0⅛		154	172	Ditto.
10	6	762	2⅜	4⅜	0$\frac{1}{16}$		168	180	Ditto.
11	6	690	2⅛	4¾	0$\frac{1}{10}$		168	168	Little shaky.
12	6	720	2	3⅝	0⅛		168	203	Very sound.
13	6	690	2	3¾	0⅛		176	186	Sound.
14	6	708	2⅝	4⅛	0⅛		128	128	Very cross-grained.
15	6	726	2⅛	3¾	0⅛		209	214	Sound.
16	6	702	2⅛	4	0¼		214	214	Ditto.
17	10	606	4¼	10¼	0⅛		133	133	Very shaky and cross-grained.
18	10	720	3¼	12	1		112	112	These broke very slowly.
19	10	642	2⅝	5⅜	0¼		159	159	These broke very slowly.
20	10	666	2⅞	6⅜	0⅜		123	132	Shaky.
21	10	540	3⅛	8¼	0¾		117	117	Coarse-grained.
22	10	528	3⅝	9⅛	0¾		132	132	Cross-grained.
23	8	648	4⅛	12			112	112	Coarse-grained.
24	8	552	3¼	6⅞	0⅝		151	153	
25	29	738	2¼	4¾	0¼	13	146	160	
26	10	684	3⅛	10⅜			112	135	
27	10	684		12⅛	1¼			137	
28	6	615	2¾	4⅛	0⅜	13⅛	248	261	
29	10	492	4¾	9	1⅛	13⅛	128	135	Very shaky.
30	6	594	2¾	4¾	0⅛	12⅛	224	233	
31	8	564	4¼	7⅛	1	13⅛	140	161	
32	13	534	3¼	6	0¼	14	126	158	Coarse soft grain.
33	13	495	4¾	11⅛	1⅛	16⅛	112	149	Shaky and knotty.
34	29	639	3	5	0	12⅛	197	207	
35	8	600	3¼	5¾	0¼	14	168	199	
36	13	510	3⅛	5	0⅛	15	147	147	
37	13	555	2¾	5		15	153	208	

Sum, 37)23390

Mean, 632 wet.
Mean, 633 dry.
Mean, 537 of both.

3)1802

Mean, 600

Sum, 37)6371

Mean, 172 wet.
Mean, 153 dry.
Mean, 161 Colonel Beaufoy.

3)486

Mean of the three, 162

TABLE—(CONTINUED).

Containing similar Experiments on Battens of the same Dimensions, of different kinds of Wood.

No. of experiment.	Years in store.	Specific gravity.	Arc received by the battens under the weight of 56 lbs.	Arc received by the battens under the weight of 112 lbs.	Arc remaining after the last weight was removed.	Total curvature.	Weight under which the beam crippled.	Weight under which it broke.	REMARKS.
			VIRGINIA YELLOW PINE.						
			inches.	inches.	inches.	inches.	lbs.	lbs.	
1	Time unknown.	564	4¾	..	..	10	98	98	Dry and defective.
2	do.	720	2⅝	4⅝	0⅝	16½	246	251	Ditto.
3	do.	498	6	..	..	15¾	233	233	Ditto.
4	do.	618	4¾	3⅛	0⅝	26¾	206	234	Ditto.
5	do.	498	3¼	6⅝	0⅝	..	126	135	Part of old topmast.
6	do.	522	3⅞	8⅝	0⅞	11⅜	133	133	Dry.
7	do.	492	3⅛	6⅛	0⅝	..	140	147	Ditto.
			PITCH PINE.						
8	do.	816	2	3¾	0¼	9¼	196	203	Dry.
9	do.	816	1¼	2⅝	0⅛	..	336	365	Ditto.
10	do.	996	2⅛	3⅝	0¼	12⅛	238	244	From Lukin's kiln.
11	do.	738	2	4	0⅛	12⅛	224	332	Dry.
12	do.	732	2	3¼	0⅛	11¾	308	308	Ditto.
13	do.	696	2¼	3⅞	0⅛	14	287	303	Ditto.
14	do.	708	2⅛	4⅛	0⅛	17	273	293	Ditto.
15	do.	720	2¼	4¼	0⅛	..	140	..	Defective.
			CANADIAN WHITE PINE.						
16	1	648	4¼	..	..	14	98	123	Wet.
17	10	672	4⅝	..	..	14	98	119	Ditto.
18	8	714	4	..	..	14	84	103	Ditto.
19	8	660	5¾	..	..	14	84	108	Ditto.
20	4	720	3¾	..	..	14	84	91	Ditto.
21	4	714	3¾	..	..	10⅛	84	96	Ditto.
22	8	618	3⅝	10⅝	1⅝	18¼	116	122	Dry.
			LARCH.						
23	4	526	7⅝	16⅛	4	34	..	170	Dry.
24	4	540	3⅛	7⅛	0⅞	14⅞	133	133	Ditto.
25	4	570	5⅛	10⅛	1	15	..	137	Ditto.
26	4	526	3⅝	6⅝	0¾	16¼	160	162	Ditto.
			DANTZIC FIR.						
27	4	690	2⅜	4¾	0⅛	..	158	158	Wet.
28	4	648	2⅛	4¾	0¼	12⅛	140	140	Ditto.
29	4	630	2⅛	4¾	0⅛	12⅛	140	140	Ditto.
30	3	624	3¾	6¼	0¾	11¾	186	192	Ditto.
			ASH.						
31	1	858	2	4⅛	0⅝	16	224	239	Quite green.
32	1	828	2⅛	4⅞	3⅝	18⅛	..	217	Ditto.
33	..	660	3⅞	6⅛	0⅛	12¾	..	196	Old capstan bar.
			TEAK.						
34	2	672	2⅝	4¾	..	16⅛	224	271	Old bowsprit.
35	2	606	2¾	4⅛	..	12½	224	257	Old bowsprit.

74. The preceding Table furnishes the following means, viz.:—each bar being

5 feet long, and 2 inches square.	Riga { Dry 153 / Wet 172 }	162 mean	sp. gr. 633
	Virginia yellow pine	189	558
	Pitch pine	256	777
	Canadian white pine	109	678
	Larch	150	540
	Dantzic ditto	156	648
	Ash	217	782
	Teak	264	639

It may be remarked, that the strength of pitch pine, according to these experiments, exceeds very considerably what was found by Colonel Beaufoy; while that of the Riga fir, taking a mean between the wet and dry, is exactly the same in both: but it is to be observed, that in the experiments by Messrs. Peake and Barrallier, the bending of the pieces over the arc, as above described, shortens the ultimate radius; and therefore they ought to be stronger than with the unifom radius of 5 feet: consequently the specimens of Riga fir in these experiments were really weaker than those of Colonel Beaufoy, although they apparently agree with each other.

Experiments on Triangular Oak Beams, &c., by Mr. Couch.

75. In a preceding chapter, we have given the detail of several valuable experiments by Mr.

Couch, of Plymouth Dockyard; and the two following Tables are due to the same gentleman. They exhibit the detail and results of experiments on the lateral or transverse strength of triangular prisms of Canadian oak, the sections of which were equilateral triangles, the sides being 3 inches; and also on some pieces reduced to the form of trapezoids, by cutting off the vertex, or upper angle, to one-third of the depth.

The short pieces, viz. those 3 feet 3 inches, Table I., were fixed by one end horizontally in a 3-inch mortise; the others, as given in Table II., which were 6 feet 6 inches, were fixed at each end into 3-inch mortises, so as to prevent the ends from rising; and in both cases they were so well fitted as to require slight blows of the mallet to drive them in.

These experiments were made in order to obtain data connected with mast-making, and to ascertain how far the commonly received notion was correct —namely, that if the vertex, or upper edge of a triangular prismatic beam, be cut off to one-third of the depth, the pieces will be stronger than before; or, in other words, that a part opposes more resistance than the whole;—which assertion, as anticipated, was satisfactorily contradicted by the following results.

These experiments are also very conclusive on another point, viz. that the strength of triangular prisms does not follow the law laid down either by

Leibnitz or Galileo; for, according to the former, the weights required to break a beam of this kind, with its base upwards, ought to be three times greater than in the reverse position; and according to the latter, it ought to be double. Now, the mean of the first seven experiments is 306, and of the next four 348; which is very far from the weight required in either of the above theories.

TABLE I.

76. *Experiments on Triangular Oak Beams, by Mr. B. Couch. Pieces 3 feet 3 inches long, fixed by one end horizontally into a pillar; 3 feet beyond the prop.*

Weight placed on the end.

Order of the experiments.	Position, form, &c.	Deflection below the first position.	Weight in lbs. supported.	Weight of the pieces.	The same pieces placed end for end, after altering their position or form.	Deflection below the first position.	Weight in lbs. supported.	Weight of the pieces.
				lbs. oz.				lbs. oz.
1	Angle upward.	9	290	3 1				
2		9	313	3 3½				
3		9	290	3 3	Reduced to trapezoids, narrow end upward.	9	261	2 13
4		9	333	3 6		9	271	2 15½
5		9	309	3 6		11	248	2 15½
6		9	308	3 5				
7		10	298	3 4		9	270	2 15
8	Angle downward.	16	332	3 10				
9		11	349	3 7	Angle upward.	9	286	3 7
10		11	351	3 3				
11		11	360	3 4				
12	Trapezoid, narrow end up.	8	283	3 4				
13		11	285	3 1½				

Sum of the first seven weights = 2141
Sum of the next four = 1392

7)2141   4)1392

306 mean.   348 mean.

Sum of the six trapezoids = 1618

6)1618

269 mean.

TABLE II.

77. *Experiments, by Mr. Couch, on pieces 6 feet 6 inches long, each end fixed into pillars horizontally; 6 feet between the props.*

Weights placed on the middle.

Order of the experiments.	Position, form, &c.	Deflection below the first position.	Weight in lbs. supported.	Weight of the pieces. lbs.	Weight of the pieces. oz.	Remarks.
1	Angle upward.	6	980	7	5	
2	Angle upward.	6	896	6	9	
3	Angle upward.	6	1008	7	3	
4	Angle upward.	5	1116	6	14	
5	Angle upward.	6	1288	6	15	
6	Angle downward.	3	1056	6	14	Fractured ⅜ inch on the angle.
		4	1166			Ditto 1 inch on the angle.
		7	1257			Broke nearly off.
7	Angle downward.	2	870	7	2	Sprung a little on the angle.
		3	947			Broke nearly off.
8	Angle downward.	3	1003	7	3	Sprung ½ inch on angle, and continued breaking with the addition of every ¼ cwt., fibre after fibre, ⅜ inch at a time, till all gave way.
		5	1366			
9	Angle downward.	2¼	1285	7	14	Sprung without giving warning, from angle to half the depth.
10	Angle downward.	2¼	1395	9	2	Sprung ½ an inch on angle.
		3½	1686			
11	Trapezoid, narrow face upward.	6	1319	8	6	Coarse, strong grain.
12	Trapezoid, narrow face upward.	7	1099	6	0	Fine, weak grain.

78. The following Table exhibits the detail and results of experiments carried on also by Mr. Couch, on the lateral or transverse strength of Riga, Norway, and Halifax spars; as also on pieces of timber wrought to the shape of the said spars, (viz. frustrums of cones,) converted from large logs of red pine, yellow pine, and oak, all the growth of Canada.

The spars and other pieces were all of the same dimensions, viz. 27 feet long, 3¼ inches diameter at the butt, and to the distance of 5 feet from the butt: the upper end was 1¼ inch in diameter.

They were fixed by the greater end horizontally into a mortise, the prop or fulcrum being 5 feet from the butt; and the weights were placed 1 foot from the smaller end, leaving a lever or purchase of 21 feet.

TABLE III.

79. *Experiments on Riga, Norway, and Halifax Spars, Red and Yellow Pine, &c., by Mr. Couch.*

Order of experiments.	Species of wood.	Weight of each spar.	Deflection.	Weights which broke them.	Remarks.
		lbs.	feet.		
1	Riga spar	29¼	11	130	
2	Riga spar	29⅛	11	137	Upset or compressed,
		..	12¾	144	very much broken.
3	Norway spar ..	32	12	168	Upset, lower part.
		..	13¾	172	
4	Norway spar ..	36⅛	11	180	Upset, very much.
		..	12¾	206	
5	Halifax spar	37¾	11¼	115	
6	Halifax spar	34⅛	12¾	188	The tension of the fibres of this spar was much increased by being placed near a large fire for several days.
7	Red pine timber ..	40¼	14	150	
8	Red pine timber ..	42¾	14	180	
9	Red pine timber ..	42¼	14	202	
10	Yellow pine timber	33¼	rapid deflection	56	Fibres undulated.
11	Yellow pine timber	32	14	146	
12	Yellow pine timber	33⅛	rapid deflection	56	Fibres undulated.
13	Oak timber......	52⅛	16	231	
14	Oak timber......	53½	18	254	

The experiments which have been now detailed relative to the transverse strains, are, it is presumed, all that are historically deserving of any particular notice in this place; we shall, therefore, now proceed to describe the experiments from which the data given in a subsequent part of this work have been obtained.

Experiments made at the Royal Military Academy.

80. The foregoing were the principal experiments which had been made on the strength of timber, when I undertook to enter upon an investigation of this subject. They each furnished certain results; but there was no attempt at generalizing and connecting one set of results with another, by certain rules. Some rules, indeed, were to be found in different authors; but they differed in most cases the one from the other, not only numerically, but in principle. My object, therefore, has been to endeavour to examine these points of difference by independent and distinct experiments, and ultimately to furnish such practical rules as might be had recourse to by practical men.

An Explanation of the Method of making the Experiments on the Transverse Stress and Strength of Battens of different Woods, with a Description of the Apparatus, &c.

81. These experiments may be divided into four classes, viz. 1st, When the battens were supported on two props, as shown in Plate IV. 2dly, When they were fixed horizontally, with one end in a wall, as in fig. 3, Plate V. 3dly, When they were fixed at any given angle, as shown in figs. 1 and 2,

Plate V.; and, lastly, When both ends were firmly fixed, as in fig. 4 of the same Plate.

Plate IV. represents an experiment on a fir batten, A B, 7 feet in length and 2 inches square, resting on the two props C D, E F, 6 feet asunder: the two weights P P are 1 lb. each, and were used to keep the fine silk line, to which they were attached, stretched in a horizontal position between the props; to facilitate which, the line was made to pass over two small brass rollers, one of which is shown at G. By means of this line, and the several small scales, *s s s*, &c., each divided into 10ths of inches, the deflection of the batten might be observed with great accuracy; and in this manner those given in the detail of the experiments were taken.

The number of these scales was varied at pleasure; commonly there was only one in the centre; at other times we had from three to ten, or even more; and in some few cases a board was placed against the batten, and the curve traced upon it with a pencil.

The small ivory scale at H was intended to measure the successive lengthening or stretching of the lower fibres, and was thus adjusted:

A fine silk line was fixed at the end A of the batten A B, and brought under the whole length of A to B: the scale, which had two fine steel points attached to it, was fixed by them into the under side of the batten, as shown at H: at the top of the scale was a small brass wheel or roller, over which

the silk passed, and to the end of this was hung a small semicylindrical brass weight, with its flat side towards the scale: two fine grooves were also cut, one in each of the brass plates, with which the tops of the props C D, E F, were defended, in order to allow the silk line to pass freely in them under the piece.

The batten thus furnished was now rested on the two props, with the line placed so as to pass in the two grooves above mentioned; and by means of a screw, by which the line was attached to the piece at A, the weight at H was adjusted to 0, on the same scale, which was divided from 0 upwards into 40ths of inches.

It is obvious now, that after the weights begin to give the batten any deflection, the small weight at H will be raised along the scale by a quantity exactly equal to the difference between the original length of the bottom fibres and the length to which they are stretched at the time of making the observation; and in this manner the stretching of the fibres at several different degrees of deflection was measured in a few experiments; but as it did not appear that any useful application of this datum could be made in the theory, and as it required a longer time to adjust, &c., it was employed, comparatively, but in a few cases.

It would be useless to enter more minutely into an explanation of these experiments, as the process will be obvious from an inspection of the Plate: it

will be therefore sufficient merely to observe further, that the artist has chosen to represent the apparatus as if the experiments were performed in the open air; and the consequence is, that the props do not appear sufficiently steady: they were, however, performed under cover, on a substantial floor; and the trestles or props were made to slide in grooves, and were firmly fixed in them, so as to render the whole perfectly secure and steady: and to prevent any momentum in loading the scale, this was always made stationary by wedges, when the larger weights were introduced.

82. In order to make the experiments on those pieces which were fixed by one end in a wall, the following means were employed. A block of hard wood, A B C D, fig. 3, Plate V., about 18 inches long and 12 inches in breadth and depth, was cut through at about 5 inches from each end, as at *a b c d*, for the convenience of forming a hole 2 inches in breadth and depth, or rather more; the one with the side of the square vertical, and the other with the diagonal vertical, as shown in the figure. The parts of the block were then screw-bolted together; and an iron socket, exactly 2 inches square on the inside, was made to fit these holes very accurately, but so that it might be taken out and put in at pleasure: a hole was then cut out of a very heavy solid wall, a little larger than the block, into which the latter was fixed by means of

inverted wedges, whereby the whole was rendered perfectly firm and immoveable.

The pieces of timber on which the experiments were made were 2 inches square, and therefore fitted tight into the iron sockets above mentioned, the edges of which are shown in the figure; the under side being made slightly curving, to prevent the cutting of the lower face of the piece after the weight was hung on: and as the deflection would have rendered the scale liable to slip off, an iron plate, with two studs riveted to it, was screwed on the end of the batten, as shown at E and F, the former being bent into a right angle to fit its upper edge.

In the same manner the blocks of figs. 1 and 2 were made and fixed, differing from the former in nothing except the hole being made to form an angle of 26° with the horizon; the first ascending, and the other descending.

Those of fig. 4 were precisely the same as the lower part of fig. 3, and were fixed into two walls exactly 6 feet asunder.

Every thing being thus adjusted, the scale was hung on, as shown in Plate IV., but which, for simplicity, is merely represented, in Plate V., by a single ball W.

83. It may not be amiss to add, that the walls in which the blocks were fixed were not less than 40 feet high, although in the Plate they are repre-

sented as if they were not above 6 feet; it being thought useless to show them in their full height.

Such were the means employed for assuring accuracy in the results, and which it has been thought right to explain at length, in order that the reader may judge of the degree of confidence to which these experiments are entitled. This has been commonly omitted by preceding authors, and has been the subject of just complaint by those who would have wished to avail themselves of their data for the purpose of theoretical investigation; so that in cases where a disagreement was found to have place between the theoretical and practical results, it was always doubtful to which the error belonged, and was therefore attributed to either, as best suited the views of the writer.

The following are the results of the different experiments made on the transverse strain, arranged according to the dimensions of the battens.

TABLE I.

84. *Experiments on Fir Battens, supported at each end.*

No. of experiments.	Length in inches.	Depth in inches.	Breadth in inches.	Specific gravity.	Weight in lbs.	Weight reduced to sp. gr. 600.	Mean wt. corresponding to sp. gr. 600.
1				504	360	428	
2				533	388	436	
3				564	418	444	
4	15	1	1	646	453	421	439
5				588	453	462	
6				600	441	441	
7				552	318	346	
8				647	364	338	
9				724	436	371	
10	18	1	1	719	404	337	342
11				648	353	327	
12				672	376	336	

The above experiments were made principally in order to determine what relation there might be between the ultimate strength and the specific gravity of the rods: they were therefore selected out of those which had been the same time in store, and that differed the most from each other in their specific gravity, and principally from the fragments of those that had been broken in preceding experiments, of which the detail is given in the subsequent pages.

The reduced weight in the seventh column above is found on a supposition that the strength is as the specific gravity; a reduction which is adopted throughout.

We can see no physical reason for the circumstance of the strength being so nearly proportional to the specific gravity. It ought rather, one would have supposed, to have been as the $\frac{2}{3}$rd power; for, supposing the number of particles to be as the specific gravity, the number of them in any section would be as the $\frac{2}{3}$rd power of the latter. Upon the whole, however, the simple ratio of the strengths being as the specific gravities seems to answer better than any other.

TABLE—(CONTINUED).

85. *Experiments on Fir Battens, supported at each end.*

No. of experiments.	Length in inches.	Depth in inches.	Breadth in inches.	Deflection.	Specific gravity.	Weight in lbs.	Weight reduced to sp. gr. 600.	Mean Weight sp. gr. 600.
1	24	1	1	1·25	..	270	..	265
2				1·25	..	262	..	
3				1·25	..	262	..	
4				..	560	261	279	288
5				..	560	283	303	
6				..	540	256	284	
7	30	1	1	1·80	..	242	..	237
8				1·80	..	234	..	
9				1·80	..	235	..	
10	36	1	1	1·85	577	229	237	196
11				3·12	505	162	192	
12				3·00	505	148	160	
13				2·2	553	181	196	
14				3·2	553	181	196	
15				2·2	553	181	196	

The specific gravities of Nos. 1, 2, and 3 were not observed, nor the deflections of 3, 4, and 5. The deflections of 1, 2, and 3 were all the same, viz. for 220 lbs. $\frac{11}{16}$ inch; for 250 lbs. one inch; for 260 lbs. $1\frac{1}{4}$ inch.

The specific gravities of Nos. 7, 8, and 9, not observed; these, with Nos. 1, 2, and 3, were broken before it was thought necessary to introduce that consideration.

Nos. 11 and 12 were both off a very light plank, and were very elastic.

Nos. 13, 14, and 15 were very uniform rods. Nos. 13 and 15 were each bound to two pieces of the same thickness as themselves, each piece occupying half the whole length, to prevent any curving; while No. 14 was broken as usual. It seems, therefore, that the curving of the batten does not weaken it, although it increases the deflection.

TABLE—(CONTINUED).

86. *Experiments on Fir Battens, supported at each end.*

No. of experiments.	Length in inches.	Depth in inches.	Breadth in inches.	Deflection.	Specific gravity.	Weight in lbs.	Reduced to sp. gr. 600.	Mean Weight sp. gr. 600.
1	24	$1\frac{1}{4}$	$\frac{3}{4}$	..	646	420	390	397
2				..	646	424	393	
3				..	646	441	409	
4				·70	746	557	448	435
5				·70	709	501	424	
6				·70	734	531	434	
7	30	$1\frac{1}{4}$	$\frac{3}{4}$	1·12	733	412	337	336
8				1·12	733	411	336	
9				..	646	360	334	

No. 1 was a very complete fracture, showing very distinctly the part of the section which had been compressed, and that which had acted by tension; the latter rather exceeded $\frac{1}{3}$rd of the whole depth. In Nos. 2 and 3 the same appearance might be observed, but not so perfectly. No. 3 hung two hours and a half before breaking; the others only ten minutes.

Nos. 4, 5, and 6 were remarkably sound pitch pine, full of turpentine. No. 5 would probably have borne as much as No. 4 or No. 6, but that the upper part, on which the weight hung, was more tender, and was much crippled in the experiment.

Nos. 7 and 8 were part of the same plank as Nos. 4, 5, and 6; and No. 9 was part of the specimen from which Nos. 1, 2, and 3 were made.

It appears from the first of the above set of experiments, that the strength is in a higher ratio than that of the specific gravities.

TABLE—(CONTINUED).

87. *Experiments on Fir Battens, supported at each end.*

No. of experiments.	Length in inches.	Depth in inches.	Breadth in inches.	Deflection.	Specific gravity.	Weight in lbs.	Reduced sp. gr. 600.	Mean weight sp. gr. 600.
1	24	2	1	·625	613	1190	1164	1119
2				..	563	1000	1066	
3				..	600	1128	1128	
4	30	2	1	..	586	882	903	900
5				..	581	871	901	
6				1·08	571	852	895	
7	36	2	1	1·00	..	600	..	600
8				1·12	..	622	..	
9				1·12	..	680	..	
10				1·12	..	595	..	
11				1·52	..	552	..	
12				1·50	..	550	..	
13	36	2	1	1·12	606	722	715	745
14				1·12	606	752	744	
15				1·12	564	730	776	

No. 1 was left for twenty-four hours, with 865 lbs. hanging upon it, without any deflection beyond what it had acquired in a few minutes.

The successive deflections of No. 6 were

520 lbs. = $\frac{5}{16}$ inch, 620 lbs. = $\frac{6}{16}$, 720 lbs. = $\frac{11}{16}$.

Nos. 7, 8, and 12 were broken before it was thought necessary to introduce the specific gravities; they were lighter and weaker wood than the preceding; and Nos. 5 and 6 were obviously damaged, by being exposed to wet.

The successive deflections and stretching of Nos. 13 and 14 were as follows; viz.

220 lbs. deflection $\frac{2}{8}$ stretching 0
420 .... $\frac{4}{8}$ .... $\frac{1}{30}$
520 .... $\frac{5}{8}$ .... $\frac{3}{40}$
580 .... $\frac{6}{8}$ .... $\frac{1}{10}$

TABLE—(CONTINUED).

88. *Experiments on Fir Battens, supported at each end.*

No. of experiments.	Length in inches.	Depth in inches.	Breadth in inches.	Specific gravity.	Weight in lbs.	Successive Deflections.	Mean weight reduced to sp. gr. 600.
					421	·175 ·266 ·300	
					848	·350 ·566 ·660	
					1054	·450 ·700 ·900	
1	44	2	2	630	1166	·530 ·900 1·025	1255
					1211	·600 1·00 1·15	
					1226	·650 1·10 1·30	
					1288	·900 1·57 1·95	
					1317	.. .. 2·35	
					421	·175 ·275 ·350	
2	44	2	2	..	848	·366 ·633 ·763	
					1054	.. .. 2·00	
					421	·15 ·25 ·33 ·36	
					711	·27 ·47 ·60 ·66	
3	48	2	2	601	920	·40 ·60 ·90 1·02	1116
					1020	·53 ·90 1·23 1·4	
					1125	.. .. .. 2·3	
4	48	2	2	601	1110	The same deflection.	

The deflections in the above experiments were measured by scales fixed on the pieces at equal distances, from one end to the middle, as explained in Art. 81.

It was remarked, in the experiment No. 1, that the deflection of the piece was very sensibly affected, after 1240 lbs. were on, by the addition and subtraction of a 7 lb. weight.

No. 2 was part of the same plank as No. 1, and only parted from it by the saw, although it was so much weaker; it was sappy and light, but the account of its specific gravity was lost, or not taken.

In Nos. 3 and 4 seven scales were used, placed at equal distances, viz. one at every six inches. The deflections are only given above from the middle to one end.

TABLE—(CONTINUED).

89. *Experiments on Fir Battens, supported at each end.*

No. of experiments.	Length in inches.	Depth in inches.	Breadth in inches.	Specific gravity.	Weight in lbs.	Successive Deflections and Lengthening. Deflections.	Length.	Mean weight reduced to sp. gr. 600.
1	60	..	2	..	788			
2	60	2	2	..	421 521 711 811	·33 ·56 ·75 ·40 ·70 ·96 ·73 1·30 1·80 ·93 1·70 2·37	·087 ·125 ·162	770
3	60	2	2	..	711	not observed		
4	72	2	2	563	221 421 521 621 682	·35 ·60 ·75 ·70 1·2 1·45 ·90 1·55 1·87 1·30 2·30 2·80 .. .. 4·30	·062 ·125 ·150 ·187 ·200	744
5	72	2	2	600	221 421 521 621 760	·30 ·53 ·65 ·60 1·03 1·20 ·76 1·33 1·50 1·00 1·70 2·00 .. .. 3·50	·075 ·162 ·187 ·225 ·350	

But little dependence can be placed upon the experiments Nos. 1, 2, and 3. No. 1 was part of a weak plank; and Nos. 2 and 3 were cut from one piece, which was at first 8 feet 6 inches: after breaking it at 5 feet, the remnant, which was then 6 feet, was broken again at 5 feet, breaking with the weight stated in No. 3: the latter part was nearest the root end. The specific gravities were not taken.

Nothing particular was noticed in experiments 4 and 5. The lengthening of the piece was measured by means of the instrument described Art. 81. And, in order to protect the battens against the splintering which commonly happened in the preceding experiments, they were bound round with twine on each side of the place of fracture, leaving about two inches clear in the middle.

Observations relative to the preceding Experiments.

90. It is proper here to observe, that the preceding results must not be considered as furnishing any data that are applicable to fir in general; for as the object was principally to ascertain the relation which exists between the strength and the dimensions of the pieces, the greatest care was taken in selecting the best and most perfect specimens of the kind that could be procured: several of the planks had been in store for a considerable time, and were perfectly seasoned, which accounts for their specific gravities being less than is usually found for Riga fir and Christiana deals, of which the specimens principally consisted. By this means a greater uniformity was found in the results, and a greater strength than is generally due to this kind of wood; but the results were obviously so much the better adapted for eliciting a correct idea of the nature of the straining and resisting forces. The medium strength of Riga fir will be found in the general Table of Data.

TABLE II.

91. *Miscellaneous Experiments on Fir Beams, cross-cut in the centre, and supported at each end.*

No. of experiments.	Length in inches.	Depth in inches.	Breadth in inches.	Specific gravity.	Weight in lbs.	Deflection.	Mean weight reduced to sp. gr. 600.
1	30	2	1	581	808	1·00	
					220	·250	
					420	·440	
2	30	2	1	581	520	·500	856
					620	·625	
					780	·750	
					846	·875	
3	30	2	1	580	835	·875	Same deflections as No. 2.

The preceding experiments having shown pretty clearly the situation of the neutral axis; viz. that it was at about $\frac{3}{8}$ths of the depth of the section from the bottom; these bars, which were part of the same specimens as those of the same dimensions (Art. 87), were cut down $1\frac{1}{4}$ inch, or $\frac{5}{8}$ths of the depth, and the saw-groove filled up by a thin slip of pear-tree, sufficiently tight to preserve the stiffness of the battens, but without straining them. They were then loaded as usual, and were broken with the weights above stated.

On examining the wedges, or slips of pear-tree, after the experiments, it was found that No. 1 was a little longer than No. 3, and No. 3 than No. 2; and the wedge of another batten, that broke with a considerable less weight, was $\frac{1}{16}$th of an inch longer than any of them. The impression of the fibres was very distinctly marked on the wedges; strongest at top, and gradually weakening towards the bottom, where they could scarcely be distinguished.

These experiments seem to indicate that the neutral axis was very nearly at $\frac{5}{8}$ths of the depth of the batten. The deflection of No. 1 exceeded that of Nos. 2 and 3 by $\frac{1}{8}$th throughout.

TABLE II.—(CONTINUED.)

92. *Miscellaneous Experiments on Fir Battens, grooved out in the centre, and supported at each end.*

No. of experiments.	Length in inches.	Depth in inches.	Breadth in inches.	Specific gravity.	Weight in lbs.	De-flection.	REMARKS.
1	36	2	1½	564	421 711 1095	·25 ·43 1·0	Whole beam.
2	36	2	1½	564	421 711 985	·300 ·566 1·10	Groove downwards ⅓rd in. deep, and ½ in. broad.
3	36	2	1½	538	421 621 780	·366 ·630 1·50	Groove upw^{ds}. ⅓rd inch deep, & ½ in. broad.

These weights, reduced to specific gravity 600, gave No. 1, 1164; No. 2, 1047; No. 3, 870.

The experiments in the preceding page having nearly pointed out the position of the neutral axis, these experiments were made with a particular view. Nos. 2 and 3 were grooved out, in the centre of their breadth, from end to end; the former to ⅓rd of the depth, and the latter to ⅔rds, and each ½ an inch broad; viz. ⅓rd of the breadth. The idea was, that what No. 2 broke short of the weight required in the whole batten, would be the measure of ⅓rd of the tension; and what No. 3 broke short of the same, would be the measure of ⅓rd of the compression. This view of the subject was afterwards found to be erroneous; but the experiments were retained, on a supposition that they might still form some standard of comparison.

TABLE II.—(CONTINUED.)

93. *Miscellaneous Experiments on Triangular Fir Battens.*

No. of experiments.	Length in inches.	Depth in inches.	Breadth in inches.	Specific gravity.	Weight in lbs.	Position of the battens.	Mean weight reduced to sp. gr. 600.
1	24	$\frac{1}{2}\sqrt{2}$	$\sqrt{2}$	..	118	Base upwards.	
2				..	97	Do. downwards.	
3	24	$\sqrt{3}$	2	613	740	Base upwards.	740
4				588	740	Do. do.	
5				559	680	Do. do.	720
6				574	680	Do. do.	
7	24	$\sqrt{3}$	2	619	637	Base downwards.	626
8				603	637	Do. do.	
9	20	$\sqrt{3}$	2	..	907	Base upwards.	
10				630	843	Do. downwards.	

These pieces were made out of the fragments of the 2-inch square battens; viz.

3 and 8 out of No. 3, art. 88.
4 and 7 out of No. 4, art. 88.
5 and 6 out of No. 4, art. 89.
9 out of No. 2, art. 88.
10 out of No. 1, art. 88.

All these pieces, except Nos. 5 and 6, were rested in triangular saddles of hard wood, cut very exactly to the angle of the batten, when they were broken with their edge down; but when the edge was upward, a similar one was placed on the centre, in order that the weight might not break down its edge. This latter saddle was about half an inch thick.

Nos. 5 and 6 had pieces glued and screwed on at their ends, in order to render their bearings solid; but it did not appear to make any difference: they were weaker than Nos. 3 and 4; but the piece from which they were made, viz. No. 4, Art. 89, was itself comparatively weak, as appears by that experiment.

TABLE II.—(CONTINUED.)

94. *Experiments on Fir Battens, fixed at each end.*

No. of experiments.	Length in inches.	Depth in inches.	Breadth in inches.	Specific gravity.	Deflection.	Weight in lbs.	Reduced to sp. gr. 600.	Remarks.
1	72	2	2	581	·45 1·00 1·30 2·1	220 620 822 1024	1058	The whole time of the experiment 34 min.; after last weight 6 m.
2	72	2	2	581	·41 ·95 1·25 2·1	220 620 822 1139	1174	Whole time 28 m.
3	72	2	2	611	·40 ·87 1·35 2·2	220 620 822 1090	1070	Whole time 45 m.
4	72	2	2	600	·45 1·00 1·30 2·3	220 620 822 1120	1120	Whole time 18 m.
					Mean weight		1105	

Nothing remarkable occurred in making these experiments. We have before (Art. 82) explained the methods that were employed in order to insure a permanent fixing of the two ends, which was done with the greater care, as experiment and theory differed very materially in the comparative strength of equal battens, when *fixed* at each end, and when only *supported:* all former theories make the strength in the two cases as 2 to 1, while most experimentalists state it as in the ratio of 3 : 2. According to the former, the mean strength of these beams, as compared with those at Art. 89, ought to have been 1442 lbs., and according to the latter, 1116 lbs.: the mean is 1105 lbs.; which is consistent with what has been shown, Art. 20.

TABLE II.—(CONTINUED.)

95. *Experiments on Fir Battens, fixed at one end, at different angles of inclination and in different positions.*

No. of experiments.	Length in inches.	Depth in inches.	Breadth in inches.	Specific gravity.	Weight in lbs.	Deflection in inches.	Weight reduced to length 36, & sp. gr. 600.	Position of the beams, &c.
1	36	2	2	560	317	5·0	400	Side parallel to the horiz^{n}.
2	32	2	2	609	432	6·0	400	
3	32	2	2	571	417	6·0	389	
4	30	$\sqrt{8}$	$\sqrt{8}$	600	462	4·9	385	Diagonal vertical.
5	30	$\sqrt{8}$	$\sqrt{8}$	613	469	4·7	391	
6	30	$\sqrt{8}$	$\sqrt{8}$	620	466	4·9	389	
7	24	2	1	620	279	4·1	180	Horizontal.
8	24	2	1	600	276	3·9	184	
9	24	2	1	596	273	4·3	183	Angle 26° upwards.
10	24	2	1	581	281	4·1	193	
11	24	2	1	600	294	3·9	196	Angle 26° downwards.
12	24	2	1	601	290	4·0	193	

The first six of the above pieces were the fragments of the first two and last specimens of the preceding page; care having been taken, in those experiments, to prevent the weights from going quite down, which would have endangered the breaking of the pieces at the ends where they were fixed in the wall. By blocking the scale as soon as the fracture commenced in the middle, the ends were left perfectly whole, the parts recovering completely their original rectilinear form.

The first three of the above were broken in the same position; viz. with the sides parallel and perpendicular to the horizon; the next three angle-ways, viz. with the diagonal vertical.

Nos. 7 and 8 were fixed in the usual horizontal position; Nos. 9 and 10, which were the same two pieces inverted, or turned end for end, were fixed at an angle of inclination upwards of 26°; and Nos. 11 and 12 at the same angle downwards.

TABLE II.—(CONTINUED.)

96. *Experiments on Oak Battens, supported at each end.*

No. of experiments.	Length in inches.	Depth in inches.	Breadth in inches.	Specific gravity.	Deflection.	Weight in lbs.	Reduced to sp. gr. 800.	Mean reduced weight.
1	18	1	1	767	..	323	337	358
2				768	..	353	368	
3				768	..	339	368	
4	24	1	1	764	..	266	278	269
5				774	..	251	260	
6				774	..	260	268	
7	30	1	1	777	..	196	202	202
8				777	..	196	202	
9				777	..	196	202	
10	36	1	1	..	2·95	158		180*
11				..	4·20	190		
12				..	..	176		

Nos. 1, 2, 5, and 4 were all from one piece, near the root end, and rather cross-grained, particularly Nos. 1 and 5. Nos. 2 and 4 were cut from the ends of these. Nos. 7, 8, and 9, each bore 286 lbs. without any appearance of fracture; but each broke immediately with the addition of 15 lbs.: it was therefore only taken as 10 lbs.

No. 11 was remarkably elastic; and, just before its fracture, its curve was traced on a plane-board placed against it, and the ordinates, carefully measured at every inch, were found as follow:

Ordinates,	·26,	·53,	·85,	1·13,	1·4,	1·7,	1·93;	2·2,	2·45.
Abscisses,	1,	2,	3,	4,	5,	6,	7,	8,	9.
Ordinates,	2·65,	2·87,	3·1,	3·3,	3·46,	3·63,	3·75,	3·82,	3·9.
Abscisses,	10,	11,	12,	13,	14,	15,	16,	17,	18.

* The specific gravities of 10, 11, 12, were not noted; the mean 180 is found by assuming them at 777, being part of the same plank as the above.

TABLE II.—(CONTINUED.)

97. *Experiments on Oak Battens, supported at each end.*

No. of experiments.	Length in inches.	Depth in inches.	Breadth in inches.	Specific gravity.	Deflection.	Weight in lbs.	Reduced to sp. gr. 800.	Mean reduced weight.
1 2 3	24	$1\frac{1}{2}$	$\frac{3}{4}$	768 784 777	1·1 1·1 1·1	387 408 395	403 416 406	408
4 5 6	30	$1\frac{1}{2}$	$\frac{3}{4}$	777 784 768	1·5 1·5 1·5	316 327 300	325 333 311	323
7 8 9	30 30 30	2 2 2	1 1 1	777	1·4 1·4 1·4	721 736 736	742 758 758	753
10 11 12	36	2	1	764	.. .. ..	598 607 612	626 635 641	634

The successive deflections of Nos. 1, 2, 3 were measured as follow, viz.

Weights.	Deflection of No. 1.	No. 2.	No. 3.
321	·65	·62	·65
366	·85	·72	·85
380	1·05	·95	1·05
387	1·10	1·05	1·08

The deflections of Nos. 7 and 9 were exactly equal, and were measured on three equidistant ordinates: the lengthening of the fibres was also in both cases equal: the particulars are as below, viz.

Weights.	Deflections.			Lengthened.
421 Nos. 7 and 9	·10	·25	·366	·075
521	·13	·35	·466	·100
621	·19	·50	·700	·125
671	·20	·60	·800	·150
721	..	..	1·400	

The deflections of No. 8 were not observed.

TABLE II.—(CONTINUED.)

98. *Experiments on Ash Battens, fixed at one end in a wall.*

No. of experiments.	Length in inches.	Depth in inches.	Breadth in inches.	Specific gravity.	Deflection.	Weight in lbs.	Weight reduced to 36 inches length, and 2 in. square.*	Position of the beams.
1	36	2	2	658	$11\frac{1}{3}$	436	436	Side parallel to the horizon.
2	36	2	2	730	$14\frac{1}{3}$	431	431	
3	30	$\sqrt{8}$	$\sqrt{8}$	658	$5\frac{1}{3}$	471	392	Diagonal vertical,
4	30	$\sqrt{8}$	$\sqrt{8}$	730	$6\frac{3}{4}$	466	388	
5	24	2	1	730	5	352	470	Fixed at an < 26° down.
6	24	2	1	730	not obs.	321	428	Ditto, ditto, upwards.
7	24	2	1	730	6	332	441	Horizontal.
8	24	2	1	730	6	321	428	Horizontal.
9	24	2	1	730	not obs.	302	403	Angle 26° upwards.

No. 1 was the same piece as No. 3.
No. 2 ,, ,, No. 4.
No. 5 ,, ,, No. 7.
No. 8 ,, ,, No. 9.

Nos. 1, 2, 5, and 8 were first broken at one end (but not so as to completely separate the parts); after which they were turned end for end, and broken again, as stated in Nos. 3, 4, 7, and 9. No. 6 was so fractured in the first experiment, that it could not be submitted to a second trial. The same thing always occurred when the beam was first broken at an angle upwards: it appeared, in these cases, to turn on a point, about 6 inches from the wall, where the strain and curvature seemed to be the greatest, and from which point the fracture commenced, splitting the piece through its whole length.

In the above experiment, No. 2, the neutral line was remarkably well defined, and appeared to be very nearly, or exactly, at $\frac{3}{8}$ths of the whole depth; the same as in fir.

* The reduction in column 8 is made on a supposition that the strength is inversely as the length.

TABLE II.—(CONTINUED.)

99. ***Experiments on Beech Battens, fixed at one end in a wall, at different inclinations and in different positions.***

No. of experiments.	Length in inches.	Depth in inches.	Breadth in inches.	Specific gravity.	Deflection.	Weight in lbs.	Weight reduced to 36 inches length, and 2 in. square.	Position of the beams.
1	36	2	2	700	11	401	401	Side parallel to the horizon.
2	36	2	2	690	8	401	401	
3	36	2	2	700	11	401	401	
4	30	$\sqrt{8}$	$\sqrt{8}$	690	5	466	388	Diagonal vertical.
5	30	$\sqrt{8}$	$\sqrt{8}$	700	6	451	376	
6	24	2	1	740	4½	371	495	Fixed at an < 26° down.
7	24	2	1	740	5	352	469	Ditto < 26° upwards.
8	24	2	1	740	5	352	469	Horizontal.
9	24	2	1	740	5	352	469	Ditto.
10	24	2	1	740	5½	317	463	At an < 26° upwards.

No. 2 was the same piece as No. 4.
No. 3 „ „ No. 5.
No. 6 „ „ No. 8.
No. 9 „ „ No. 10.

Nos. 1 and 7 were so much splintered in the first experiments that they could not be submitted to a second trial, as was done with Nos. 2, 3, 6, and 9. These, after being broke at one end, (without a total separation,) were turned end for end, and then broken with the weights indicated in Nos. 4, 5, 8, and 10.

It should be observed here, that the deflections were not, in these experiments, measured so accurately as in those that were supported at each end: the apparatus not being so convenient, we were generally satisfied with measuring it to the nearest ⅛ of an inch: the successive deflections, however, seemed to follow, while the weights were small, the ratio of the weights, as was observed in the preceding experiments. The deflections from first to last were as follow:

121 lbs.	No. 1 = 1⅞	No. 2 = 1 9/16	No. 3 = 1⅞
221 lbs.	= 3⅝	= 3	= 3⅜
271 lbs.	= 5	= 3⅞	= 4½
321 lbs.	= 7	= 5	= 6⅛
401 lbs.	= 11	= 8	= 11

TABLE II.—(CONTINUED.)

100. *Experiments on Solid and Hollow Cylinders, supported at each end.*

No. of experiments.	Names of woods.	Specific gravity.	Length in inches.	Diameter external.	Diameter internal.	Breaking weight.	Deflections in inches.
1	Fir.	581	48	2	solid.	740	2·0
2		603	48	2	do.	796	2·1
3		580	48	2	do.	780	1·9
4	Ash.	590	46	2	solid.	700	2·7
5		590	46	2	solid.	730	2·5
6		586	46	2	½ inch.	650	3·0
7		540	46	2	½ inch.	664	3·0
8		601	46	2	¾ inch.	646	3·1
9		601	46	2	¾ inch.	654	2·9
10		580	46	2	1 inch.	631	2·8
11		580	46	2	1 inch.	630	3·6

The fir pieces were part of the same plank as those of 4 feet, given in Art. 88, viz. Nos. 3 and 4, which was a very fine specimen of Christiana deal, and had been in store a considerable time.

The ash cylinders were obviously of a much weaker quality than those of which the detail is given at Art. 98; but the results were very uniform, and they therefore furnish a good comparison between the strength of solid and hollow cylinders amongst themselves, although we cannot compare them with our square battens, as they were of a much inferior quality to the preceding square pieces. The fir cylinders, on the contrary, furnish no comparison between solid and hollow cylinders; but they may be correctly compared with like pieces of the same dimension square, being, as stated above, precisely the same wood as Nos. 3 and 4, Art. 88.

101. Similar experiments to those last described were made on battens of elm and teak; but the results of the latter were so irregular, that it would be useless to give the detail of them: it will be sufficient to observe, that one of the pieces of teak bore 478 lbs., which was more than equal to the load borne by the ash pieces of the same dimensions; viz. 3 feet long by 2 inches square; while the other two pieces broke with little more than 300 lbs., the deflection in each case being about 7 inches: and one piece 2 feet long, 2 inches deep, and 1 inch in breadth, fixed at one end, and at an angle of 26° upwards, broke with 422 lbs., which is considerably more than was found to be necessary for breaking an equal piece of ash.

The elm battens gave much more uniform results, although the pieces were found very weak in comparison with those of ash and beech. The mean weight which broke the three pieces 3 feet long and 2 inches square, was 216 lbs.; and the mean of the same three pieces inverted and fixed diagonally, was 296 lbs., the latter being broken at 30 inches: the mean specific gravity was 570.

Remark.—If the same reduction be made here as in the pieces of ash and beech, we shall have

$$36 : 30 :: 296 : 246,$$

which shows that the strength of elm is the same whether it be fixed direct or diagonally; whereas it was found that ash and beech were both weakest in the latter position.

Determination of Practical Data.

102. It has been observed, that all the preceding specimens of wood were selected from deals, planks, and battens which had been in store a considerable time, and that only the best, or those of the most uniform texture, were chosen for the purpose; the object of the experiments not having been to furnish practical data, but to compare, under the most favourable circumstances, the theoretical formulæ with experimental results. This having been effected, and the agreement having been found generally perfectly satisfactory, it became necessary to make another series of experiments on woods of more common quality, in order to furnish data for practical cases. The author therefore applied to the Admiralty, and obtained permission to select specimens for experiment, from all the timber in store in Woolwich Dockyard; in which selection he was kindly assisted by Mr. Hockey, assistant builder in that establishment.

It has been shown, (Art. 28,) that as regards the absolute strength of a beam, we ought to find,

When the beam is fixed at one end and loaded at the other,

$$\frac{l\,W}{a\,d^2} = S,$$

a constant quantity for all wood of the same quality, whatever may be the length l, the breadth a, or the

depth d; consequently, S once determined, remains the same, and serves for computing the strength of any sized beam of the same wood, or the dimensions necessary to insure a given strength in a given direction. That is, of the four quantities, l, a, d, W, any three being given, the fourth may be found: thus,

$$W = \frac{S a d^2}{l}$$

$$l = \frac{S a d^2}{W}$$

$$\left.\begin{array}{l} a = \dfrac{l W}{S d^2} \\ d = \sqrt{\dfrac{l W}{a S}} \end{array}\right\} \text{In square beams } a = d = \sqrt[3]{\frac{l W}{S}}.$$

When supported at one end and loaded in middle,

$$\frac{l W}{4 a d^2} = S.$$

In this case, therefore,

$$W = \frac{4 \overset{b}{a} d^2 S}{l}$$

$$l = \frac{4 a d^2 S}{W}$$

$$\left.\begin{array}{l} a = \dfrac{l W}{4 d^2 S} \\ d = \sqrt{\dfrac{l W}{4 a S}} \end{array}\right\} \text{In square beams } a = d = \sqrt[3]{\frac{l W}{4 S}}.$$

When the beam is fixed at both ends and loaded in the middle,

$$W = \frac{6\,a\,d^2\,S}{l}$$

$$l = \frac{6\,a\,d^2\,S}{W}$$

$$\left.\begin{aligned} a &= \frac{l\,W}{6\,d^2\,S} \\ d &= \sqrt{\frac{l\,W}{6\,a\,S}} \end{aligned}\right\} \text{In square beams } a = d = \sqrt[3]{\frac{l\,W}{6\,S}}.$$

When the beam is supported at both ends and loaded at an intermediate point,

$$W = \frac{l\,a\,d^2\,S}{m\,n}$$

$$l = \frac{m\,n\,W}{a\,d^2\,S}$$

$$\left.\begin{aligned} a &= \frac{m\,n\,W}{l\,d^2\,S} \\ d &= \sqrt{\frac{m\,n\,W}{l\,a\,S}} \end{aligned}\right\} \text{In square beams } a = d = \sqrt[3]{\frac{m\,n\,W}{l\,S}}.$$

When the beam is fixed at both ends and loaded at an intermediate point,

$$W = \frac{3\,l\,a\,d^2\,S}{2\,m\,n}$$

$$l = \frac{2\,m\,n\,W}{3\,a\,d^2\,S}$$

$$\left.\begin{aligned} a &= \frac{2\,m\,n\,W}{3\,l\,d^2\,S} \\ d &= \sqrt{\frac{2\,m\,n\,W}{3\,l\,a\,S}} \end{aligned}\right\} \text{In square beams } a = d = \sqrt[3]{\frac{2\,m\,n\,W}{3\,l\,S}}.$$

When the weight is uniformly distributed, the

same formulæ will apply; but W in this case will represent only half the required or given weight.

103. Again, it has been found, (Art. 66, &c.,) using a for b, in reference to elasticity and deflection, that

When a beam is fixed at one end and loaded at the other,

$$\frac{l^3 W}{a d^3 \delta} = E,$$

a constant quantity for all woods of the same quality.

When fixed at one end uniformly loaded,

$$\frac{3 l^3 W}{8 a d^3 \delta} = E.$$

When supported at each end and loaded in the middle,

$$\frac{l^3 W}{16 a d^3 \delta} = E.$$

When supported at each end and uniformly loaded,

$$\frac{5}{8} \times \frac{l^3 W}{16 . a d^3 \delta} = E.$$

E therefore being determined for any given wood, the other quantities may be found by a proper inversion of these formulæ, as in the preceding cases of strength. These several values of S and E have been found experimentally on the several specimens as stated in the following Table.

(COPY OF A REPORT TRANSMITTED TO THE HON. THE PRINCIPAL OFFICERS AND COMMISSIONERS OF HIS MAJESTY'S NAVY.)

TABLE OF DATA,

CONTAINING THE

(104.) *Results of Experiments on the Elasticity and Strength of various Species of Timber, selected from Woolwich Dockyard.*

No. of experiments.	Names of the woods, and dimensions.	Specific gravity.	Greatest weight & deflection while the elasticity remained perfect.		Breaking weight in lbs.	Ultimate deflection in inches.	Depth of neutral axis in inches.	Value of E, from the formula *$E=\frac{l^3 W'}{16 a d^3 \delta}$	Value of S, from the formula $S=\frac{l W}{4 a d^2}$.
			Weight in lbs.	Deflect^n. in inches.					
1	Teak, 7 ft. by 2 in. sq.	742	300 300	1·065 1·093	1020	4·75	1·2		
2		749	300 300	1·150 1·130	975	4·20	1·2		
3		744	300 300	1·276 1·192	820	4·00	..		
	Mean Results..	745	300	1·151	938	4·32	1·2	603600	2462
4	Poon, 7 ft. by 2 in. sq.	600	150 150	·830 ·780	860	6·00	1·25		
5		570	150 150	·820 ·837	848	5·75	..		
6		568	150 150	·837 ·830	830	6·00	1·20		
	Mean Results..	579	150	·822	846	5·92	1·225	422400	2221
7	English Oak, 1st specimen,	986	150 150	1·420 1·420	470	6·00	1·3		
8	7 ft. by 2 in. sq.	998	150 150	1·700 1·700	421	5·90	1·3		
9	inferior specimen	925	150 150	1·650 1·650	460	5·80	1·3		
	Mean Results..	969	150	1·590	450	5·90	1·3	218400	1181

* NOTE.—For the sake of simplifying the calculations, the value of E is not carried on exact beyond the nearest fourth figure.

TABLE—(CONTINUED).

No. of experiments.	Names of the woods, and dimensions.	Specific gravity.	Greatest weight & deflection while the elasticity remained perfect. Weight in lbs.	Deflectⁿ. in inches.	Breaking weight in lbs.	Ultimate deflection in inches.	Depth of neutral axis in inches.	Value of E, from the formula $E=\frac{l^3 W'}{16 a d^3 \delta}$.	Value of S, from the formula $S=\frac{l W}{4 a d^2}$.
10	English Oak, 2nd specimen, 6 ft. by 2 in. sq. reduced to 7 ft.	942	200 200	1·260 1·280	640	7·90	1·2		
11		900	200 200	1·290 1·290	623	8·30	1·2		
12		960	200 200	1·275 1·285	649	8·10	1·2		
	Mean Results..	934	200	1·280	637	8·10	1·2	362800	1672
13	Canadian Oak, 7 ft. by 2 in. sq.	865	225 225	1·150 1·150	660	5·70	1·1		
14		885	225 225	1·009 1·011	708	6·20	..		
15		867	225 225	1·070 1·070	651	6·10	1·15		
	Mean Results..	872	225	1·080	673	6·00	1·125	536200	1766
16	Dantzic Oak, 7 ft. by 2 in. sq.	767	200 200	1·710 1·690	520	5·00	1·2		
17		787	200 200	1·260 1·300	580	4·10	1·2		
18		713	200 200	1·855 1·715	580	5·50	..		
	Mean Results...	756	200	1·590	560	4·86	1·2	297800	1457
19	Adriatic Oak, 7 ft. by 2 in. sq.	941	150 150	1·070 1·070	560	6·00	1·20		
20		948	150 150	1·550 1·450	500	5·50	1·25		
21		1090	150 150	1·720 1·720	520	5·70	1·15		
	Mean Results..	993	150	1·430	526	5·73	1·2	243600	1383
22	Ash, 7 ft. by 2 in. sq.	760	225 225	1·270 1·250	777	9·00	1·35		
23		758	225 225	1·300 1·270	760	9·10	1·30		
24		762	225 225	1·240 1·270	780	8·66	1·25		
	Mean Results..	760	225	1·266	772	8·92	1·3	411200	2026

TABLE—(CONTINUED).

No. of experiments.	Names of the woods, and dimensions.	Specific gravity.	Greatest weight & deflection while the elasticity remained perfect. Weight in lbs.	Deflectn. in inches.	Breaking weight in lbs.	Ultimate deflection in inches.	Depth of neutral axis in inches.	Value of E, from the formula $E = \frac{l^3 W'}{16 a d^3 \delta}$.	Value of S, from the formula $S = \frac{l W}{4 a d^2}$.
25	Beech, 7 ft. by 2 in. sq.	712	150 150	1·075 1·025	565	6·00	1·2		
26		628	150 150	1·009 1·024	600	5·70	..		
27		688	150 150	1·025 1·000	615	5·50	1·2		
	Mean Results..	696	150	1·026	593	5·73	1·2	338400	1556
28	Elm, 6 ft. by 2 in. sq. reduced to 7 feet	583	125 125	1·620 1·610	368	7·00	1·2		
29		540	125 125	1·420 1·460	398	6·93	1·1		
30		535	125 125	2·070 1·930	394	6·86	..		
	Mean Results..	553	125	1·685	386	6·93	1·15	174960	1013
31	Pitch Pine, 7 ft. by 2 in. sq.	712	150 150	1·133 1·166	650	6·25	1·2		
32		628	150 150	1·140 1·110	595	5·75	1·2		
33		641	150 150	1·166 1·091	620	6·00	..		
	Mean Results..	660	150	1·134	622	6·00	1·2	306400	1632
34	Red Pine, 7 ft. by 2 in. sq.	655	150 150	·825 ·825	473	5·70	1·3		
35		667	150 150	·700 ·725	530	5·83	1·25		
36		650	150 150	·725 ·730	530	5·96	1·25		
	Mean Results..	657	150	·755	511	5·83	1·26⅔	460000	1341
37	New England Fir, 7 ft. by 2 in. sq.	560	150 150	·862 ·862	446	4·50	1·36		
38		560	150 150	·970 ·970	403	4·70	1·30		
39		540	150 150	·960 ·960	411	4·78	1·33		
	Mean Results..	553	150	·931	420	4·66	1·33	547800	1102

TABLE—(CONTINUED.)

No. of experiments.	Names of the woods, and dimensions.	Specific gravity.	Greatest weight & deflection while the elasticity remained perfect. Weight in lbs.	Deflec^n. in inches.	Breaking weight in lbs.	Ultimate deflection in inches.	Depth of neutral axis in inches.	Value of E, from the formula $E = \frac{l^3 W'}{16 a d^3 \delta}$	Value of S, from the formula $S = \frac{l W}{4 a d^2}$
40	Riga Fir, 1st specimen, 7 ft. by 2 in. sq.	730	125, 125	·812, ·837	420	5·80	1·35		
41		765	125, 125	·912, ·912	440	6·10	1·33		
42		763	125, 125	·937, ·910	406	6·10	..		
	Mean Results..	753	125	·870	422	6·00	1·35	332200	1108
43	Riga Fir, 2nd specimen, 6 ft. by 2 in. sq.	714	150, 150	·794, ·794	567	5·50	..		
44		768	150, 150	·907, ·909	367	6·00	..		
45		732	150, 150	·950, ·950	467	6·50	..		
	Mean Results..	738	150	·883	467	6·00	..	247600	1051
46	Mar Forest Fir, 1st specimen, 7 ft. by 2 in. sq.	715	125, 125	1·560, 1·500	360	5·50	1·3		
47		616	125, 125	1·370, 1·250	463	5·50	1·3		
48		684	125, 125	1·370, 1·370	465	7·00	1·3		
49		769	125, 125	1·560, 1·560	457	6·00	1·3		
	Mean Results..	696	125	1·442	436	6·00	1·3	161340	1144
50	Mar Forest, 2nd specimen, 6 ft. by 2 in. sq.	720	150, 150	1·150, 1·150	600	7·00	1·3		
51		756	150, 150	1·250, 1·150	517	6·00	1·3		
52		603	150, 150	0·675, 0·675	567	6·25	..		
	Mean Results..	693	150	1·006	561	6·42	1·3	217400	1262
53	Mar Forest, 3rd specimen, 6 ft. by 2 in. sq.	700	150, 150	1·150, 1·150	561	6·5	1·3		
54		710	150, 150	1·230, 1·170	570	6·5	1·3		
55		698	150, 150	0·675, 0·675	552	6·25	1·3		
	Mean Results..	703	150	1·006	561	6·42	1·3	217400	1262

TABLE—(CONTINUED.)

No. of experiments.	Names of the woods, and dimensions.	Specific gravity.	Greatest weight & deflection while the elasticity remained perfect. Weight in lbs.	Deflecn. in inches.	Breaking weight in lbs.	Ultimate deflection in inches.	Depth of neutral axis in inches.	Value of E, from the formula $E=\frac{l^3 W'}{16 a d^3 \delta}$.	Value of S, from the formula $S=\frac{l W}{4 a d^2}$.
56	Larch, 1st specimen, 7 ft. by 2 in. sq.	504	125 / 125	1·930 / 1·910	300	8·60	..		
57		576	125 / 125	1·740 / 1·760	340	8·60	..		
58		514	125 / 125	1·970 / 2·000	336	8·54	..		
	Mean Results..	531	125	1·885	325	8·58	..	154080	853
59	Larch, 2nd specimen, 6 ft. by 2 in. sq.	552	125 / 125	0·750 / 0·750	300	6·00	..		
60		480	125 / 125	0·812 / 0·812	412	4·50	..		
61		534	125 / 125	0·875 / ..	398	4·50	..		
	Mean Results..	522	125	0·812	370	5·00	..	224400	832
62	Larch, 3rd specimen, 6 ft. by 2 in. sq.	546	150 / 150	0·750 / 0·750	417	4·70	1·25		
63		552	150 / 150	0·825 / 0·825	497	4·90	1·25		
64		552	150 / 150	0·750 / 0·750	537	5·00	1·20		
65		576	150 / 150	1·050 / ·950	552	5·40	1·20		
	Mean Results..	556	150	0·831	501	5·00	1·225	263200	1127
66	Larch, 4th specimen, 6 ft. by 2 in. sq.	552	150 / 150	·831 / ·831	500	4·8	1·2		
67		581	150 / 150	·900 / ·864	515	5·2	1·2		
68		548	150 / 150	·762 / ·798	515	5·0	1·2		
	Mean Results..	560	150	·831	510	5·0	1·2	263200	1149
69	Norway Spar, 6 ft. by 2 in. sq.	600	200 / 200	·800 / ·800	667	4·0	1·35		
70		600	200 / 200	·760 / ·740	617	4·0	1·25		
71		580	200 / 200	·840 / ·860	680	4·0	1·30		
	Mean Results..	577	200	·800	655	4·0	1·30	364400	1474

To the Hon. the Principal Officers and Commissioners of His Majesty's Navy.

105. *Additional Experiments made in the Royal Arsenal, by P. W. Barlow, Civil Engineer, on the Strength and Elasticity of various Woods of English and Foreign growth.*

No. of experiments.	Names of woods.		Specific gravity.	Weight in lbs. which produced 1 in. deflectn.	Breaking weight in lbs.	Value of E, from the formula $E=\frac{l^3 w}{32\,a\,d^3\,\delta}$.	Value of S, from the formula $S=\frac{l w}{4\,a\,d}$.	Remarks.
1	Acacia, English growth.		710	..	1195	144000	1867	
2	——, ditto		710	bore	1084	rope broke, the piece little injured.		
3	Oak, fast grown		903	660	999	161100	1561	Specimens supplied by W. Withers, Esq.
4	—, slow grown		856	414	677	101100	1058	
5	—, fast grown		972	550	999	135800	1561	
6	—, slow grown		835	439	943	107400	1473	
7	—, superior quality		748	896	1447	218700	2261	Very fine specimen; been in store two years.
8	—, ditto		756	680	1304	166000	2037	been in store 16 yrs.
9	Tonquin Bean	middle	1036	1388	2414	338900	3850	This timber was sent from Berbice by Captain Gipps, R. E.
10		outside	1080	1332	2228	325200	3481	
11	Locust	middle	972	1052	2116	250200	3303	
12		outside	936	940	2284	229500	3568	
13	Bullet Tree	middle	1029	1360	1724	332000	2696	
14		outside	1029	1332	1668	325200	2606	
15	Greenheart	middle	1015	1332	1892	325200	2956	
16		outside	986	1388	1612	338900	2562	
17	Cabacally	middle	907	952	1668	232400	2606	
18		outside	892	940	1556	229500	2431	
19	African Oak	middle	972	1168	1447	285400	2261	From a very fine timber long in store.
20		outside	972	1168	1657	285100	2589	
21		middle	1015	1288	1643	314400	2567	
22		outside	972	1097	1643	267800	2567	
23	American Black Birch	middle	648	775	1279	189200	1967	A long time in store, very dry; the same tree.
24		outside	633	775	915	189200	1430	
25		middle	648	644	1027	157200	1604	
26		outside	669	831	1433	202900	2239	
27	Common Birch	middle	792	800	1164	195300	1820	
28		outside	630	884	1304	215900	2037	
29	Ash	middle	727	660	1304	161100	2037	Dry, and of the same plank.
30		outside	702	660	1304	161100	2037	
31	Elm	middle	554	436	772	106400	1206	Dry, and of the same tree.
32		outside	532	324	660	79100	1031	
33	Christiana Deal	middle	698	856	1052	209000	1644	Dry, and of the same deal.
34		outside	680	772	940	188400	1480	
35	Memel Deal	middle	590	786	1108	191900	1731	Dry, and of the same deal.
36		outside	590	856	1108	209000	1731	

Note.—In these experimenst the bearing distance was 50 inches, and the bars 2 inches square.

Experiments on the Strength of Bent Timber.

106. In naval architecture it is always necessary to make use of a great quantity of bent timber. This, as far as can be done, is selected out of natural grown pieces, as nearly as possible of the required form, and is commonly known in the dock-yards by the term *compass timber*, which was formerly contracted for at a higher rate than that of straight growth; but both compass and straight timber is now, I believe, sent in at the same price. The great call for the former, however, during the war, rendered it very scarce, and much time and labour were employed in examining the stacks, in order to select pieces proper for each required purpose; and as the pieces, when they could be obtained, generally exceeded the requisite dimensions, much was necessarily cut away, and a great difference was always found between the *first* and the *converted* contents: the pieces were also, frequently, very much grain-cut, which necessarily diminished their strength very considerably.

These inconveniences, and particularly the great difficulty in obtaining compass timber, led Mr. Hookey, at that time master boat-builder in Woolwich Dockyard, but now assistant builder, to extend a method which he had long practised for bending boat timbers, to the bending of the largest ship timbers; and having obtained permission to have

a machine constructed for the purpose, it was found to answer every possible expectation that could be formed of it; the largest timbers, viz. pieces 18 inches square, being brought to any required curve in about fifteen minutes after being placed upon the machine: a description of which, in its original state, (but it has since received some improvements,) may be seen in vol. xxxii. of the 'Transactions of the Society of Arts.'

The method of preparing the timber is as follows: a fine saw-cut is made from one end, or both, according to the form into which the timber is to be bent; the length of it being also different, according to the length of the piece and the degree of curvature: but commonly, in a curve the height of which is about $\frac{1}{6}$th or $\frac{1}{8}$th of the whole length, the saw-cut from each end is about $\frac{1}{3}$rd of the length. The piece is then boiled for some hours, depending upon its lateral dimensions, and placed upon the machine, when the screws, &c., being applied, the required curvature is obtained, as above stated, in about twelve or fifteen minutes; after which it is screw-bolted, and is then ready for use. The reader, by referring to figs. 11 and 12, Plate III., will readily understand the above description; these figs. representing the fragments of two pieces bent for the following experiments. It is only necessary to observe, that the keys *k*, *k*, and K, are no part of the original plan; but were suggested during our experiments.

The advantages attending this method of bending timber for the purposes of ship-building, are, 1st, That it dispenses with the use of compass timber, should it again become very scarce; and therefore no impediment would arise to the service, if the necessary quantity of timber of this kind could not be in any way procured. 2dly, It saves a deal of the time and labour necessary for unstacking and restacking piles of timber, to procure pieces of requisite compass; any piece of the proper length and squarage being at once available with the application of the machine. 3dly, It saves a great quantity of timber, which is necessarily cut to waste in bringing compass timber to its required dimensions; the conversion, in some cases, taking away a considerable part of the original contents; while, in bending timber, the original and converted contents are nearly the same. But, notwithstanding these recommendations in its favour, there appears to be a prejudice, well or ill founded, against the adoption of it, and some objections have been offered to the practice; the first of which is, that boiling the timber, and the strain impressed upon it, have a tendency to weaken the pieces, and, consequently, the ship into which such timbers are introduced: and, secondly, that the bolts are not sufficient to keep the two parts in a proper degree of contact, so as to prevent the introduction of damp and moisture. The latter point must be left to the decision of the practical builder; but with regard to the strength,

this may be otherwise determined, and I therefore solicited permission of the Navy Board to be allowed to make experiments on bent pieces of natural growth, grain-cut, and others, bent on the principle of Mr. Hookey, and the results of these experiments will be seen in the following Table: from which it will appear, that, taking the medium between the natural grown pieces and those which are partly so and partly grain-cut, no defect in point of strength will be found on the side of those bent upon the above plan. I also wished to try what effect boiling and steaming timber had upon the ultimate strength without bending; the account of which is given in my third Report, from which it appears, that although there is an obvious falling off in the strength of those pieces boiled for a long time, the defect is very small while the boiling or steaming is not continued beyond the proportion of an hour to an inch in thickness, which is the usual practice in the dockyard.

(COPY OF A REPORT TRANSMITTED TO THE HONOURABLE THE PRINCIPAL OFFICERS AND COMMISSIONERS OF HIS MAJESTY'S NAVY; CONTAINING)

107. ***Experiments on the Strength of Bent Oak Scantlings: 1st, Of Natural Growth; 2dly, Grain-cut; and 3dly, On those bent according to the Plan of Mr. Hookey. The latter with a saw-cut, and without it. Also the former of these with and without keys.***

Note.—The pieces were each 6 feet long and 2 inches square, but they were broken on props 5 feet apart.

No. of experiments.	Nature of the pieces.	Arch up or down.	First curve in inches.	Specific gravity.	Breaking weight.	Last deflection below the props.	Strength computed from the formula $S = \frac{l W \sec^2 \Delta}{4 a d^2}$.
1	Natural growth.	up	6	804	680	− 2	1312
2	Do.	up	8	820	764	− 0	1504
3	Do.	down	6	822	768	10	1600
4	Do.	down	8	874	762	13	1647
5	Grain-cut.	up	7½	980	505	− 3	1161
6	Do.	up	8½	830	568	− 2	1122
7	Do.	down	7½	938	546	10	1137
8	Do.	down	8½	840	550	10	1146
9	Bent whole.	up	7½	798	667	− 1	1314
10	Do.	down	7½	810	617	13	1353
11	Saw-kerf, but no keys.	up	8½	886	517	+ 2	——
12		down	8½	856	517	15	——
13	Saw-kerf, with square keys.	up	8½	754	712	+ 2	1407
14		down	8½	732	662	14	1470
15	Saw-kerf, with cylindrical keys.	up	6	873	717	+ 5	1447
16		down	6	873	762	12¾	1657

Note 1.—The last deflection, having the sign *plus* + prefixed, indicates that the pieces arched so many inches the contrary way before breaking; and those marked *minus* —, wanted the number of inches following, of coming down to the level of the props.

Note 2.—The pieces laid with the arch up were necessarily supported by the outside of the props; these, therefore, must be considered as being broke at 5 feet 3 inches, which was the distance from the outside of one prop to that of the other; and this is the case even where the pieces bent the contrary way; for, notwithstanding the middle of the piece came below the props, the half-lengths were still sufficiently curved to throw the principal bearing on the outside.

In each of the figs. 11 and 12, Plate III., A B C D represents a fragment of the scantlings; *aa*, *bb*, *cc*, the screw-bolts, and *m n* the saw-cut; which latter is 2 feet, or one-third the length of the piece. In fig. 12, K represents the form of the key, which was of oak, 1 inch long and ½ an inch deep, let in ¼ of an inch into each part; and in fig. 11, *k* and *k* are copper bolts, of ½-inch diameter; which, therefore, also laid ¼ of an inch into each part; and in both figures the keys passed through the whole thickness of the scantling.

The idea of this mode of keying was suggested in our first experiments on pieces of this description; viz. Nos. 11 and 12, in which it was found that the screw-bolts were not sufficient to prevent the part above and below *m n* from sliding upon one another. This defect may not have place when pieces of this kind are introduced into a ship, in consequence of the number of tree-nails with which the futtocks are pierced, which have necessarily a tendency to prevent that slipping of the parts noticed above. But, even in this case, I am convinced that considerable stiffness would be gained by keying the pieces after the manner of fig. 11, where it may be observed that hard wood, as sound oak or lignum-vitæ, would answer equally as well as copper bolts; and farther, that as the neutral axis in any section of fracture is generally at about ⅝ths of the depth, there would be no loss of strength in the piece, provided the key did not exceed ¼th of the whole depth.

N.B. Mean strength { Nos. 1, 2, 3, 4, of natural growth, 743 ; Nos. 13, 14, 15, 16, bent and keyed, 713 }

Additional Experiments.

108. In order to form a comparison between the strength of a piece of timber bent upon Mr. Hookey's principle, and a straight piece in its natural state, two pieces were formed from the same scantling, having been only parted by the saw; the bent piece was brought to a curve of 9½ inches, and keyed, as in fig. 11, Plate III.; the two pieces were then broken at the same distance, viz. 5 feet; their other dimensions being also the same as those above. The results of these experiments are as follow:

Straight piece, not boiled . . .	deflected 5¼ inches; broke with 667 lbs.
Bent to a curve of 9½ in., arch down	deflected to 14¼ inches; broke with 727 lbs.

By a comparison with all the above results, we obtain the following proportional breaking weights, viz.:

Natural growth	743 lbs.
Bent on Hookey's principle, and keyed . .	713
Bent, without a saw-cut	632
Grain-cut	562
Bent on Hookey's plan, without keys . .	517
Straight, and in natural state, deduced from the results of the 2nd specimen of the first Report	764

Note.—In comparing the first two of the above numbers with the last, it should be remembered, that although the former were broken with less weight, it does not indicate a less degree of strength; the same weight producing a greater strain upon a bent than upon a straight piece, proportional to the secant squared of the angle of deflection.

To the Hon. the Principal Officers and Commissioners of His Majesty's Navy.

(COPY OF A REPORT TRANSMITTED TO THE HONOURABLE THE PRINCIPAL OFFICERS AND COMMISSIONERS OF HIS MAJESTY'S NAVY; CONTAINING)

109. *Experiments on the Strength of Oak Timber, in its natural state, compared with similar pieces boiled and steamed for different periods.*

Note.—The following pieces of oak were all cut from the same log, the mean specific gravity of which was 822.

No. of experiments.	Boiled or steamed.	No. of hours.	Length in feet.	Square in inches.	Deflection with 100 lbs.	Ultimate deflection.	Breaking weight, lbs.	Mean breaking weight, lbs.
1	Natural state	0	6	2	·425	6·0	722	669
2	Natural state	0	6	2	·500	6·5	617	
3	Steamed	5	6	2	·450	6·0	617	669
4	Steamed	5	6	2	·425	7·0	722	
5	Steamed	10	6	2	·430	6·0	662	614
6	Steamed	10	6	2	·475	5·0	567	
7	Boiled	2	6	2	·500	5·0	567	614
8	Boiled	2	6	2	·425	6·5	662	
9	Boiled	4	6	2	·462	7·5	662	614
10	Boiled	4	6	2	·525	4·0	567	
11	Boiled	6	6	2	·550	6·0	597	589
12	Boiled	6	6	2	·425	5·5	582	
13	Boiled	8	6	2	·475	5·5	647	639
14	Boiled	8	6	2	·500	7·0	632	
15	Boiled	10	6	2	·550	5·5	567	607
16	Boiled	10	6	2	·500	6·0	647	

Nos. 17 and 18, bent and keyed on Hookey's plan, part of the same log, and broke at the same length, viz. 6 feet; and the same squarage, viz. 2 inches.

17	Boiled	3 hs.	1st curve 10 in.	Arch up.	Breaking wt. 632
18	Boiled	3 ,,	1st curve 10 in.	Do. down	Breaking wt. 636

To the Hon. the Principal Officers and Commissioners of His Majesty's Navy.

There is not in the above experiments that degree of uniformity that we might have expected, considering the pieces were all cut from the same log. It should be observed, however, that the two experiments, 11 and 12, ought not to be considered as equally conclusive with the others, as they each broke at a knot about 6 inches from the centre of the beam.

Rejecting these, therefore, there appears, generally, to be a slight loss in strength from boiling and steaming; but it is not very perceptible while that process is not continued beyond the time usually allowed in the dockyards.

In several experiments which I made on pieces boiled only for two or three hours, there was no apparent defect in strength; some of them even exceeding, and others falling a little short of, similar unboiled pieces: but as they were not all from the same timber, they would not, probably, be thought conclusive if they were detailed; on which account they are omitted.

On Trussed Girders.

110. We shall now conclude this course of experiments with the four following, on girders, trussed and plain: the two former, viz. No. 1 and No. 3, were very accurately made, and constructed on a scale of 2 inches to the foot, from the drawing given by Nicholson (Plate XXXIX., '*Carpenter's New Guide*'); the former being supposed to denote a 34-feet, and the other a 25-feet girder.

On the Deflection and Strength of Girders, trussed and plain.

No. of experiments.	Distance between the props.	Depth of the girder.	Breadth of the girder.	Weight in lbs.	Deflection in inches.	REMARKS.
	ft. in.	ft. in.	ft. in.			
1	5 8	0 2	0 1⅛	100 200 300 400 450 500	·35 ·67 1·05 1·47 1·75 2·25	Truss in 3 pieces; length of centre piece 1 ft. 6 in. Distance of extreme butments 4 ft. 10 in. 2 king-bolts, 2 plate-bolts, and 5 screw-bolts.
2	5 8	0 2	0 1⅛	100 200 300 400 450 500	·30 ·60 ·90 1·20 1·35 1·55	Without a truss.
3	4 2	0 2	0 1⅛	100 200 400 600 700 743 803 903 953	·15 ·30 ·57 ·87 1·20 1·30 1·45 1·50 broke	Truss in 2 pieces; distance of extreme butments 3 ft. 4 in. 1 king-bolt, 2 pl.-bolts, and 4 screw-bolts.
4	4 2	0 2	0 1⅛	100 200 300 400 500 600 717	·15 ·27 ·41 ·57 ·77 1·00 broke	Without a truss.

Nos. 1 and 2 were not broken in the experiment; but it appears that the trussed beam was the weakest; or at least it gave the greatest deflections. The wood of No. 1 was certainly inferior to the untrussed beam, but still it was to have been expected that the trusses would have been more than equivalent to the difference in the former respect; but as it was not, the experiment seems to indicate that there is but little efficacy in a truss of that description.

The trusses of No. 3 came fairly into action with each other, and certainly added very considerably to the resistance of the girder.

On the Resistance to Pressure.

111. Besides the two kinds of strains, *i. e.* the tensile and transverse strains, to which timber is exposed in building and machinery, there is another of considerable importance to which we have only at present very slightly referred, and this is the strain that pillars, columns, &c. have to sustain when supporting weights in a vertical position; and it must be admitted that it is one less supported both by theory and experience that any other branch of the general subject of strength and resistance. It has indeed been found experimentally, according to Mr. Tredgold, in his 'Treatise on Carpentry,' "that when a piece of timber is compressed in the direction of its length, it yields to the force in a different manner according to the proportion between its length and area of its cross section;" and that in case of a cylinder whose length is less than seven or eight times its diameter, it is impossible to bend it by a force applied longitudinally, as the piece is destroyed by splitting before the bending takes place; but when the length exceeds this, the pillar will bend under a certain load and be ultimately destroyed by a similar kind of action to that which has place in the transverse strain.

112. *Crushing force.*—A few experiments have been made on the resistance which different woods offer to a crushing force when their length is incon-

siderable, principally by M. Rondelet, in his '*Art de Bâtir*,' and by George Rennie, Esq., in the 'Philosophical Transactions' for 1818; but unfortunately the results differ very widely from each other.

According to M. Rondelet, it required a force of from 5000 to 6000 lbs. to crush a piece of oak of 1 inch base, and from 6000 to 7000 lbs. to crush a similar section of fir; whereas Mr. Rennie gives the following specific numbers, which are so much less than the former in the two cases which admit of comparison, as to throw considerable doubt on the subject.

Mr. Rennie's results are as stated below:

Base 1 *inch square.*—1 *inch in height.*

Elm, crushed with	1284 lbs.
American pine	1606
White deal	1928
English oak	3860
Do., length 4 inches, same base	5147
African oak, base 3 in., side length 81 in.,	60480
Or 6720 per square inch.	

This seems to prove that the resistance increases in a much higher ratio than the area, but without further experiments it is impossible to derive any general rule.

113. *Resistance of columns to flexure.*—This is the most important question connected with the inquiry, but it is, like the former, one on which few experiments have been made, and in which little has been

derived from theory, although it has engaged the attention of some of the most distinguished mathematicians of the last and present centuries. Experiments on this subject are, as we have said, very few indeed; those given by M. Girard, in his '*Traité Analytique de la Résistance des Solides*,' are the only ones of any importance to which we can refer, and the results in these are by no means so uniform as might be desired.

The following is an abstract from M. Girard's first and second Tables. Table I. contains the results of his experiments on the vertical pressure of oak beams. The first column contains the number of the experiment; the second, the dimensions and specific gravity[14] of the several pieces; the third and fourth, the height from the bottom to the point of greatest curvature; the former in the direction of the least thickness, and the latter in that of the greatest. The fifth and sixth exhibit the measure of the greatest deflection; the former in the direction of the least, and the latter in that of its greatest dimension: the seventh column shows the several weights under which those deflections were observed; the eighth, the time from the commencement; and the ninth contains remarks, &c.

We have only shown the effect of four different weights for each beam; but the author himself has

[14] M. Girard gives only the weight of the pieces; but we have preferred changing the weights into the specific gravities, as furnishing a readier means of comparing one piece with another.

in some cases employed ten, twelve, or more different pressures, measuring the deflection, &c. for each; but as it was thought unnecessary to follow him through the whole, the results of his first two and last two charges in the first eighteen experiments have been given. Those columns also, which M. Girard has drawn from computation founded on his theory, are omitted.

Table II., which is an abstract from the author's second Table, contains the results of his experiments on the transverse deflection of such of the beams as were not broken in the experiments above referred to: they were supported at each end at different lengths, and in different positions, viz. first with their greatest thickness vertical, and then with their less.

The formulæ M. Girard employs to compute the weight under which a piece of timber ought to begin to bend when pressed vertically, from the deflection being given when charged with any weight horizontally, are as follow:

Let P represent half the weight when the piece is charged horizontally in the middle, and b the corresponding deflection; f, half the length of the piece, and π, the semicircumference of a circle to diameter 1.

Then $\frac{Pf^3}{3b}$ = *absolute elasticity*,

And $Q = \frac{\pi^2 Pf}{12b}$ = *the weight*

under which the same piece will begin to bend when the pressure is vertical.

If, therefore, for the same depth and thickness, $\frac{Pf^2}{3b}$ were constant, the weight Q, under which a piece begins to bend, would be inversely as the square of the length: but M. Girard finds $\frac{Pf^2}{3b}$ nearly as the length, or as f; and consequently Q varies, *cæteris paribus*, as f inversely.

The formula given by Dr. Young, in his 'Natural Philosophy,' differs from this. See Prob. VI. Art. 115.

Note.—In the following Table, where two heights and two versed sines are connected by a { with one weight, it shows that the piece bent in two places, in opposite directions.

TABLE I.

114. *Girard's Experiments on the Vertical Pressure and Resistance of Oak Beams or Columns.*

No. of experiments.	Dimensions and specific gravity of the pieces.	Height of the greatest curvature from the foot. In the direction of the thickness.	Height of the greatest curvature from the foot. In the direction of the breadth.	Versed sine of greatest curvature. In the direction of the thickness.	Versed sine of greatest curvature. In the direction of the breadth.	Weight in kilogrammes.	Time in hours.	REMARKS.
	Metres.	Metres.	Metres.	Metres.	Metres.			
1	Length 2·5979	1·0689	1·2989	·0068	..	17320	0·83	Recovered its first form.
	Breadth 0·1580	..	..	·0070	·0068	29691	2·08	
	Thickness 0·1285	..	..	·0090	·0090	37429	2·91	
	Sp. gr. 1038	..	..	·0113	·0090	42418	9·58	
2	Length 2·5979	1·1907	1·2989	·0056	·0045	11993	2·50	Broke under the last weight.
	Breadth 0·1624	0·9742	..	·0068	·0079	25664	9·16	
	Thickness 0·1060	..	..	·0068	·0135	42514	10·83	
	Sp. gr. 984	..	..	·0068				
3	Length 2·5979	0·6495	..	·0023	..	11991	0·83	Recovered its first form.
	Breadth 0·1579	1·9484	..	·0017				
	Thickness 0·1050	1·7861	..	·0113	..	28575	9·58	
	Sp. gr. 1010	1·6237	..	·0282	..	31339	10·41	
4	Length 2·5979	1·2989	1·2989	·0113	·0079	11993	6·66	Did not recover its first form.
	Breadth 0·1330	1·2989	1·2989	·0180	·0124	17341	8·33	
	Thickness 0·0992	1·2989	1·2989	·0497	·0124	22939	10·00	
	Sp. gr. 1000							
5	Length 2·5979	1·2989	1·1366	·0169	·0068	11996	6·66	Broke under the last weight.
	Breadth 0·1308	1·2989	0·9742	·0372	·0113	17341	8·33	
	Thickness 0·1060	..	..	..	..	22931	8·75	
	Sp. gr. 923							
6	Length 2·2731	1·2989	1·1366	·0023	·0045	28619	9·58	Broke under the last weight.
	Breadth 0·1556	1·2989	1·1366	·0023	·0045	33115	16·66	
	Thickness 0·1308	1·4613	1·2989	·0090	·0034	47073	19·58	
	Sp. gr. 920	..	..	..	..	52270	22·50	
7	Length 2·2731	1·6237	0·9742	·0040	·0040	22934	2·08	Recovered its first form.
	Breadth 0·1579	1·6237	0·9742	·0040	·0040	28612	2·91	
	Thickness 0·1285	1·2989	1·2989	·0169	·0045	47047	20·83	
	Sp. gr. 973	1·2989	1·2989	·0186	·0090	47032	23·33	
8	Length 2·2731	1·6237	1·2989	·0062	·0034	17320	12·08	Broke under the last weight.
	Breadth 0·1556	1·6237	1·2989	·0096	·0034	22936	12·91	
	Thickness 0·1038	1·6237	1·2989	·0181	·0045	28616	13·75	
	Sp. gr. 972	..	..	..	..	33120	13·95	
9	Length 2·2731	1·4613	1·2989	·0068	·0068	17321	12·08	Broke under the last weight.
	Breadth 0·1579	1·2989	1·1366	·0124	·0045	22940	12·58	
	Thickness 0·1015	..	..	..	..	26626	14·50	
	Sp. gr. 926							

TABLE I.—(CONTINUED.)

No. of experiments.	Dimensions and specific gravity of the pieces.	Height of the greatest curvature from the foot. In the direction of the thickness.	Height of the greatest curvature from the foot. In the direction of the breadth.	Versed sine of greatest curvature. In the direction of the thickness.	Versed sine of greatest curvature. In the direction of the breadth.	Weight in kilogrammes.	Time in hours.	REMARKS.
	Metres.	Metres.	Metres.	Metres.	Metres.			
10	Length 2·2631	1·4613	1·1366	·0079	·0062	11999	10·00	Recovered its first form.
	Breadth 0·1262	1·4613	0·9742	·0079	·0062	15025	12·91	
	Thickness 0·1015	1·4613	0·9742	·0113	·0062	17320	22·91	
	Sp. gr. 1038	1·2989	0·9742	·0135	·0068	20326	25·83	
11	Length 1·9484	0·9742	0·9742	·0045	·0079	17321	7·08	Recovered its first form.
	Breadth 0·1556	0·9742	0·9742	·0045	·0090	22940	10·00	
	Thickness 0·1330	0·9742	0·9742	·0051	·0101	33105	26·66	
	Sp. gr. 1102	..	..	..	..	39644	27·50	
12	Length 1·9484	0·9742	0·9742	·0079	..	22940	20·00	
	Breadth 0·1579	0·9742	0·9742	·0079	..	33123	25·00	
	Thickness 0·1308	0·6495	0·9742	·0068	·0146	39637	27·91	
	Sp. gr. 935	1·6237		·0023				
13	Length 1·9484	0·6495	0·6495	·0045	·0056	17321	2·08	Recovered its first form.
	Breadth 0·1579	0·6495	0·6495	·0062	·0068	22939	3·33	
	Thickness 0·1015	0·6495	0·6495	·0068	·0108	39456	33·33	
	Sp. gr. 987			·0023				
14	Length 1·9484	1·4613	0·9742	·0045	·0040	11973	10·00	Broke under the last weight.
	Breadth 0·1601	1·4613	0·9742	·0045	·0045	17274	27·50	
	Thickness 0·1015	1·6237	1·2989	·0113	·0090	28509	40·41	
	Sp. gr. 1035	..	..	..	..	32996	50·41	
15	Length 1·9484	1·2989	0·6495	·0056	·0045	17294	10·00	Recovered its first form.
	Breadth 0·1330	0·9742	0·6495	·0051	·0045	22899	28·33	
	Thickness 0·1060	1·6237	1·4342	·0068	·0118	46952	86·66	
	Sp. gr. 1032	0·3247		·0011				
16	Length 1·9484	0·9742	0·9742	·0045	·0056	11998	18·33	Broke under the last weight.
	Breadth 0·1285	0·9742	0·9742	·0056	·0079	17317	20·00	
	Thickness 0·1082	0·6495	0·6742	·0045	·0135	37273	92·50	
	Sp. gr. 993	0·2435		·0011				
17	Length 2·2731	0·6495	0·9742	·0029	·0028	11998	10·00	Recovered its first form.
	Breadth 0·1579	..	0·6495	..	·0034	17320	20·00	
	Thickness 0·1082	1·6237	0·9742	·0056	·0045	33120	52·50	
	Sp. gr. 920	1·4613	0·9742	·0113	·0051	39630	57·50	
18	Length 2·5979	0·9742	1·2989	·0051	·0034	11999	10·00	Broke under the last weight.
	Breadth 0·1579	0·9742	1·2989	·0068	·0056	17321	20·00	
	Thickness 0·1353							
	Sp. gr. 916	1·6237	1·2989	·0146	·0079	37305	50·83	

TABLE II.

Girard's Experiments on the Deflection of Oak Beams, when supported at the ends, and loaded in the middle of their length.

Note.—These Beams were the same as those in the preceding Table.

No. of experiments.	Dimensions in metres, and specific gravity of the pieces.	Deflection in metres.	Weight in kilogrammes.	Absolute elasticity, computed from $\frac{Pf^3}{3b}$.*	REMARKS.
1	Length 2·5978	·0180	1884	38250	These two experiments were performed on the same piece of wood, which was also the same as No. 1, first Table.
	Depth ·1579	·0238	2379	36524	
	Breadth ·1285	·0238	2932	37876	
	Sp. gr. 1038	·0373	3467	33954	
2	Length 2·2731	·0158	1884	29174	
	Depth ·1579	·0215	2395	27266	
	Breadth ·1285	·0248	2930	28909	
	Sp. gr. 1038	·0300	3470	28304	
3	Length 1·9484	·0045	1882	64461	These were all the same piece of wood, viz. No. 7 of the first Table.
	Depth ·1579	·0056	2393	65864	
	Breadth ·1285	·0113	4007	54634	
	Sp. gr. 973	·0153	4542	45744	
4	Length 1·6237	·0056	1877	29893	
	Depth ·1579	·0068	2388	31312	
	Breadth ·1285	·0119	4976	37288	
	Sp. gr. 973	·0141	5512	34844	
5	Length 1·6237	·0056	1876	29877	
	Depth ·1285	·0079	2388	26953	
	Breadth ·1579	·0119	3463	34313	
	Sp. gr. 973	·0158	4000	29973	
6	Length 1·9484	·0090	1874	32101	This piece was No. 13 of Table I.
	Depth ·1579	·0135	2385	27228	
	Breadth ·1015	·0271	3786	22669	
	Sp. gr. 987	·0316	4519	22038	
7	Length 1·9484	·0135	1874	21395	
	Depth ·1015	·0226	2383	16313	
	Breadth ·1579	·0271	2919	16500	
	Sp. gr. 987	·0474	3448	11212	
8	Length 1·9484	·0158	1871	18257	This piece was No. 15 of Table I.
	Depth ·1330	·0180	2380	20381	
	Breadth ·1060	·0282	2917	15942	
	Sp. gr. 1032				
9	Length 1·9484	·0232	1870	12437	
	Depth ·1060	·0893	2916	13267	
	Breadth ·1330				
	Sp. gr. 1032				

* P = half the charge, f half the length, and b the deflection.

Solution of Practical Problems.

115. Having in the foregoing pages established the requisite data and formulæ for determining the dimensions of beams under every variety of transverse strain, it is proposed to give a few examples by way of illustration, in which I shall confine myself to the woods given in the preceding Table of Data; these having been carefully selected, and the experiments made with this particular object. The numbers for direct cohesion are drawn from Art. 14.

PROBLEM I.

To determine the Strength of Direct Cohesion of a piece of Timber of any given Dimensions.

Rule.—Multiply the area of the transverse section, in inches, by the cohesion per square inch, Art. 14, and the product is the strength required.

In practice, the weight the timber has to support should not exceed one-fourth of the strength as calculated by the rule.

Example I.—What weight will be required to tear asunder a piece of teak 3 inches square?

In this case the tabular value is . . .	15000
The area of the section $3 \times 3 =$ . . .	9
The weight required	135000 lbs.

Example II.—The diameter of a rod of ash being

2 inches, and its specific gravity 700, what weight will be required to tear it asunder?

The tabular value is	17000
The area of the section 2 × 2 × ·7854 =	3·1416
The product	53407·2 lbs.

Note.—If the weight be given and the area of section required, it is only necessary to divide the given weight by the tabular value of cohesion.

PROBLEM II.

To find the Strength of a Rectangular Beam of Timber, fixed at one end and loaded at the other.

Rule.—Multiply the value of S, in the Table of Data, by the area, and the depth of the section in inches, and divide that product by the leverage in inches, and the quotient will be the weight required in lbs.

Note 1.—In case the beam is inclined, the leverage is the distance I L, or F′ L′, fig. 6, Plate III. When the beam is horizontal, the leverage is usually called the length.

Note 2.—In practice, the load ought not to be greater than one-fourth of the weight found by the rule; for permanent stretching or displacement of the fibres begins to take place as soon as the load exceeds about one-fourth of the breaking weight. This will be perceived by comparing the weights which the specimens bore, without loss of elasticity,

with the weights that broke them, in the Table of Data.

Note 3.—If the load be distributed in any manner over the length of the beam, the horizontal distance between the point of support and a vertical line drawn through the centre of gravity of the load, must be taken for the leverage.

Example I.—A beam projecting 5 feet over the point of support, is 6 inches deep and 4 inches in breadth of Riga fir, and is intended to support a load at its extremity; it is required to determine the greatest load it would bear, and the load it may be exposed to without injury.

For Riga fir, $S = 1108$, and the area being $6 \times 4 = 24$, the depth 6 inches, the leverage 5 feet $= 60$ inches, we have $\frac{1108 \times 24 \times 6}{60} = 2659 \cdot 2$ lbs. the greatest or breaking load; and $\frac{2659 \cdot 2}{4} = 664 \cdot 8$ lbs. for the load it would bear without injury.

Example II.—A cistern to contain 36 cubic feet, or one ton of water, is to be supported by two cantilevers: the projection of the cistern from the face of the wall being 4 feet, it is required to determine the size for the cantilevers.

Let the cantilevers be of larch, such as the 3rd specimen, then we find by the Table of Data, $S = 1127$, and the depth 5 inches. The load on them will be 1 ton $= 2240$ lbs., and the weight will be uniformly distributed over the length; therefore, the distances of the centre of gravity from the wall

will be half the length, or 2 feet = 24 inches, which is the leverage. This is the reverse of the preceding operation, on account of the weight being given.

$\frac{2240 \times 24}{1127 \times 5} = 9{\cdot}54$ inches, nearly, for the area of both cantilevers, or $\frac{9{\cdot}54}{2} = 4{\cdot}77$ inches for the area of one of them; and if the section be rectangular, the depth being 5 inches, the breadth will be ·954 inch for each cantilever.

PROBLEM III.

To determine the Strength of a Rectangular Beam of Timber when it is supported at the ends, and is loaded in the middle of its length.

Rule.—Multiply the value of S, in the Table of Data, by four times the depth in inches, and by the area of the section in inches, and divide the product by the distance between the supports, in inches, and the quotient will be the greatest weight the beam will bear in lbs.

Note 1.—If the beam be not horizontal, the distance between the supports must be the horizontal distance.

Note 2.—One-fourth of the weight found by the rule should be the greatest weight upon a beam in practice.

Note 3.—If the load be applied at any other point than the middle, it will be as the rectangle of the

segments, into which the point divides the distance between the supports, is to the square of half that distance; so is the weight found by the rule, to the weight the beam will sustain at the given point.

Note 4.—If the load be distributed in any manner whatever over the beam, the centre of gravity of the load must be considered its place, and its stress equal to the whole weight; unless part of such weight be sustained by the supporting points independently of the resistance of the beam.

Example I.—Required the weight a beam of Riga fir, 1 foot square, would sustain in the middle, its length being 20 feet?

In this case the tabular value of S is 1108, and the depth 12 inches, and the area 144 inches, the length 240 inches; consequently,

$$\frac{1108 \times 4 \times 12 \times 144}{240} = 32010 \text{ lbs.}$$

And the beam may be loaded in practice with $\frac{32010}{4} = 8002\frac{1}{2}$ lbs., without injury to its texture.

If the load were applied at 8 feet distance from the end, instead of being applied in the middle, then it would be 12 feet from the other end; and by Note 3, we have $8 \times 12 : 10 \times 10 :: 8002\frac{1}{2} : 8336$ lbs. nearly, for the weight the beam 12 inches square would support at 8 feet from the end; showing the advantage of applying the load as far from the middle as possible.

Example II.—To determine the size of a girder of Riga fir for a warehouse, where the distance between

the points of support is 18 feet = 216 inches, and the greatest probable stress at the middle, including the weight of the floor itself, 20 tons.

The tabular number is

$$S = 1108, \text{ and } 20 \text{ tons} = 44800 \text{ lbs.}$$

Let us further suppose that the greatest depth of the timber intended for the purpose is 20 inches. By reversing the rule, we have

$$\frac{4 \times 44800 \times 216}{1108 \times 4 \times 20 \times 20} = 21{\cdot}83 \text{ inches}$$

for the breadth of the girder, which would be obtained by bolting together two pieces, each 20 inches by 11 inches; or much better by putting the two pieces at the most convenient distance apart that would admit of both resting on the sustaining piece.

If there be only 20 tons distributed uniformly over the surface of the floor, then a girder of 20 inches by 11 inches would be sufficient.

PROBLEM IV.

To determine the Dimensions of a Beam capable of supporting a given Weight with a given degree of Deflection, when fixed at one end.

Rule.—Divide the weight in lbs. by the reduced tabular value of E,[15] multiplied by the breadth and deflection, both in inches; then the cube root of the quotient, multiplied by the length in feet, will be the depth required in inches.[16]

Example I.—A beam of Riga fir is intended to bear a load of 665 lbs. at its extremity, its length being 5 feet, its breadth 4 inches, and the deflection not to exceed $\frac{1}{4}$ of an inch.

In this case the tabular value of E is 192; hence, $\frac{665}{192 \times 4 \times \frac{1}{4}} = 3{\cdot}44$; the cube root of which is $1{\cdot}5096$; hence, $5 \times 1{\cdot}5096 = 7{\cdot}548$ inches, the depth required.

By reference to Example I. of Prob. II. it will be found that a beam of 6 inches depth would be suf-

[15] The value of E in these rules is the tabular value divided by 1728, which renders it unnecessary to reduce the length in feet into inches.

For English oak, $E = 210$
For Riga fir, $E = 192$

[16] Owing to the imperfect fixing which obtains in practice, the deflection will in ordinary cases be greater than that given by the rule, in the proportion of $1 : \sqrt{2}$.

ficient to bear the load; but when, from the nature of the construction, only a limited degree of flexure can be allowed, this mode of calculation becomes necessary.

Note 1.—When the weight is uniformly distributed over the length of the beam, the deflection will be only $\frac{5}{8}$ths of the deflection from the same weight applied at the extremity, and in the rule consider the weight reduced in this proportion.

Note 2.—If the beam be a cylinder, the deflection will be 1·7 times the deflection of a square beam, other circumstances being the same.

Note 3.—In the above examples the reduction of results to the differences depending on the specific gravity is not shown, neither is it applicable in practice; but for theoretical comparison it is important, and may always be performed by stating, as the specific gravity of the tabular specimen is to the load supported in any example, so is the actual specific gravity of the specimen to the load it would support under similar circumstances.

PROBLEM V.

To find the Dimensions of a Beam capable of sustaining a given Weight with a given degree of Deflection, when supported at both ends.

Rule.—Multiply the weight to be supported in lbs. by the cube of the length in feet. Divide this product by 16 times the reduced tabular value of E, (see Note 1, Prob. IV.,) multiplied into the given deflection in inches, and the quotient is the breadth multiplied by the cube of the depth in inches.

Note 1.—If the beam be intended to be square, then the breadth is equal to the depth, and the fourth root of the quotient is the depth required.

Note 2.—If the beam be a cylinder, multiply the quotient by 1·7, and then the fourth root will be the diameter of the cylinder.

Note 3.—When the load producing the depression is greater than one-fourth of the greatest stress the beam would bear, it is too great to be trusted in construction; but in timber this limit is seldom exceeded, on account of its flexibility.

Note 4.—If the load be uniformly distributed over the length, the deflection will be $\frac{5}{8}$ths of the deflection from the same load collected in the middle. And in the rule, employ $\frac{5}{8}$ths of the weight of the load instead of the whole load.

Example I.—The length of the fir shaft of a water-wheel being 20 feet, and the stress upon it

7 tons, it is required to determine its diameter so that its deflection may not exceed ·2 of an inch.

The reduced tabular value of E = 192, or more exactly 16 E = 3075, and 7 tons = 15680 lbs.; hence (by the Rule and Note 2) $\frac{1 \cdot 7 \times 15680 \times 20^3}{3075 \times \cdot 2} = 346730$, nearly. The fourth root of this sum is 24·3 inches, the diameter required.

Shafts which are to be cut for inserting arms, &c. will require to be larger, in a degree equivalent to the quantity destroyed by cutting.

The flexure of shafts ought not to exceed $\frac{1}{100}$ of an inch for each foot in length, this being considered the limit; and it will be always desirable to make shafts as short as possible, to avoid bending.

Example II.—The greatest variable load on a floor being 120 lbs. per superficial foot, it is required to determine the depth of a square girder to support it, the area of the floor sustained by the girder being 160 feet, the length of the girder 20 feet, and the deflection not to exceed half an inch.

The reduced value of E for Riga fir is 192, or 16 E = 3075, and the weight is 120 × 160 = 19200 lbs. uniformly distributed; hence (by Note 4) we have

$$\frac{\frac{5}{8} \times 19200 \times 20^3}{3075 \times \frac{1}{2}} = 62440.$$

The fourth root of this number is 15·8 inches, the depth required.

The deflection of $\frac{1}{40}$th of an inch for each foot in length is not injurious to ceilings; indeed, the

usual allowance for settlement is about twice that quantity. Ceilings have been found to settle about four times as much without causing cracks, and have been raised back again without injury.

The variable load on a floor seldom can exceed half the quantity of 120 lbs. on a superficial foot, unless it be in public rooms; hence, the number may be taken from 60 to 120, according to circumstances.

The same rule applies to joists of different kinds for floors; the area of the floor supported by the joists being multiplied by from 60 to 120 lbs. per superficial foot, according to the use the room is designed for.

Example III.—To determine the size of a rafter for a roof to support the covering of slate, the distance between the supports being 6 feet, and the weight of a superficial foot, including the stress of the wind, being 56 lbs., and the deflection not to exceed $\frac{1}{40}$th of an inch for each foot in length.

The tabular value gives

$$16\,E = 3075, \text{ the weight} = 56 \times 6 = 336 \text{ lbs.};$$

hence (by Note 4),

$$\frac{5 \times 336 \times 40 \times 6^3}{8 \times 3075 \times 6} = 98{\cdot}34.$$

If the breadth be made $2\frac{1}{2}$ inches, then

$$\frac{98{\cdot}34}{2{\cdot}5} = 39{\cdot}3\,;$$

and the cube root of 39·3 is 3·4 inches, the depth required.

PROBLEM VI.

To determine the Dimensions of a Pillar or Column to bear a given Stress in the direction of its Axis, without sensible Curvature.

Rule.—Multiply the weight to be supported in lbs. by the square of the length of the pillar in feet, and divide the product by 40 times the tabular value of E, (Art. 104,) reduced as in Prob. IV., the quotient will be equal to the breadth multiplied by the cube of the least thickness; therefore, either the breadth or thickness will require to be fixed upon, before the other can be found.[17]

Note 1.—If the pillar be square, its side will be the fourth root of the quotient.

[17] The rule is derived as follows:—The force f, which a column will bear without sensible flexure is

$$f = \cdot 8225 \frac{d^2 m}{l^2}; \text{ and } m = \frac{l^3 \mathrm{W}}{4 d^2 \delta}$$

(see Dr. Young's Nat. Phil. ii. pp. 47, 48); hence, when l is in feet, we have

$$f = \frac{2\cdot 4675\, l \mathrm{W}}{\delta}. \quad \text{But we have } \mathrm{W} = \frac{16\, \mathrm{E}\, a\, d^3\, \delta}{l^3};$$

$$\text{consequently, } f = \frac{39\cdot 48\, \mathrm{E}\, a\, d^3}{l^2}.$$

In the rule the number 40 is used for 39·48. If the above expression be divided by 1·7, it becomes a rule for a cylinder,

$$\text{or } \frac{1\cdot 4508\, \mathrm{E}\, d^4}{l^2} = f, \text{ or } \frac{1\cdot 5\, \mathrm{E}\, d^4}{l^2} = f, \text{ for simplicity.}$$

Note 2.—If the column be a cylinder, multiply the tabular value of E by 24 instead of 40. The fourth root of the quotient in the rule will be the diameter of the cylinder.

Example I.—What should be the least thickness of a pillar of oak to bear a ton without sensible flexure, its breadth being 3 inches, and its length 5 feet?

The reduced tabular value of E for oak is 210, and 1 ton = 2240 lbs.; hence

$$\frac{2240 \times 5^2}{40 \times 210 \times 3} = 2{\cdot}222.$$

The cube root of 2·222 is 1·31, nearly, which is the side as required.

Example II.—Required the side of a square post of Riga fir to support 10 tons, the pressure being in the direction of the axis, and the height of the post 12 feet.

The reduced tabular value of E is 192; hence

$$\frac{22400 \times 12^2}{40 \times 192} = 419{\cdot}6, \text{ nearly};$$

the fourth root of which is 4·53 inches, the side of the post as required.

The dimensions given by this rule are obviously too small to be used in practice. The rule only shows the extreme load that can be supported by a pillar under the theoretical condition that the pressure exactly coincides with the axis of the pillar; but this pressure will overpower the resistance of the pillar if it has the smallest deviation from the axis.

(See Dr. Young's Nat. Phil. ii. p. 47.) It is the more necessary to point out this circumstance, because it is the same in Girard's Rules, quoted in p. 164; and Poisson's Equation ('Traité de Mécanique,' Art. 160, tome i.). For the case where the force is applied at a distance from the axis, Poisson has left the solution incomplete. Dr. Young has given a solution of this case in his work above quoted; but it is not quite so convenient for application as one which may be obtained by assuming certain data that are difficult to obtain in a simple form by calculation.

In the former editions of this work, other problems and questions were given connected with this subject; but the data are so uncertain, that it has been thought better to omit them,—no rule being preferable to one which may be erroneous.

ON THE TRANSVERSE STRENGTH OF BRICK, STONE, CEMENT, ETC.

116. THERE are but few cases in which it is important to know the transverse strength of the above materials, and we have but scanty information on the subject. The following includes nearly all I have seen.

Cohesive Power of Stone.

The first experiments, I know of, relative to the cohesion of stone, are those of M. Gauthey, a German engineer; who found, from the results of several trials, that a piece of stone, of what he denominated soft *givry*, 1 foot square and 1 foot long, required a weight of 5000 lbs. to break it across, one end being fixed in a rock, and the weight hung on at the other; and that hard *givry* required, under similar circumstances, 5600 lbs. to produce fracture. Taking our dimensions, therefore, in feet, we have

$$\text{Soft givry, } \frac{l\,W}{a\,d^2} = 5000.$$

$$\text{Hard ditto, } \frac{l\,W}{a\,d^2} = 5600.$$

Or taking, as we have done in timber, the dimensions in inches,

$$\text{Soft givry, } S = \frac{l\,W}{a\,d^2} = 35.$$

$$\text{Hard ditto, } S = \frac{l\,W}{a\,d^2} = 39.$$

I am not acquainted with the nature of this stone; but its power is very inferior to three specimens of stone tested by George Rennie, Esq., at the London Docks. These specimens, which I saw, were certainly very fine; but the difference between the strength of them, and the above, is very extraordinary, particularly the Welsh slate.

Experiments made by Mr. G. Rennie, upon the following Stones, generally used for paving.

The dimensions were, length 12 inches; breadth 2½ inches; depth 1 inch. The stones were laid flat on two bearings, 10 inches apart, and the weights suspended from the middle of the stones.

Kinds of Stones.	Weight it bore. cwt. qr. lbs.	Weight of stone. lbs. oz.	Value of $S = \frac{l\,W}{4\,b\,d^2}$.
Green Moor Yorkshire Blue Stone	2 3 27	2 12	335
Ditto Ditto White Do.	3 0 23	2 12	359
Caithness—Scotland	7 2 17	3 0	857
Valentia—Ireland	7 3 3	3 2	871
Welsh	17 0 12	3 2	1961

On the Cohesive Power of Brick.

117. In order to ascertain the cohesion of brick, three common bricks were procured, which had been exposed to the weather for two years at least; and three of the same kind of recent make; and three of the best stock. These were supported between two props, 8 inches apart, and then loaded in the middle till they broke. The least thickness of the bricks was $2\frac{1}{2}$ inches, and the greatest 4 inches; and they were placed with their less dimension vertical. The following are the results of these experiments:

Common old brick.	Common new brick.	Best stock.
1...... 384 lbs.	1...... 411 lbs.	1...... 434 lbs.
2...... 298	2...... 411	2...... 479
3...... 347	3...... 387	3...... 420
Mean 3(1029	3)1209	3)1333
343	403 lbs.	444 lbs.

Hence, taking the dimensions in feet:

$$\text{Common old brick } \frac{l\,W}{4\,a\,d^2} = 3939$$

$$\text{Do. of recent make } \frac{l\,W}{4\,a\,d^2} = 4631$$

$$\text{Best stock} \ldots\ldots \frac{l\,W}{4\,a\,d^2} = 5115.$$

Strength of different Cements.

118. I am indebted for the following experiments, on the strength of different cements, to M. I. Brunel, Esq., who made them in reference to the construction of the tunnel under the Thames.

Experiment 1.—Against a brick wall a brick was attached by cement, its broadest surface to the wall, and with its length vertical to this brick, another was added; to this a third; and so on till thirteen bricks were thus cemented to each other: to the thirteenth brick another was added endwise; and, lastly, a fifteenth brick to the end of this, in the same position as the first thirteen. The cement supported this length of column without any appearance of breaking. Two bricks were then laid on the farthest extremity; and, lastly, four others in front of these: in laying on the last brick the column or arm broke at the wall.

Experiment 2.—In this experiment twelve bricks were cemented to each other exactly as above; and then nine bricks more were laid on, viz. by placing one over each of the last seven; and, lastly, two at the farthest extremity. The arm was left in this state without breaking.

These experiments were made with Parker and White's cement, which was perfectly dry in both cases before the additional bricks were placed.

Experiment 3.—Eleven bricks were attached in the same manner, and several weeks after, twenty-one bricks were piled upon the farthest extremity. Adding the last brick caused the arm to break off at the wall.

Experiment 4.—Eleven bricks were attached to the wall edgewise; in this state the arm supported four bricks, and then broke at the wall.

These two experiments were made with Messrs. Turner and Montague's cement.

Experiment 5.—A column was built 6 feet high and 14 inches square, and when dry was laid lengthwise on two props, 5 feet 6 inches asunder; in this position a weight of 896 lbs. was laid over the centre, which it supported without breaking. It continued to bear this a considerable time.

Experiment 6.—Exactly the same experiment was tried on a column, using half cement and half sand; this bore the same weight for half an hour, and then broke.

These experiments were made with Mr. Shepherd's cement.

It may be proper to add, that in every case of fracture the brick itself gave way before the cement.

CRUSHING FORCE.

119. *Experiments on the resisting Power of various Building Materials, Stone, Brick, &c., to a Crushing Force.*

No. of experiments.	MATERIALS.	Specific gravity.	Crushing weight.
1	Portland stone, 1 inch cube............	....	1284
2	Ditto 2 inches long...........	....	805
3	Statuary marble, 1 inch	....	3216
4	Craigleith do. do................	....	8688
5	Chalk, cube of 1¼ inch.....	....	1127
6	Brick, pale red, do......	2085	1265
7	Roe stone, Gloucestershire, do......	....	1449
8	Red brick do......	2168	1817
9	Ditto, Hammersmith paviors', do......	....	2254
10	Burnt do. do......	....	3243
11	Fire brick do......	....	3864
12	Derby grit do......	2316	7070
13	Ditto, another specimen, do......	2428	9776
14	Killaly white freestone, do......	2423	10264
15	Portland do. do......	2428	10284
16	Craigleith do. do......	2452	12346
17	Yorkshire paving, with the strata, do......	2507	12856
18	Ditto do. against strata, do......	....	12856
19	White statuary marble do......	2760	13632
20	Bramley Fall sandstone do......	2506	13632
21	Ditto against strata, do......	....	13632
22	Cornish granite do......	2662	14302
23	Dundee sandstone do......	2530	14918
24	Portland, a 2-inch cube	2423	14918
25	Craigleith, with the strata, 1¼ inch cube...	2452	15560
26	Devonshire red marble do......	....	16712
27	Compact limestone do......	2584	17354
28	Granite, Peterhead do......	....	18636
29	Black compact limestone do......	2598	19924
30	Purbeck do......	2599	20610
31	Black Brabant marble do......	2697	20742
32	Freestone, very hard do......	2528	21254
33	White Italian marble do......	2726	21783
34	Granite, Aberdeen, blue kind do......	2625	24556

See Experiment by G. Rennie, Esq., Phil. Trans. 1818.

120. *On the Force necessary to overturn Walls and Columns.*

A column of soft givry (assuming the specific gravity 2000) is erected on a base 2 feet square, and its height is 20 feet. Required the force, acting perpendicular to its end, necessary to overturn it.

It is obvious here that the force necessary to produce the fracture will consist of two parts, viz. 1st, that which is necessary to produce an equilibrium with the weight of the wall, independent of the cohesion; and, 2nd, of a part sufficient to overcome the cohesion, independent of the equilibrium. The latter will vary with the area of the base of fracture and the point of application of the force; and the former with the weight of the column and the situation of its centre of gravity.

Generally, if W denote the weight of the wall, l, the distance of the point of application of a direct force from the fulcrum about which the wall is to turn, and r, the distance of the centre of gravity from the same, both in feet; then, by the property of the lever, $F = \frac{W r}{l}$, the force necessary to produce an equilibrium.

And from the theory of the strength of materials,

$$\frac{F l}{a d^2} = C, \textit{ a constant quantity,}$$

where a is the breadth, and d the depth of the section of fracture in feet; whence $F' = \frac{a d^2 C}{l}$, the

force requisite to produce the fracture: therefore, $F + F' = \frac{W r}{l} + \frac{a d^2 C}{l}$, the whole force required.

In the present case,

$$W = 2000 \text{ oz., or } 125 \text{ lbs., and } 125 \times 2^2 \times 20 = 10000, r = 1, l = 20,$$
$$a = 2, d = 2, \text{ and let } C = 500;$$

whence,

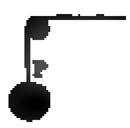
$$F + F' = \frac{10000}{20} + \frac{8 \times 500}{20} = 500 + 200 = 700 \text{ lbs.},$$

the force sought.

On the Pressure of Banks and the Dimensions of Revetments.

121. Having established above (at least approximatively) certain data relative to the resistance and cohesion of walls and columns, it remains now to ascertain the pressure of earth against revetments, in order thence to determine their requisite dimensions, that an equilibrium may be established between those two forces.

For this purpose, let CBHE (in the annexed figure)



denote a bank of earth, the natural slope of which is

E B. Let the weight of the part C B E, 1 foot thick, $= \mathrm{W}$, and make $\mathrm{B\,E} = l$, $\mathrm{C\,B} = h$, $\mathrm{C\,E} = b$. From the theory of the inclined plane,

$$\text{as } l : h :: \mathrm{W} : \frac{h}{l}\mathrm{W} = \mathrm{W}',$$

the weight which, attached to the centre of gravity of the sliding solid, would preserve it in equilibrio on the plane EB, supposing no friction between the two surfaces. The weight W' will, therefore, under this supposition, denote the quantity; FI, the direction; and I, the effective point of application of the force of the bank against the wall ABCD. And now, to find the horizontal force at I: since the triangles KFI and BEC are similar, we have by the resolution of forces

$$l : b :: \mathrm{W}' : \frac{b\,\mathrm{W}'}{l} = \frac{b\,h\,\mathrm{W}}{l^2}$$

for the horizontal effect at I: also, since K A, from the nature of the centre of gravity $= \frac{1}{3}$ of DA, or $\frac{1}{3}h$;

$$\text{and } \mathrm{K\,I} = \frac{h\,x}{b}, \text{ and } \mathrm{A\,I} = \tfrac{1}{3}h - \frac{h\,x}{b},$$

(x being taken to denote the breadth of the wall at bottom), the whole effect of the above pressure to turn the wall as a lever about a fulcrum at A, will be expressed by

$$\left(\frac{1}{3}h - \frac{h\,x}{b}\right)\frac{b\,h\,\mathrm{W}}{l^2}, \text{ or } \left(\frac{1}{3}h - \frac{h\,x}{b}\right)\frac{b^2 h^2 s}{2\,l^2},$$

s denoting the specific gravity of the earth.

Now, to find the dimensions of the revetment requisite to keep this force in equilibrio, let h' denote

the given height of the wall; S, its specific gravity, or the weight of 1 cubic foot; x, as above, the thickness of the wall at the bottom; y, the distance of the perpendicular, let fall from its centre of gravity upon its base, from the outward edge of the wall at bottom, viz. the point about which the wall turns; and a, the area of its transverse vertical section; then, since we are only considering 1 foot in length, the same quantity, a, will also denote the solid content of the wall opposed to the bank; and, consequently, a S will be its weight.

Therefore, by the preceding proposition,

$$F = y\,a\,S,$$

the resistance which the wall opposes in consequence of its weight, and

$$F' = C\,x^2,$$

the resistance from cohesion, C being a constant quantity, $\frac{1}{10}$th of which we may take = 500, as in the preceding article; whence

$$y\,a\,S + C\,x^2$$

will be the whole resistance opposed to the bank; and, consequently, in case of an equilibrium, or of an equality between the force of pressure of the bank and the resistance of the wall, we shall have

$$y\,a\,S + C\,x^2 = \frac{b^2\,h^3\,s}{6\,l^2} - \frac{b^2\,h^3\,s\,x}{2\,b\,l^2};$$

a general formula, from which x, the breadth of the wall, in all cases may be determined.

If the wall be rectangular, then $y = \frac{1}{2}x$, and $a = h'x$, and the above becomes

$$\tfrac{1}{2}h'\mathrm{S}x^2 + \mathrm{C}x^2 = \frac{b^2h^3s}{6l^2} - \frac{b^2h^3sx}{2bl^2}$$

$$\text{or, } x^2 + \frac{bh^3sx}{\mathrm{S}h'l^2 + 2\mathrm{C}l^2} = \frac{b^2h^3s}{6\mathrm{C}l^2 + 3h'l^2\mathrm{S}}.$$

If the wall be triangular, then $y = \frac{2}{3}x$, and $a = \frac{1}{2}h'x$, and the above becomes

$$\tfrac{1}{3}h'\mathrm{S}x^2 + \mathrm{C}x^2 = \frac{b^2h^3s}{6l^2} - \frac{b^2h^3sx}{2bl^2};$$

$$\text{or, } x^2 + \frac{bh^3sx}{\frac{2}{3}h'l^2\mathrm{S} + 2\mathrm{C}l^2} = \frac{b^2h^3s}{2h'\mathrm{S}l^2 + 6\mathrm{C}l^2}.$$

Example I.—As an example, let the natural slope of a given soil, when unsupported, be 45°, and its specific gravity 2000, or the weight of a cubic foot, 125 ℔s.; and let it be required to determine the breadth of a rectangular wall of soft givry necessary to support it: the wall and bank being both 12 feet high; and the specific gravity of the wall 2500, or 156 ℔s. to the cubic foot.

Here $h' = 12$, $h = 12$, $b = 12$, $l = 12\sqrt{2}$, $\mathrm{S} = 156$, $s = 125$, and $\mathrm{C} = 500$.

Whence,

$$x^2 + 3{\cdot}794\,x = 15{\cdot}176;$$

$$\text{or, } x = -1{\cdot}897 \pm \sqrt{(1{\cdot}897^2 + 15{\cdot}176)} = 2{\cdot}435 \text{ feet.}$$

Example II.—Let all the data remain the same, to find the breadth at bottom of a triangular wall, that will keep the same bank in equilibrio.

Here, putting our second formula into numbers, we have

$$x^2 + 5{\cdot}148\,x = 20{\cdot}594; \text{ or,}$$

$$x = -2{\cdot}574\,x + \sqrt{(2{\cdot}574^2 + 20{\cdot}594)} = 2{\cdot}643 \text{ feet.}$$

This is but little different from the former, as ought obviously to be the case, because a great part of the resistance is due to the cohesion of the bottom section, that arising from the weight being comparatively small: it is singular, therefore, that the former datum has never (I believe) been introduced into the solution of the problem. Prony, who has attempted an elaborate solution of this proposition, has no reference to the wall's cohesion. It will be observed, also, that in the above investigation we have not considered the friction of the two surfaces: this is, of course, very considerable, and will reduce the thickness of the wall to a quantity less than the above. Experiments are, therefore, necessary to establish this point: in the mean time it may be observed, that as it is always desirable that the resistance of the wall should be more than equal to the pressure it has to sustain, it will be safer to omit it entirely than to introduce it without very correct data, drawn from the results of experiments carried on upon a large scale.

Example III.—Supposing the wall to be built of the best stock brick, which weighs 100 lbs. to the cubic foot, and that a cubic foot of the earth weighs 96 lbs.; also that the bank is 12 feet high, and the natural slope of the soil is 30°: what must be the thickness of the rectangular wall that will just prevent the bank from slipping?

Example IV.—With the same data, required the thickness of the wall at bottom, supposing it in the

form of a triangular wedge, as in the second example above.

Example v.—To find the thickness of an upright rectangular wall necessary to support a body of water, the depth being 10 feet, and the wall 12 feet high, the specific gravity of water being 1000, and the best stock brick 2000.

Example vi.—Required the thickness of the wall at bottom, supposing the data the same as in the preceding example, but the wall to be in the form of a triangle, as in examples ii. and iv.

Note.—The pressure in the last two examples is to be estimated on the principles of the pressure of fluids.

122. *Remark.*—The above can only be considered as a very imperfect sketch of the theory of Revetments, at least as relates to its practical application, for want of the proper experimental data; being merely given, in connexion with our general theory of the strength of materials, for the sake of introducing considerations relative to the cohesion of walls, &c., which have been commonly omitted: and the consequence has been, that, according to all theories, (and there have been several,) the computed thickness of the wall has very far exceeded what was ever considered to be practically necessary.

To render the theory complete, with respect to its practical application, it is necessary to institute a course of experiments upon a large scale: first, upon

the strength of common cement and mortar; and, secondly, upon the force with which different soils tend to slide down, when erected into the form of banks. A well-conducted set of experiments of this kind would blend into one what many writers have divided into several distinct data. Thus some authors have considered first, what they call the natural slope of different soils, by which they mean the slope that the surface will assume when thrown loosely in a heap; very different, as they suppose, from the slope that a bank will assume that has been supported, but of which that support has been removed or overthrown. This, therefore, leads to the consideration of the friction and cohesion of soils, and what is denominated the slope of maximum thrust: but however well this may answer the purpose of making a display of analytical transformations, I cannot think it is at all calculated to obtain any useful practical results. I should conceive that a set of experiments, made upon the absolute thrust of different soils, which would include or blend all these data in one general result, would be much more useful, as furnishing less causes of error, and rendering the dependent computations much more simple and intelligible to those who are commonly interested in such deductions.

We may further observe, that the method of resolving the force of the bank at the point I, instead of the point F, which former is obviously the effective point as regards the lever by which the wall

turns, shows, that while the continuation of the slope falls within the base of the wall, the soil which forms it will add to the stability of the revetment; which is conformable to the experiments of Major-General Pasley. (See vol. iii. of that author's 'Course of Military Instruction.')

ON THE STRENGTH OF CAST IRON.

Direct Cohesion.

123. CAST iron is but seldom employed to act as a tie, or to resist by its direct cohesive power, for which purpose it is not considered well calculated; not perhaps because it has not sufficient strength, but because its strength is not certain, and that it accommodates itself less to any cross strain than malleable iron. A bar of malleable iron will admit of considerable torsion without any great diminution of its direct strength, but in cast iron this is not the case, and any twist brought on a bar with a direct strain is pretty sure to produce fracture long before the whole of its direct strength is called into action.

The three following experiments give a mean of 8·14 tons, or about 18,000 lbs. per square inch, viz.:

Experiment 1.—By Captain Brown, on a bar	tons.
1¼ inch square, which was broken with 11·35 tons, or per square inch	7·26
Experiment 2.—By George Rennie, Esq., on a bar	
¼ inch square, cast horizontally, which was broken with . . . . 1193 lbs., or per square inch	8·52
Experiment 3.—By the same on another bar	
¼ inch square, cast vertically . . . 1218 lbs., or per square inch	8·66
	3) 24·44
Mean . . .	8·14

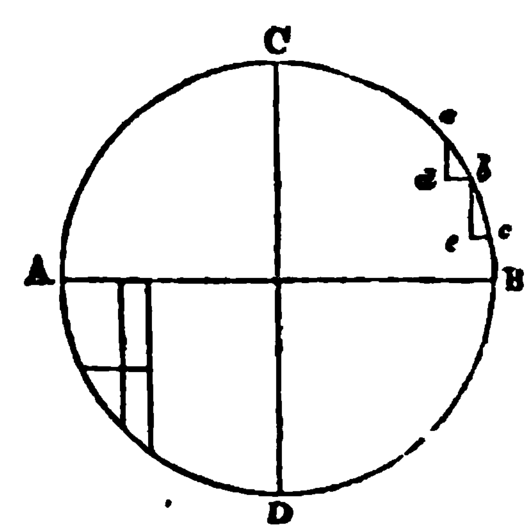
Numerous experiments upon iron of various manufactures, conducted by Mr. Hodgkinson, and recorded in the Report on the application of cast iron to railway structures, give for the lowest quality a direct cohesive strength of 5·667 tons; for the best, a strength of 11·502 tons; and for the mean, 7·29 tons.

On the Strength of Hydrostatic Presses.

124. It has been remarked that cast iron is seldom employed to resist a direct strain, but there are some cases in which this is unavoidable, and amongst others, in hydrostatic presses and water pipes; for the tendency of the internal pressure is here obviously to rend open the cylinder longitudinally, and its power of resistance is only the direct cohesion of the particles of metal in its longitudinal section. It would at first sight appear that the strength of a

cylinder exposed to an internal pressure must be proportional to its thickness, but practically this is not the case, it being found necessary to increase the thickness in a much higher proportion than in that of the strain. My attention was called to this apparent mystery some years back, by Mr. Kier, who was engaged in the manufacture of hydrostatic presses, and it led me to the following investigation of the subject, which was presented to the Institution of Civil Engineers, and has been since published in the first volume of their 'Transactions.'

Let *a b*, *b c*, be any small elementary parts of the circumference, which may be taken as right lines, and let the pressure on each of them be called *p*, which, being proportional to them, may be represented by the elements themselves, *ab*, *b c*, these being perpendicular to the direction in which the pressure acts.



Resolve these pressures or forces each into two rectangular forces, *ad*, *db*, and *b e*, *e c*, of which, *a d* and *b e* will represent forces acting perpendicular to their direction or parallel to AB, and *db* and *ec* forces parallel to DC. Confining ourselves at present to the former, if we conceive the semi-circumference D B C to be divided into its component elements, it is obvious that the sum of all the forces acting parallel to AB, will be equal to the sum of all the perpendiculars, *a d*, *b e*, or to the

whole diameter D C. That is, the sum of all the forces acting parallel to A B, will be to the sum of all the forces or pressure on the semi-circumference D B C, as the diameter to the semi-circumference. But the pressure on the semi-circumference is equal to the number of inches in the same, multiplied by the pressure per square inch; consequently the force or pressure exerted parallel to A B, will be equal to the inches in the diameter multiplied by the pressure per square inch, the ring being here supposed, for the purpose of simplification, only an inch deep. But to resist this pressure, we have the two thicknesses of the ring at D and C; therefore the direct strains on the circumference at any one point, as D, will be equal to the pressure of the fluid per square inch multiplied by the number of inches in the radius.

We should come to the same result more simply, but perhaps not so satisfactorily, by conceiving a section passing through the diameter D C; then it follows that the pressure on this section, which is directly resisted at D and C, is equal to the number of square inches in the section multiplied by the pressure per square inch. Therefore the strain on D or C singly, is equal to the pressure per square inch multiplied by the inches in the radius; the same as above.

Having thus found the strain at D and C, it would appear at first, as is stated above, only to be necessary to ascertain the thickness of metal required to

resist this strain when applied directly to its length: this, however, is by no means the case, for if we imagine, as we must do, that the iron, in consequence of the internal pressure, suffers a certain degree of extension, we shall find that the external circumference participates much less in this extension than the interior; and as the resistance is proportional to the extension divided by the length, according to the law *ut tensio sic vis*, it follows, that the external circumference, and every successive circular lamina, from the interior to the exterior surface, offers a less and less resistance to the interior strain: the law of which decrease of resistance it is our present object to investigate.

In the first place, it is obvious that whatever extension the cylinder or ring may undergo, there will be still in it the same quantity of metal; or, which is the same, the area of the circular ring, formed by a section through it, will remain the same, which area is proportional to the difference of the squares of the two diameters.

Let D be the interior diameter before the pressure is exerted, and $D + d$ its diameter when extended by the pressure. Let also D' be the external diameter before, and $D' + d'$ the diameter after, the pressure is exerted; then, from what is stated above, it follows that we shall have

$$D'^2 - D^2 = (D' + d')^2 - (D + d)^2;$$

$$\text{or, } 2D'd' + d'^2 = 2Dd + d^2;$$

$$\text{or, } 2D' + d' : 2D + d :: d : d';$$

or, since d' and d are very small in comparison with D′ and D, this analogy becomes $D' : D :: d : d'$. That is, the extension of the exterior surface is to that of the interior as the interior diameter to the exterior.

But the resistance is as the extension divided by the length; therefore the resistance of the exterior surface is to that of the interior as $\frac{D}{D'} : \frac{D'}{D}$, or as $D^2 : D'^2$. That is, the resistance offered by each successive lamina is inversely as the square of the diameter, or inversely as the square of its distance from the centre; by means of which law the actual resistance due to any thickness is readily ascertained.

Let r be the interior radius of any cylinder, t the whole thickness of the metal, and x any variable distance from the interior surface. Let also s represent the strain exerted at the interior surface. Then by the law last illustrated we shall have

$$(r + x)^2 : r^2 :: s : \frac{r^2 s}{(r + x)^2}$$

for the strain at the distance x from the interior surface; and consequently $\int \frac{r^2 s\, dx}{(r + x)^2} + \text{Cor.} =$ the sum of all the strains, or the sum of all the resistances. This becomes, when

$$x = t, \ R = r^2 s \left(\frac{1}{r} - \frac{1}{r + t}\right) = s \frac{r t}{r + t}.$$

That is, the sum of all the variable resistances due to the whole thickness t, is equal to the resistance that would be due to the thickness $\frac{rt}{r+t}$ acting uniformly with a resistance s.

Application of this Rule for computing the proper Thickness of Metal in a Cylindric Hydraulic Press of given Power and Dimensions.

125. Let r be the radius of the proposed cylinder, p the pressure per square inch on the fluid, and x the required thickness: let also c represent the cohesive strength of a square inch rod of the metal.

Then from what has preceded it appears, that the whole strain due to the interior pressure will be expressed by $p\,r$, and that the greatest resistance to which the cylinder can be safely opposed is $c \times \frac{rx}{r+x}$: hence, when the strain and resistance are in equilibrio, we shall have

$$rp = \frac{rx}{r+x} \times c \qquad . \quad . \quad . \quad . \quad . \quad . \qquad (1)$$

$$\text{or, } pr + px = cx;$$

$$\text{whence } x = \frac{pr}{c-p} \text{ (the thickness) sought.}$$

Hence the following rule in words for computing the thickness of metal in all cases; viz., multiply the pressure per square inch by the radius of the cylinder, and divide the product by the difference between the cohesive strength of a square inch rod

of the metal and the pressure per square inch, and the quotient will be the thickness required.

At present we have only considered the circumferential strain: to find the longitudinal strain, we have to multiply the area of the piston by the pressure per inch; while the resistance in this direction will be equal to the cohesive power of the metal multiplied by the area of the transverse section of the cylinder; so that when these are equal to each other, we shall have

$$3 \cdot 1416\, r^2 p = 3 \cdot 1416\, (2rx + x^2)\, c,$$

$$\text{which gives } x = r \left\{ \sqrt{\left(\frac{p}{c} + 1\right)} - 1 \right\} \quad . \quad . \quad (2)$$

On the Strength of Direct Cohesion of various Metals.

126. As but few applications of other metals than iron are called for under circumstances of great importance, the number of experiments upon them are very limited, and these are generally upon their direct strength. We shall content ourselves, therefore, with giving the following Table of Results, from experiments made in Woolwich Dockyard; and others by George Rennie, Esq. (Phil. Trans. 1818.)

TABLES OF THE DIRECT COHESIVE POWERS OF VARIOUS METALS.

TABLE I.

The following Experiments were made by order of the Admiralty, with the Testing Machine in Woolwich Dock-yard (described in a subsequent article), on King's Copper, Greenfell's Copper, and on the Patent Yellow Metal, by Mr. John Kingston.

KING'S COPPER.

Diameter of bolt.	Quantity stretched in four feet.	Breaking weight in tons.	Reduced to square inch.
	with tons .. inch.		
1⅛	15 .. ·062	22 tons.	51189
1⅛	15 .. ·100	22	51189
1⅛	15 .. ·125	21¾	50607
1	12 .. ·125	16¼	48578
1	12 .. ·137	17	50050
1	12 .. ·125	17¼	50786
0⅞	9 .. ·125	12¾	51286
0⅞	9 .. ·125	12¾	49135
0⅞	9 .. ·085	13¼	51062
0¾	6 .. ·125	9	47104
0¾	6 .. ·137	8¾	45797
0¾	6 .. ·137	9	47104
		Mean ..	49499 = 22·1 tons.

GREENFELL'S COPPER.

Diameter of bolt.	Quantity stretched in four feet.	Breaking weight in tons.	Reduced to square inch.
1⅛	15 .. ·137	19½ tons.	45372
1⅛	15 .. ·125	19½	45372
1⅛	15 .. ·125	18	41881
1	12 .. ·125	15¾	46369
1	12 .. ·150	15½	45633
1	12 .. ·150	14¾	43425
0⅞	9 .. ·100	13	50098
0⅞	9 .. ·112	13	50098
0⅞	9 .. ·087	13½	52989
0¾ $\frac{1}{40}$	6 .. ·100	9¾	47727
0¾ $\frac{1}{40}$	6 .. ·125	9	44150
0¾ $\frac{1}{40}$	6 .. ·125	8¾	42832
		Mean ..	46329 = 20·7 tons.

TABLE—(CONTINUED).

PATENT YELLOW METAL.

Diameter of bolt.	Quantity stretched in four feet.	Breaking weight in tons.	Reduced to square inch.
	with tons inch.	tons.	
$1\frac{1}{32}$	15 .. ·150	$23\frac{1}{4}$	51750
$1\frac{1}{32}$	15 .. ·230	23	50640
$1\frac{1}{32}$	15	19	41840
1	12 .. ·250	$16\frac{3}{4}$	50628
1	12 .. ·750	$18\frac{3}{4}$	55329
1	12 .. ·500	$20\frac{1}{4}$	59617
$0\frac{7}{8}$	9 .. 2·00	13	50098
$0\frac{7}{8}$	Defective	8	30830
$0\frac{7}{8}$	9 .. 2·00	$12\frac{1}{2}$	48172
$0\frac{3}{4}$	6 .. 1·70	$9\frac{1}{2}$	49720
$0\frac{3}{4}$	6 .. 3·00	8	41870
$0\frac{3}{4}$	6 .. 2·00	$9\frac{1}{2}$	49720
		Mean..	49185 = 21·9 tons.

TABLE II.

Experiments on the Strength of Direct Cohesion of various Metals. By George Rennie, Esq. (from Phil. Trans. 1818).

No.	Metals.	Reduced to inch square.	
		lbs.	tons.
1	$\frac{1}{4}$-inch cast-iron bar, horizontal cast .. 1168	1193	8·51
2	Ditto vertical cast 1218		
3	Ditto, cast steel, previously tilted	8391	59·93
4	Ditto, blistered steel, reduced per hammer..	8322	59·43
5	Ditto, sheer ditto, ditto	7977	56·97
6	Ditto, Swedish iron, ditto	4504	32·15
7	Ditto, English ditto, ditto	3492	24·93
8	Ditto, hard gun-metal..................	2273	16·23
9	Ditto, wrought copper, reduced per hammer	2112	15·08
10	Ditto, cast copper	1192	8·51
11	Ditto, fine yellow brass	1123	8·01
12	Ditto, cast tin	296	2·11
13	Ditto, cast lead	114	0·81

On the Resistance of ¼-inch Iron Bars to a wrenching Force.

127. The following experiments were made by George Rennie, Esq., and were published by him in the Phil. Trans., Part I., for 1818. The apparatus consisted of a wrought-iron lever, 2 feet long, having an arched head of about 60°, and 4 feet diameter, of which the lever represented the radius: the centre round which it moved had a square hole, made to receive the end of the bar to be twisted. The lever was balanced, and a scale hung on the arched head; the other end of the bar being fixed in a square hole, in a piece of iron, and that again in a vice. The under-mentioned weights represent the quantity of weight put into the scale.

EXPERIMENTS

ON TWISTS CLOSE TO THE BEARING, CAST HORIZONTAL.

No.		lbs.	oz.	
1.	¼-inch bars, twisted as under, with	10	14	in the scale.
2.	¼ ditto, bad casting	8	4	
3.	¼ ditto	10	11	
	Average . . .	9	15	

CAST VERTICAL.

4.	¼	10	8
5.	¼	10	13
6.	¼	10	11
	Average . . .	10	10

ON TWISTS OF DIFFERENT LENGTHS, HORIZONTAL CAST.

7.	¼ by ½ long	7	3
8.	¼ by ¾ ditto	8	1
9.	¼ by 1 inch ditto	8	8

VERTICAL.

No.		lbs.	oz.	
10.	¼ by ½ long, twisted asunder with	10	1	in the scale.
11.	¼ by ¾ ditto	8	9	
12.	¼ by 1 inch ditto	8	5	

CAST HORIZONTAL, TWISTS AT 6 INCHES FROM THE BEARING.

13.	¼ by 6 inches long	10	9
14.	¼ by ditto ditto	9	4
15.	¼ by ditto ditto	9	7

TWISTS OF ½-INCH SQUARE BARS, CAST HORIZONTALLY.

		qrs.	lbs.	oz.	
16.	½ close to the bearing .	3	9	12	end of the bar hard.
17.	½ ditto	2	18	0	middle of the bar.
18.	½ at 10 in. from bearing, lever in the middle . .	1	24	0	

On Twists of different Materials.

128. These experiments were made close to the bearing, and the weights were accumulated in the scale until the substances were wrenched asunder:

No.		lbs.	oz.
19.	Cast steel	19	9
20.	Sheer steel	17	1
21.	Blistered steel.	16	11
22.	English iron	10	2
23.	Swedish iron	9	8
24.	Hard gun-metal	5	0
25.	Fine yellow brass	4	11
26.	Copper	4	5
27.	Tin	1	7
28.	Lead	1	0

It will of course be understood that these experiments give only the relative resistance to torsion, and not the actual resistance. On this subject the reader should consult Tredgold's 'Practical Essay on the Strength of Cast Iron.'

129. *Experiments by George Rennie, Esq., on Resistance of Cast Iron to a crushing Force; from Phil. Trans. for* 1818.

Size of the prism. Side of base.	Size of the prism. Height.	Specific gravity.	Crushing weight.	Mean from each set.	REMARKS.
inch.	inch.		lbs.	lbs.	
1/8	1/8	7033	1454	1440	These specimens were from one block.
Do.	Do.	Do.	1416		
Do.	Do.	Do.	1449		
1/8	2/8	6977	1922	2116	Iron from a block.
Do.	Do.	Do.	2310		
Do.	3/8	Do.	2363	1758	These specimens were from the same block.
Do.	4/8	Do.	2005		
Do.	5/8	Do.	1407		
Do.	6/8	Do.	1743		
Do.	7/8	Do.	1594		
Do.	8/8	Do.	1439		
1/4	1/4	6977	10561	9773	These specimens were from the same block as above.
Do.	Do.	Do.	9596		
Do.	Do.	Do.	9917		
Do.	Do.	Do.	9020		
1/4	1/4	7113	10432	10114	These specimens were from horizontal castings.
Do.	Do.	Do.	10720		
Do.	Do.	Do.	10605		
Do.	Do.	Do.	8699		
1/4	1/4	7074	12665	11136	These specimens were vertical castings.
Do.	Do.	Do.	10950		
Do.	Do.	Do.	11088		
Do.	Do.	Do.	9844		
Do.	Do.	Do.	11006		
1/4	1/8	7113	9455	9414	Horizontal casting.
Do.	Do.		9374		
1/4	1/8	7074	9938	9982	Vertical casting.
Do.	Do.		10027		
1/4	4/8	7113	9006		Horizontal castings.
Do.	5/8	Do.	8845		
Do.	6/8	Do.	8362		
Do.	7/8	Do.	6430		
Do.	8/8	Do.	6321		
1/4	4/8	7074	9328		Vertical castings.
Do.	5/8	Do.	8385		
Do.	6/8	Do.	7896		
Do.	7/8	Do.	7018		
Do.	8/8	Do.	6430		

130. *Similar Experiments on different Metals.*

Size of the prism. Side of base.	Height.	Specific gravity.	Crushing weight.	Mean from each set.	Remarks.
inch.	inch.		lbs.		
¼	¼	Cast copper.	7318	....	Crumbled by the pressure.
Do.	Do.	Brass.	10304	....	Fine yellow brass reduced $\frac{1}{10}$th with 3213 lbs.; ½ with 10304 lbs.
Do.	Do.	Wrought copper.	6440	....	Reduced $\frac{1}{16}$th with 3427 lbs.; ½ with 6440 lbs.
Do.	Do.	Cast tin.	966	....	Reduced $\frac{1}{16}$th with 552 lbs.; ½ with 966 lbs.
Do.	Do.	Cast lead.	483	....	Reduced ½ with 483 lbs.

In these experiments, after the metals had been compressed to a certain extent, the resistance is stated to have been enormous.

On the Transverse Strength of Cast Iron.

131. The form in which cast iron is most frequently employed is to resist a transverse strain, as in rafters, girders, &c., &c., and numerous experiments have been made to determine the requisite data for computing the proper dimensions in these cases. Amongst the earliest experiments of this kind were those of Mr. Banks, in his 'Treatise on the Power of Machines.' These were made by resting the ends of square inch bars on supports at 3 feet distance, and then loading them with weights at their centre till fracture took place: the results were as follow :

No. of experiment.	Distance of supports.	Depth.	Breadth.	Breaking weight.	Mean.
1	36 inches	1	1	756	756
2	36	1	1	756	
3	30	1	1	1008 reduced to 36 inches.	840
4	36	1	1	963	972
5	36	1	1	958	
6	36	1	1	994	
7	36	1	1	average of three other experiments	730
8	36	1	1	864	869
9	36	1	1	874	
10	36	1	1	by Mr. George Rennie	897
					6)5064
				Mean	844

It has been observed (Art. 40) that the position of the neutral axis is not of much importance in the case of timber, bars or beams of this material being generally rectangular where strength is required; and the strength of one being known, that of others may be computed without reference to this datum: but it is very different in cast iron, because in this, bars may be cast of various forms, and the strength of these cannot be computed without knowing the position of the axis in question. To compensate for this want of information, however, Mr. Hodgkinson has supplied us with numerous results on bars of different forms, which will be given in the sequel. From the preceding mean result we obtain for our value of S in cast-iron rectangular bars,

$$S = \frac{l\,W}{4\,a\,d^2} = 7596, \text{ or } 7600, \text{ nearly}.$$

Mr. Tredgold's Experiments.

132. In these the depth of the bar was ·65 of an inch, and the breadth 1·3 inch. They were securely fixed at one end, the load being applied at the other, the leverage being in each case 2 feet.

No. of experiment.	Kind of iron.	Length.	Breadth.	Depth.	Breaking weight.	Value of S, $S = \frac{lW}{4ad^2}$
		inches.	inch.	inch.	lbs.	
1	Old Park	24	1·3	·65	184	8040
2	Adelphi	24	1·3	·65	173	7560
3	Alfreton	24	1·3	·65	168	7341
4	Scrap iron	24	1·3	·65	174	7638
						Mean 7645

These values of S agree very nearly with that obtained from the preceding mean.

We may, therefore, with confidence state the constant (S) for rectangular cast-iron bars to be

$$S = 7620.$$

On the Deflection of Cast Iron when submitted to a Transverse Strain.

133. On this subject Mr. Tredgold[1] has furnished us with the four following results: the bars were like those given above, two of each kind having been cast for the purpose of the experiment.

[1] Treatise on the Strength of Cast Iron.

EXPERIMENT 1.

OLD PARK IRON.

Two specimens run from this kind of pig iron, each 3 feet in length; smooth, clean, and regular castings. The section of the bars rectangular, depth 0·65 inch, breadth 1·3 inch; the supports 2·9 feet or 35 inches apart, the load suspended in the middle.

Weight applied.	Deflection, 1st bar.	Deflection, 2nd bar.
60 lbs.	Bent 0·1 inch.	Bent 0·1 inch.
120	0·2	0·203
162	0·265	0·275
182	0·305 small set.	0·31 {set barely perceptible.
190	0·32 set ·005	0·33 set ·005

The iron was slightly malleable in a cold state; yielded easily to the file. The fracture dark grey, with little metallic lustre, fine-grained and compact.

We may consider 162 lbs. as the greatest load it would bear without impairing its elastic force, and 0·27 as the mean between the flexures produced by this weight, or $\delta = 0{\cdot}27$.

$$\text{Whence } E = \frac{l^3 W}{16\, a\, d^3\, \delta} = 4503600.$$

EXPERIMENT 2.

ADELPHI IRON.

The specimens of this iron were clean, good castings, of the same dimensions as the preceding; that is, depth 0·65, breadth 1·3 inch, distance of supports 35 inches.

Weight applied.	Deflection, 1st bar.	Deflection, 2nd bar.
60 lbs.	Bent 0·1 inch.	Bent 0·1 inch.
120	0·2	0·205
162	0·26 no set.	0·27 no set.
182	0·30 set ·0075	0·305 set ·005

Taking the mean deflection with 162 at ·265, we find

$$E = \frac{l^3 W}{16 a d^3 \delta} = 4588400.$$

EXPERIMENT 3.

ALFRETON IRON.

Same dimensions and distance of supports as in the preceding, viz.

$$d = \cdot 65,\ a = 1 \cdot 3,\ l = 35.$$

Weight applied.	Deflection, 1st bar.	Deflection, 2nd bar.
60 lbs.	Bent 0·1 inch.	Bent 0·1 inch.
120	0·2	0·195
162	0·27 no set.	0·28 no set.
183	0·31 small set.	0·325 small set.

Taking ·275 as the mean deflection with 162 lbs., we find

$$\dot{E} = \frac{l^3 W}{16 a d^3 \delta} = 4421600.$$

EXPERIMENT 4.

SCRAP IRON.

These bars were run from old iron; they were uneven on the surface. Dimensions as before.

Weight applied.	Deflection, 1st bar.	Deflection, 2nd bar.
60 lbs.	Bent 0·9 inch.	Bent 0·09 inch.
120	0·18	0·18
162	0·25 no set.	0·255 no set.
180	0·28 no set.	2·285 do.
190	0·30 small set.	0·30 { set not certain.
210	0·34 set ·005	0·34 set ·004

On these experiments Mr. Tredgold observes, that these bars showed no signs of a permanent set with 180 lbs.; but to whatever cause this greater range of elastic power may be owing, it would certainly be unsafe to calculate upon it. The iron was very hard to the file, and very brittle fragments flying off when hammered on the edge, instead of indenting, as the preceding specimens.

Taking ·2525 as the mean deflection with 162 lbs., we have

$$E = \frac{l^3 W}{16 a d^3 \delta} = 4815600.$$

Excluding this as an unusual specimen, we have as a mean from the other three experiments,

$$E = 4508000$$

for the mean elastic power of cast iron to the nearest fourth figure; the other places are supplied by ciphers for the sake of simplification, their real value being unimportant.

Comparison of the Strength, Stiffness, &c. of Cast Iron with good English Oak.

134. By the Table of Data, (Art. 104,) it appears that the value of S, for the best specimen of English oak, is 1672; and from the preceding experiment for cast iron, S = 7645, that is, strength of

oak	:	cast iron	: :	1	:	4·5 nearly.
Stiffness, oak	:	cast iron	: :	1	:	13 nearly.
Sp. grav. oak	:	cast iron	: :	1	:	8 nearly.

If we consider that 170 lbs. in these experiments is just within the elastic power, we find

$$S = \frac{l W}{4 a d^2} = 2075,$$

which is little more than one-third of the greatest value of S, viz. 7645. Cast iron may, therefore, be considered to have its elasticity destroyed with about one-third the weight that will produce fracture; it ought, therefore, not to be loaded in permanent constructions to more than this amount.

Of the Section of greatest Strength.

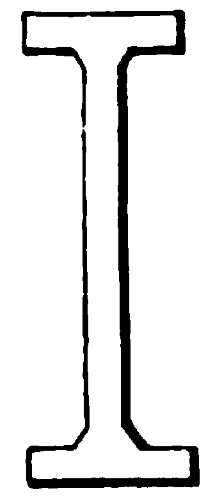
135. If cast-iron beams, rafters, &c. were employed generally of a rectangular form like timber, the above data would be all that would be required; but as this metal may be cast of any form at pleasure, it becomes an object of great importance to know which is the strongest form of section with a given load and under different kinds of strains. If the position of the neutral axis in cast iron were known, and if it were found to preserve a constant law under all circumstances, these sections might be computed; but at present this datum has not been determined, nor the direct strength of cohesion: we must be content, therefore, with the results of actual experiments on particular forms, of which a great variety have been made by Mr. Hodgkinson, of Manchester, and published by him in the 'Manchester Memoirs' (vol. v.), and from which the following abstract is made.

136. *Experiments on the Transverse Strength of Cast Iron of various Sections. By Eaton Hodgkinson, Esq.*

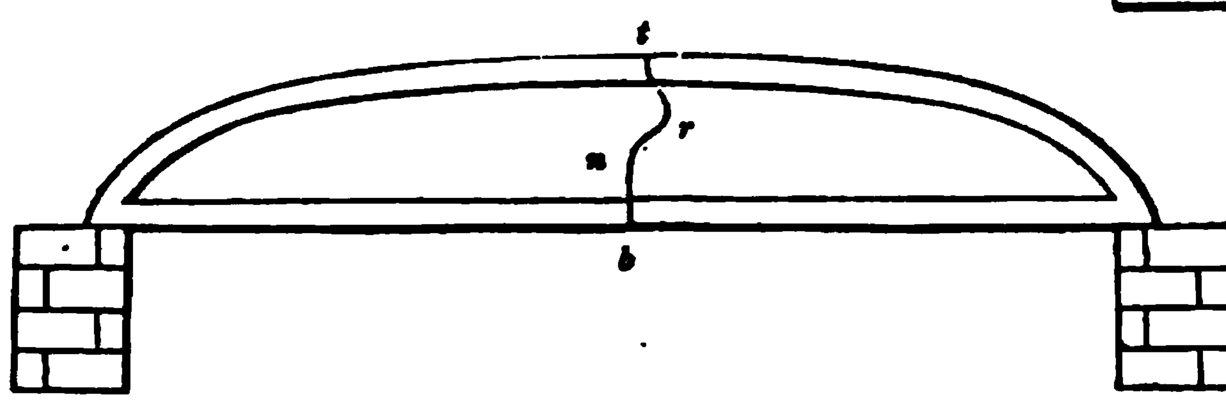
EXPERIMENT 1.

Beam with equal rib at top and bottom.

Distance between the supports 4 feet 6 inches, depth of beam $5\frac{1}{8}$ inches.



Area of top rib $= 1{\cdot}75 \times {\cdot}42 = {\cdot}735$ in.
Do. bottom rib $= 1{\cdot}77 \times {\cdot}39 = {\cdot}690$
Thickness of vertical part between the ribs . . . $= {\cdot}29$
Area of cross section . . $= 2{\cdot}82$
Weight of casting . . . . $36\frac{1}{4}$ lbs.
Breaking weight . . . . . . 6678 lbs.

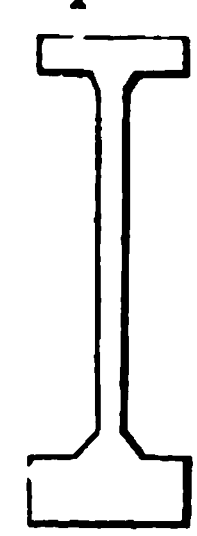


The form of fracture is represented by the line *t b n r*, where $t\,r = {\cdot}6$ inch and $b\,n$ 2·5 inches, the figure being a side view of the beam.

EXPERIMENT 2.

Beam with sectional areas of top and bottom rib as 1 : 2.

Distance between the supports 4 feet 6 inches, depth of beam $5\frac{1}{4}$ inches.



Area of top rib $1{\cdot}74 \times {\cdot}26 = {\cdot}45$ in.
Do. of bottom rib $1{\cdot}78 \times {\cdot}55 = {\cdot}98$
Thickness of vertical part . $= {\cdot}30$
Area of cross section . . $= 2{\cdot}87$
Weight of casting . . . 39 lbs.
Breaking weight . . . 7368 lbs.

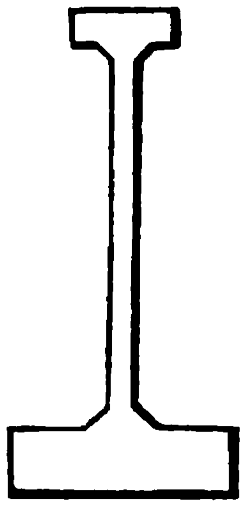
Form of fracture nearly as in experiment 1.

EXPERIMENT 3.

Beam with top and bottom rib as 1 : 4.

Distance between the supports 4 feet 6 inches, depth of beam $5\frac{1}{4}$ inches.

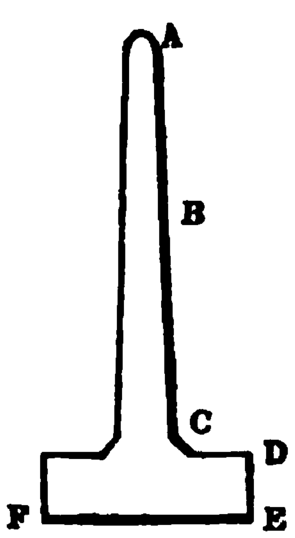
Area of top rib 1·07 × ·30 = ·32 in.
Do. of bottom rib 2·1 × ·57 = 1·2
Thickness of the vertical part = ·32
Area of cross section . . = 3·02
Weight of casting . . . 40 lbs.
Breaking weight . . . . 8270 lbs.
Fracture as in experiment 1; $tr = \cdot 6$.



EXPERIMENT 4.

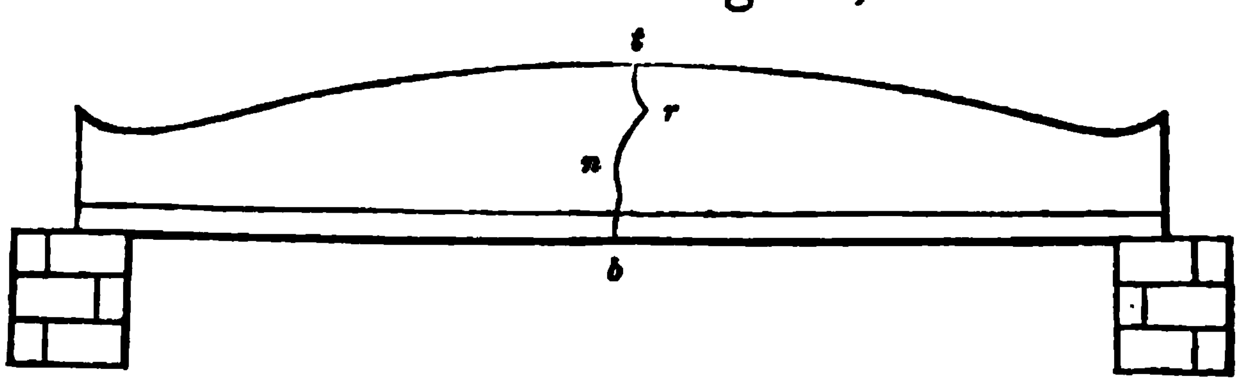
Beam cast in common form; Messrs. Fairbairn and Lillie's model.

Distance between supports and depth of beam as before.
Thickness at A = ·32
B = ·44
C = ·45
F E = 2·27
D E = ·52
Area of section = 3·2 in.
Weight of casting = 40½ lbs.
Deflection with 5758 lbs. ·25 in.
7138 ·37
Breaking weight = 8720 lbs.



The beam twisted a little before breaking; this, however, was not usually the case in the other beams of the same model.

Form of fracture as in figure; $t\,r = \cdot 75$.



All the preceding experiments were made on

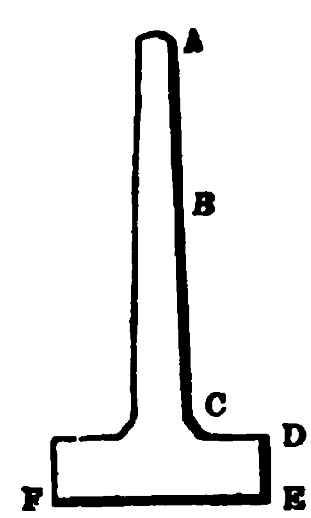
beams cast on their side from iron of which the following is a description:

$\frac{1}{3}$ of Blaina, No. 2, } Welsh.
$\frac{1}{3}$ of Blaina, No. 3, }
$\frac{1}{3}$ of W. I. S., No. 3, Shropshire.

This mixture is a strong iron, and therefore well suited for beams.

EXPERIMENT 7.[2]

This was on a beam from the same model as that in experiment 4; it was cast erect, but upside down, as usual, and therefore ought not to be compared with the preceding ones.



Distance between supports as before.
Thickness at A = ·30
B = ·37
C = ·425
FE = 2·28
DE = ·53
Area of the above section = 2·28 in.
Weight of beam = 38 lbs.

Deflection with	6679 lbs.	·37 inches.
	9495	·50
	9297	·62

Breaking weight = 9503 lbs.

It twisted in a serpentine manner before it broke. The form of fracture was nearly as in experiment 4; but here $tr = 1{\cdot}0$, and $bn = 2{\cdot}5$.

Remark.—In the future experiments, all the beams, except where otherwise stated, were cast erect, but upside down, as there is an accession of

[2] Experiments 5 and 6 are omitted, being defective.

strength from that cause. Those in experiments 8, 9, 11, 12, and 21, were elliptical, and were indeed from the model of the first three experiments, its top and bottom ribs being further changed.

EXPERIMENT 8.

Beam from the same model as that of experiment 3, the top rib in the casting being to the bottom as 1 to $3\frac{1}{2}$ nearly.

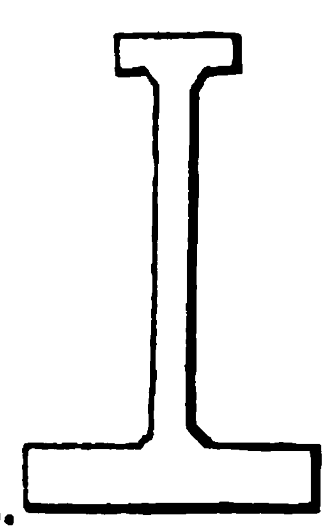
Distance between supports as before.
Area of top rib $= 1{\cdot}05 \times {\cdot}32 = 0{\cdot}34$ in.
of bottom rib $= 2{\cdot}15 \times {\cdot}56 = 1{\cdot}20$
Thickness of vertical part $= {\cdot}33$
Area of cross section $= 3{\cdot}08$ inches.
Weight of casting $39\frac{1}{4}$ lbs.
Breaking weight 8263 lbs. = 73 cwt. 89 lbs.

It broke very near to the middle.

The form of fracture was nearly as in the figure to experiment 1; but here $bn = 2{\cdot}5$ and $tr = {\cdot}55$.

EXPERIMENT 9.

In this the model of the above had 1 inch in breadth added to its bottom rib.



Ratio of the ribs 1 to $4\frac{1}{2}$, nearly.
Distance between supports as before.
Area of top rib $= 1{\cdot}05 \times {\cdot}34 = 0{\cdot}357$
Do. of bottom rib $= 3{\cdot}08 \times {\cdot}51 = 1{\cdot}570$
Thickness of vertical part $= {\cdot}305$
Area of section $= 3{\cdot}37$ inches.
Weight of beam $= 44\frac{3}{4}$ lbs.
Breaking weight $= 10727$ lbs. = 95 cwt. 87 lbs.

It broke by tension 4 inches from the middle, but slanting

towards it; and there seemed to be a small flaw in the bottom rib, at the place of fracture.

Here $t\,r =$ ·6 inch.

EXPERIMENT 10.

Common beam, cast upside down, in the usual manner. This, like the rest, was from the same model as that in experiment 4.

Distance between supports as before.

Thickness at A = ·29

B = ·425

C = ·46

F E = 2·3

D E = ·53

Area of section = 3·16 inches.

Weight of beam = 40¼ lbs.

Breaking weight = 8823 lbs.

It broke 1¼ inch from the middle. The form of fracture was nearly as in experiment 4.

Here $b\,n =$ 2·25 and $t\,r =$ ·8.

EXPERIMENT 11.

Beam from model of experiment 9, only its top and bottom rib altered as above.

Ratio of rib 1 to 4, nearly.

Distance between supports and depth as before.

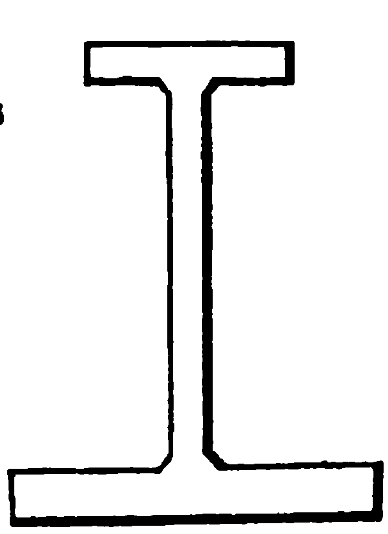
Area of top rib = 1·6 × ·315 = 0·5 in.

Do. bottom rib = 4·16 × ·53 = 2·2

Thickness of vertical part = ·38

Area of section = 4·50 inches.

Weight of beam = 57 lbs.



Deflection with 11186 lbs. = ·40 in.
12698 ·45
13706 ·52

Breaking weight = 14462 lbs.

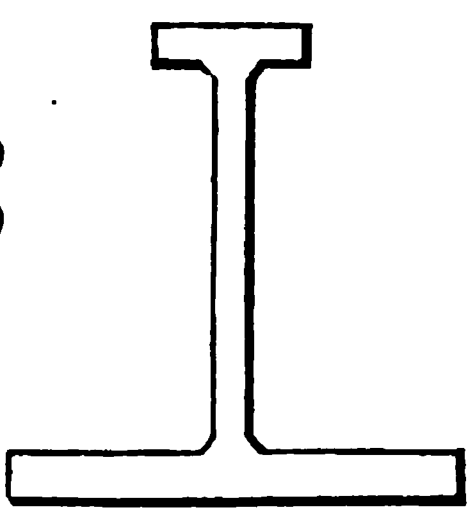
It broke by tension 1 inch from the middle.

bn = 2·5 inches.

EXPERIMENT 12.

The model of this beam differed from that of the last, in having a broader bottom flange.

Ratio of rib 1 to $5\frac{1}{2}$, nearly.
Distance of support as before.
Area of top rib = 1·56 × ·315 = 0·49
Do. bottom rib = 5·17 × ·56 = 2·89
Thickness of vertical part = ·34 in.
Area of section = 5 inches.
Weight of beam = $67\frac{1}{2}$ lbs.
Breaking weight 16730 lbs.



EXPERIMENT 13.

Distance between supports as before.

Thickness at A = ·29
B = ·425
C = ·53
DE = ·565
FE = 2·34

Area of section = 3·32 inches.
Weight of beam = 41 lbs.

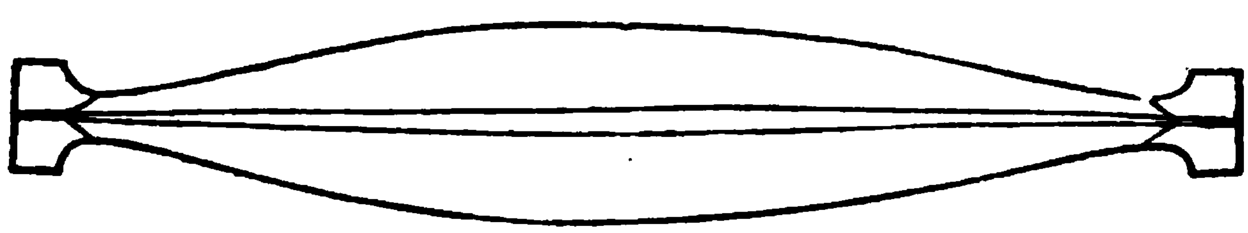
It broke at $1\frac{1}{2}$ inch from the middle with 8942 lbs.

Form of Beam altered.

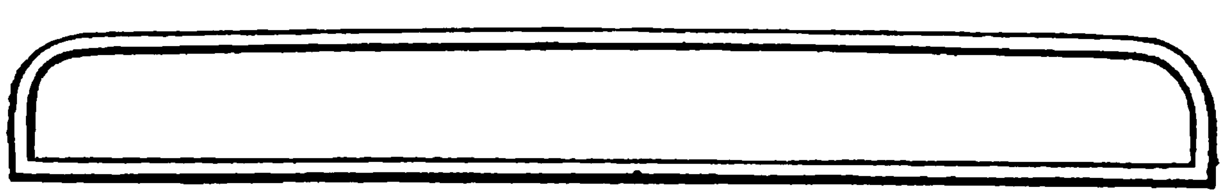
The beams in all the future experiments were of equal height through their whole length, and had

their top and bottom ribs uniform in thickness, but tapering towards the ends, the bottom rib being parabolic. They are represented below by the vertical plan and elevation, where the sections of their middle are as in the following experiments; and the sections, from their middle towards the ends, as in experiments 11, 9, 3.

PLAN.



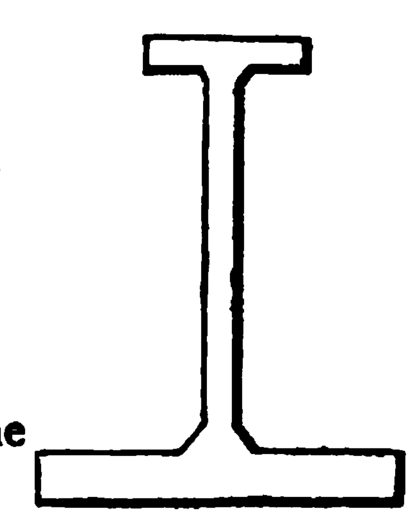
ELEVATION.



This form was adopted to save metal, by reducing the bottom rib, which was likely to become very large.

EXPERIMENT 14.

Distance between supports 4 feet 6 inches, and depth of beam 5½ inches, as before.



Area of top rib = 2·3 × ·315 = ·72
Do. bottom rib = 4·06 × ·57 = 2·314
Thickness of vertical part = ·33
Area of section = 4628 inches.
Breaking weight = 15024 lbs.

It broke by tension very near to the middle.

EXPERIMENT 15.

In this experiment the breadth of the bottom rib only was increased as before.

Distance between supports and depth as before.
Area of top rib = 2·35 × ·29 = ·68
Do. of bottom rib = 5·43 × ·537 = 2·916
Thickness of vertical part = ·35
Area of section = 5·292 inches.
Breaking weight 16905 lbs.

It broke by tension.

EXPERIMENT 16.

Beam from the same model, but with further increased bottom rib.

Distance between supports and depth as before.
Area of bottom rib = 6·8 × ·502 = 3·413 inches.
Breaking weight = 14336 lbs., nearly.

EXPERIMENT 17.

Beam of the *common form*, from the same model as the preceding one, (see fig. to experiment 4.)

Distance between supports as before.
Weight of casting 39½ lbs.

Weight.	Deflection.
6218	·28 inches.
7138	·33

Breaking weight = 7598 lbs.

EXPERIMENT 18.

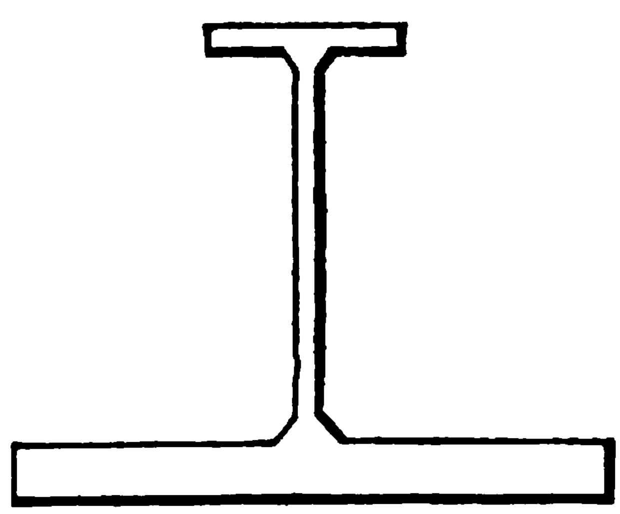
Beam from the same model as that in experiment 16.

Distance of supports as before.
Top rib = 2·3 × ·28 = ·64

Bottom rib = 6·61 × ·54 = 3·57
Thickness of vertical part = ·34
Area of section = 5·86 inches.
Weight of casting 68¼ lbs.
Breaking weight 19441 lbs.

This beam broke very nearly in the middle by tension, as before.

EXPERIMENT 19.



Distance of supports 4 feet 6 in. and depth of beam 5¼ inches, as before.
Area of top rib = 2·33 × 31 = ·72
of bottom rib = 6·67 × ·66 = 4·4
Thinckess of vertical part = ·266
Area of section = 6·4, or 6⅖ in.
Weight of beam = 71 lbs.

This beam broke in the middle by compression with 26084 lbs., a wedge separating from its upper side.

EXPERIMENT 20.

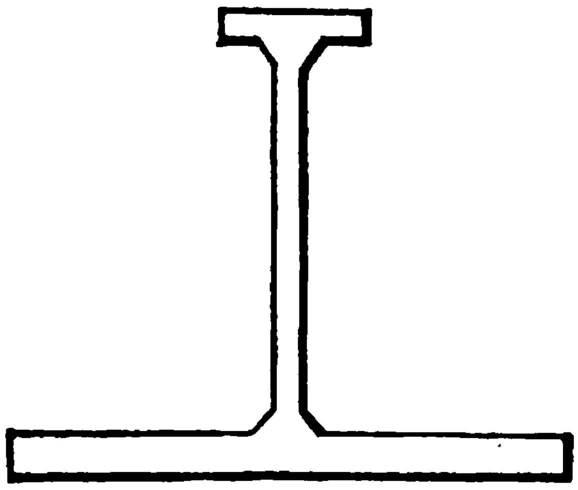
Beam from the same model as that in the last experiment.

Distance between supports as before.
Area of top rib = 2·3 × ·28 = ·64
of bottom rib = 6·63 × ·65 = 4·31
Thickness of vertical part = ·335
Area of section 6·5, or 6½ inches.
Weight of beam = 74¾ lbs.

It broke in the middle of the beam by tension, with 23249 lbs., nearly.

EXPERIMENT 21.

This was an *elliptical* beam from the same model as that in experiment 12, and those preceding it, the bottom rib being further increased, and being, like as in them, of equal breadth through the whole length of 5 feet.



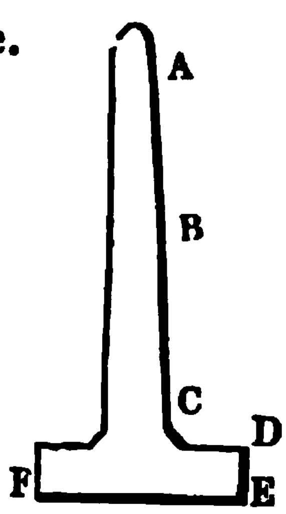
Distance between supports as before.
Area of top rib = 1·54 × ·32 = ·493
of bottom rib = 6·50 × ·51 = 3·315
Thickness of vertical part = ·34
Ratio of ribs $6\frac{1}{2}$ to 1.
Area of section = 5·41 inches.

It broke very near the middle by tension, with 21009 lbs., nearly.

EXPERIMENT 22.

This beam was of the common form, from the same model as before, for comparison with the three preceding ones.

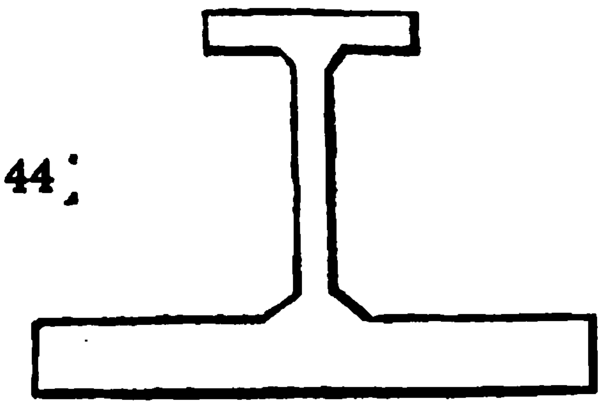
Distance between supports as before.
Thickness at A = ·30
B = ·42
C = ·45
DE = ·51
FE = 2·28
Area of section = 3·17 inches.
Weight of beam = 40 lbs.



This beam bore 8965 lbs., and broke in the middle with considerably less than 9327 lbs.

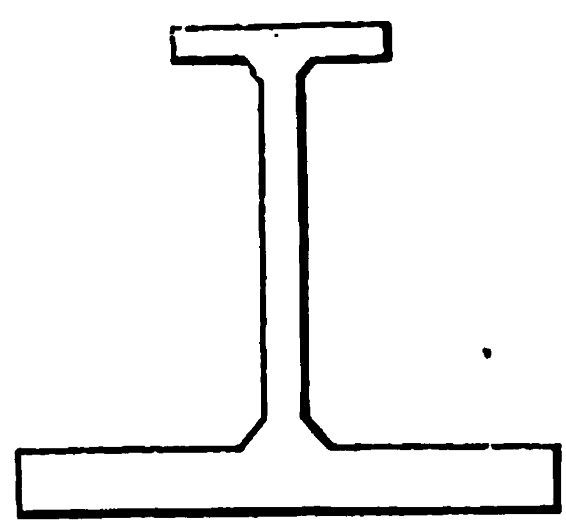
EXPERIMENT 23.

Distance between supports 7 feet.
Depth of beam 4·1 inches.
Area of top rib $=2{\cdot}25\times{\cdot}33={\cdot}74$
of bottom rib $=6{\cdot}00\times{\cdot}74=4{\cdot}44$
Thickness of vertical part $={\cdot}40$
Area of section $=6{\cdot}54$
Weight of casting $=114$ lbs.
Breaking weight 6 tons 103 lbs.



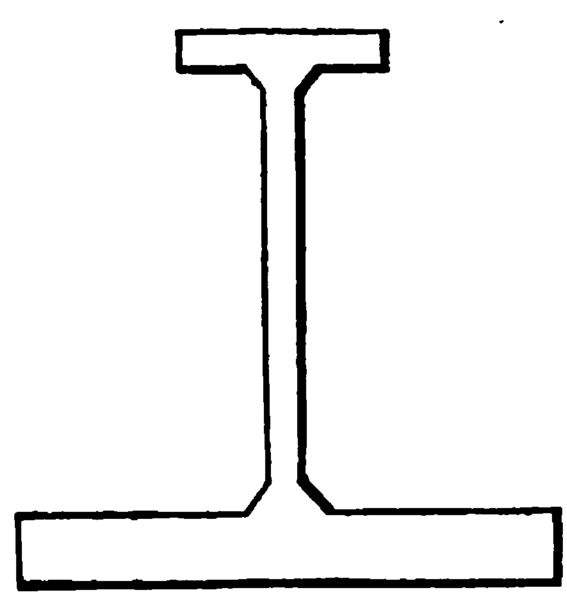
EXPERIMENT 24.

Distance between supports 7 feet.
Depth of beam 5·2 inches.
Area of top rib $= 2{\cdot}25 \times {\cdot}35 = {\cdot}79$
of bottom rib $= 6{\cdot}00 \times {\cdot}77 = 4{\cdot}62$
Thickness of vertical part $= {\cdot}34$
Area of section $= 6{\cdot}94$ inches.
Weight of casting $= 128$ lbs.
Breaking weight 6 tons 15 cwt. 9 lbs.



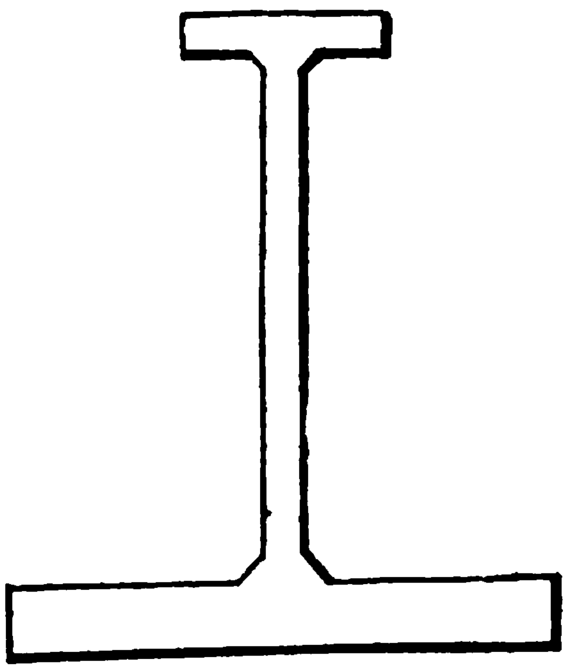
EXPERIMENT 25.

Distance between supports 7 feet.
Depth of beam 6·0 inches.
Area of top rib $= 2{\cdot}2 \times {\cdot}33 = {\cdot}73$
of bottom rib $= 5{\cdot}95 \times {\cdot}75 = 4{\cdot}46$
Thickness of vertical part $= {\cdot}355$
Area of section $= 7{\cdot}08$ inches.
Weight of casting $= 127\frac{1}{2}$ lbs.



It broke by tension in the middle with this last weight, 15129 lbs., after standing a minute.

EXPERIMENT 26.

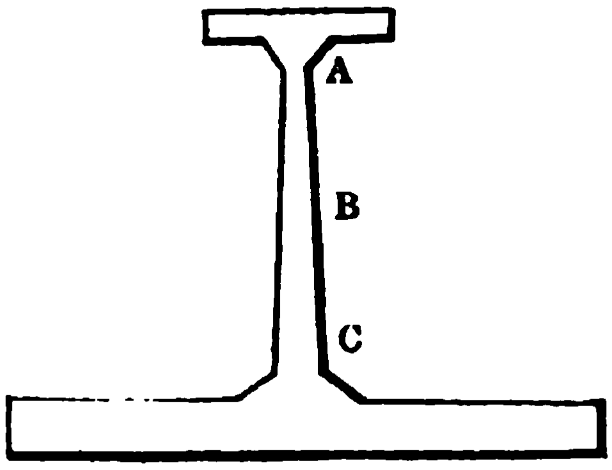


Distance between supports 7 feet.
Depth of beam 6·93 inches.
Area of top rib = 2·25 × ·34 = ·765
of bottom rib = 6·05 × ·75 = 4·537
Thickness of vertical part = ·38
Area of section = 7·67 inches.
Weight of casting = 146 lbs.
Breaking weight 9 tons 18 cwt.

EXPERIMENT 27.

Distance between supports 7 feet.
Depth of beam 6·98 inches.
Beam from the same model as the last.
Area of top rib = 2·25 × ·32 = ·72 in.
of bottom rib = 5·95 × ·73 = 4·343
Thickness of vertical part = ·37
Area of section = 7·40 inches.
Weight of beam = 141 lbs.
Breaking weight 19049 lbs.

EXPERIMENT 28.



Distance between supports 4 feet 6 inches.
Depth of beam $5\frac{1}{8}$ inches.

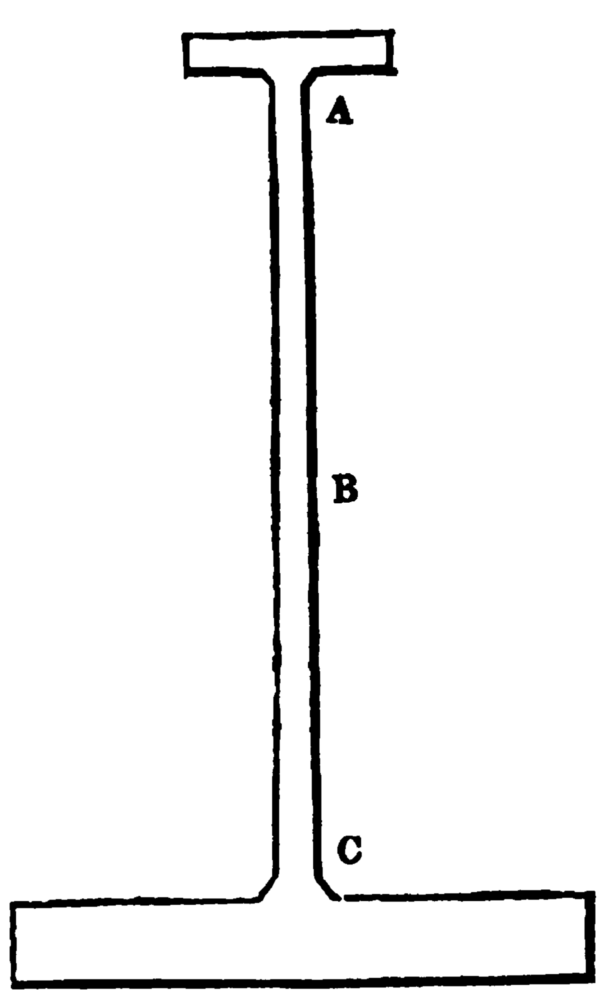
Weight of beam 81 lbs.
Area of top rib = 2·15 × ·27 = ·28
of bottom rib = 6·74 × ·71 = 4·785
Thickness at A . . . . ·25
B half-way between flanges ·37
C . . . . ·53
Area of section 7·20 inches.
Breaking weight 25144 lbs.

EXPERIMENT 29.

Distance between supports 9 feet.
Depth of beam 5¼ inches.
Weight of beam = 170½ lbs.
Area of top rib = 2·2 × ·36 = ·79
of bottom rib = 7·0 × ·69 = 4·83
Thickness at A = ·27
B = ·33
C = ·60
Breaking weight 11056 lbs.

EXPERIMENT 30.

Distance between supports 9 feet.
Depth of beam 10¼ inches.
Weight of beam 227 lbs.
Area of top rib = 2·1 × ·27 = ·57
Area of bottom rib = 6·14 × ·77 = 4·72
Thickness at A = ·20
B = ·25
C = ·35
Breaking weight 28672 lbs.



EXPERIMENT 31.

Distance between supports 4 feet 6 inches.
Depth of beam 5·1 inches.
Weight of beam 88 lbs.
Area of top rib $= 2{\cdot}15 \times {\cdot}24 = {\cdot}52$
of bottom rib $= 7{\cdot}60 \times {\cdot}72 = 5{\cdot}472$
Thickness at A $= {\cdot}27$
B $= {\cdot}44$
C $= {\cdot}48$
Area of section $= 7{\cdot}90$ inches.
Breaking weight 12 tons 11½ cwt.

EXPERIMENT 32.

Distance between supports 9 feet.
Depth of beam 5½ inches.
Weight of beam 192 lbs.
Area of top rib $= 2{\cdot}25 \times {\cdot}3 = {\cdot}67$ in.
of bottom rib $= 7{\cdot}7 \times {\cdot}76 = 5{\cdot}85$
Thickness at A $= {\cdot}36$
B $= {\cdot}42$
C $= {\cdot}60$
Breaking weight 15196 lbs.

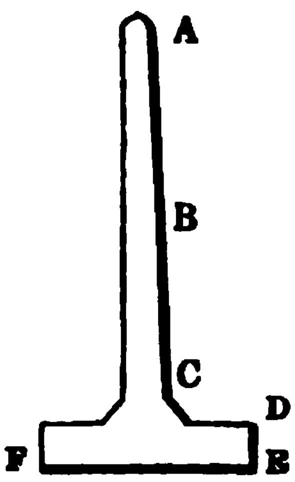
EXPERIMENT 33.

Distance between supports 9 feet.
Depth of beam 10¼ inches.
Weight of beam 244 lbs.
Area of top rib $= 2{\cdot}2 \times {\cdot}33 = {\cdot}73$ in.
of bottom rib $= 7{\cdot}6 \times {\cdot}75 = 5{\cdot}70$
Thickness at A $= {\cdot}15$
B $= {\cdot}38$
C $= {\cdot}35$
Breaking weight 32200 lbs.

EXPERIMENT 34.

Beam of common form, from the same model as before, and cast on its side for comparison.

Distance between supports 4 feet 6 inches.
Depth of beam in its middle $5\frac{1}{8}$ inches.
Weight of beam $36\frac{1}{2}$ lbs.
Thickness at A = ·27
B = ·40
C = ·44
F E = 2·27
D E = ·46
Area of section = 2·921 inches.
Breaking weight 8792 lbs.



EXPERIMENT 35.

A beam of the *common form*, and from the same model, and iron, cast erect, as usual.

Distance between supports 4 feet 6 inches.
Depth of beam in its middle $5\frac{1}{8}$ inches.
Weight of beam 37 lbs.
Thickness at A = ·27
B = ·355
C = ·43
F E = 2·26
D E = ·47
Breaking weight 9044 lbs.

To the above we may add the following experiments by George Rennie, Esq.

EXPERIMENTS

(137.) *on the Transverse Strength of Cast Iron Bars of various Figures.*

No.	Description of bar.	Weight of bar.		Distance of bearings.		Breaking weight.
		lbs.	oz.	ft.	in.	lbs.
1	Bar of 1 inch square	10	6	3	0	897
2	Ditto of 1 inch ditto	9	8	2	8	1086
3	Half the above bar			1	4	2320
4	Bar of 1 inch square through the diagonal	2	8	2	8	851
5	Half the above bar			1	4	1587
6	Bar of 2 in. deep by ½ inch thick	9	5	2	8	2185
7	Half the above bar			1	4	4508
8	Bar 3 inches deep by ⅓ inch thick	9	15	2	8	3588
9	Half the above bar			1	4	6854
10	Bar 4 inches by ¼ inch thick	9	7	2	8	3979
	Equilateral triangles, with the angle up and down:					
11	Edge or angle up	9	11	2	8	1437
12	angle down	9	7	2	8	840
13	Half the first bar			1	4	3059
14	Half the second bar			1	4	1656
	A feather-edged, or ⊥ bar was cast, whose dimensions were					
15	2 inches deep by 2 wide, edge up	10	0	2	8	3105
16	Half of ditto					

N.B. All the above bars contained the same area, though differently distributed as to their forms.

138. *Experiments made on Bars of 4 inches deep by ¼ inch thick, by giving them different forms, the bearing at 2 feet 8 inches, as before.*

17. Bar formed into a semi-ellipse weighed . 7 lbs. 4000
18. Do. parabolic on its lower edge 3860
19. Do. of 4 inches deep, ¼ inch thick 3979

Experiments on the Transverse Strain of Bars, one end made fast, the weight being suspended at the other at 2 feet 8 inches from the bearing.

lbs.

20. An inch square bar bore 280
21. A bar 2 inches deep by ½ inch thick 539

	lbs.
22. An inch bar, the ends made fast, bore . . .	1173
23. The paradoxical conclusion of Emerson was tried, which states, by cutting off a portion of an equilateral triangle (see page 114 of Emerson's 'Mechanics'), the bar is stronger than before, that is, a part stronger than the whole. The ends were loose, at 2 feet 8 inches apart, as before. The edge from which the part was intercepted was lowermost; the weight was applied on the base above; it broke with 1129 lbs., whereas in the other case it bore only	840

We have given the above experiment as it is reported by Mr. Rennie; but it is at variance, as well as experiments 11, 12, 13, 14, with all the similar experiments on wood, reported in pages 132-4.

139. *Experiments on the Transverse Strength of Steel. By M. Duleau.*[3]

Description of specimens.	Distance between the supports.	Breadth.	Depth.	Deflection with a wt. of 10 kilogrammes.
	Mètres.	Millemètres.	Millemètres.	Millemètres.
Cast steel, English, marked HUNTSMAN; perfectly regular, untempered, but brittle	0·98	5·9	13·3	8·4
German steel, (of cementation,) marked FORTSMAN, and three deer heads, used for razors; dimensions irregular	0·68	7·8	14·5	2·1
Same kind of steel	1·845	25·7	21·6	2·8
Same piece, on edge	1·845	21·6	25·7	2·2
Same kind of steel	1·845	21·9	28·5	1·8
Do. do.	1·35	54·8	25·5	0·55
Same piece, on edge	1·35	25·5	54·8	0·27
Same kind of steel	1·35	26·6	52·0	0·3

[3] Essai Théoretique et Experimental sur la Résistance du Fer forgé.

ON THE STRENGTH OF MALLEABLE IRON.

140. It is only since the commencement of the present century that malleable iron has been employed in situations which rendered it desirable to know with certainty its strength under different circumstances. With the exception of anchors and chains, malleable iron was seldom employed to resist by itself very great strains, its general application having been to connect and tie together different parts of a structure under circumstances which rendered it difficult, and not essentially necessary, to know with accuracy its ultimate force of resistance: all that is requisite in such cases is, that the iron shall exceed the strength of the other parts, and as the quantity thus employed in any case was inconsiderable, it was of little importance if more iron than was really necessary was used, and which, therefore, was commonly done, and its actual strength disregarded. But since the time alluded to, malleable iron has been introduced for several important purposes, in which it is employed by itself to resist enormous strains, as in the case of ships' cables, suspension bridges, and railway bars; it is, therefore, of the greatest importance that we should be able, from a knowledge of its actual

strength, to proportion the several parts, so that while we insure perfect safety on the one hand, we may not on the other unnecessarily employ more of the material than is requisite, for all the weight thus introduced beyond what safety requires is always unnecessary, and frequently injurious.

The first application of malleable iron, which rendered this knowledge indispensable, was the invention of iron cables, by Captain Brown, and he accordingly was the first person who constructed a machine capable of making experiments on a sufficiently large scale to be depended upon: this was made to work by wheel-work and a well-balanced system of levers, but subsequent experimenters have generally employed the hydrostatic press, a machine admirably suited to such a purpose: commonly, however, in these the force was estimated by the pressure on a small valve, which was very defective on two accounts; 1st, because the friction of the leathers, which is very considerable with large strains, was not included; and 2ndly, because the proportion between the valve and piston was too great. Such machines, therefore, commonly overrated the strain, and the motion of the balance weight was too small to be sufficiently perceptible.

To avoid these two evils, the Admiralty have had an excellent machine of this kind constructed in Woolwich Dockyard, for testing their iron cables, in which the strain is brought on by hydrostatic pressure, but its amount is estimated by a system

of levers balanced on knife edges, which act quite independently of the strain there is upon the machine, and exhibit sensibly a change of pressure of $\frac{1}{8}$th of a ton, even when the total strain amounts to 100 tons. It is also furnished with a valve according to the plan above alluded to, but this serves only to show the great defect of such apparatus; for while the lever scale is, as above stated, sensible to $\frac{1}{8}$th of a ton, the other will scarcely move with a change of 2 tons, its indications being less and less sensible as the strains become greater and greater.

Proving Machine in the Dockyard, Woolwich.

141. This machine was constructed by Messrs. Bramah, of Pimlico, and is doubtless one of the most perfect of the kind which has been executed; and as all the following experiments on railway bars, and many of the others on the tensile force of iron and other metals, were made with this press, the following description of it will not be unacceptable to the reader. It consists of two cast-iron sides, cast in lengths of $9\frac{1}{2}$ feet each, with proper flanches for abutting against each other and for fixing the whole to sleepers resting on a secure stone foundation. The whole length of the frame is $104\frac{1}{2}$ feet, equal to $\frac{1}{8}$th the length of a cable for a first-rate; so that the cables are tested in that number of detached lengths, which are afterwards united by shackle-bolts.—

The press is securely bolted down at one end of the frame, and the cylinder is open at both ends. The solid piston is $5\frac{1}{4}$ inches in diameter in front and $10\frac{1}{2}$ inches behind, so that the surface of pressure is the difference of the two, viz.

$$\left(\frac{21}{2}^2 - \frac{21}{4}^2\right) \times \cdot 7854 = 65\tfrac{1}{2} \text{ inches.}$$

The system of levers hung on knife edges is attached to the other end of the frame, and the cable is attached by bolt links to this and to the end of the piston-rod. The levers being properly balanced, and the cable attached to a short arm rising above the axis, this draws the other arm downwards, and at a distance equal to twelve times the short arm is a descending pin and ball: this acts in a cup placed on the upper part of the arm of the second lever, and this again acts on a third. The first two levers are under the floor, and pass ultimately into an adjacent room, where a scale carrying weights is conveniently placed, and the whole combination is such that every pound in the scale is the measure of a ton strain; and as we have stated, the whole acts with such precision that $\frac{1}{8}$th of a pound, more or less, in the scale very sensibly affects the balance. At the same place is situated a scale, acted upon by the water pressure from the charge-pipe of the press; and the valve in this pipe is of such dimensions that, together with the lever by which it acts, the

power is again such that a pound should balance a ton; but the friction is here so great that it requires several pounds to make a sensible change in the apparent balance, and for this reason this scale is never used. The forcing-pumps are in another adjacent room, and are worked by handles, after the manner of a fire engine. At first, six pistons are acting, and the operation proceeds quickly; but as the pressure and strains increase, the barrels are successively shut off, till at length the whole power of the men is employed on one pair of pumps only, and on this the action is continued till the proof strain is brought on the cable. A communication is then opened between the cistern and cylinder, and every thing is again restored to equilibrium.

The foregoing general description will be better understood by a reference to our Plates VI. and VII.;[1] the former exhibiting the pumps, and the latter the press-frame, levers, &c. Fig. 1, Plate VI., is a representation of the pumps in elevation, of which an end view is given in fig. 2. There are two pumps working in each of the frames, A, B, C, fig. 1, and

[1] The drawings here referred to were made from a very accurate and excellently executed model of this machine, constructed by an Egyptian youth, Mahomet Al Moonga, sent to this country by Mahomet Ali Pasha, for instructions as an engineer: he was placed under the tuition of Mr. John Kingston, assistant-engineer in His Majesty's Dockyard, Woolwich, under whose able instructions he not only constructed this machine, but also other models and drawings, highly creditable to his industry and talents.

the latter is also exhibited in fig. 2. Of these, the pair in the frame A are of the largest bore, viz. $1\frac{1}{4}$ inch; those in B are 1 inch, and in C, $\frac{3}{4}$ inch. The manner of acting on each of the pumps will be understood by the drawing fig. 2, and the process of working the whole by referring to fig. 1, in which D D, D D are rails for the persons employed to pump with, as in the fire engine. The holes for receiving these rails are also shown at D D, fig. 2. E E is an iron cistern containing the water, into which of course the several pump-barrels, or pipes from them, descend, and are there supplied with valves opening upwards to prevent the return of the water. The piston or plunger is solid, and by its action the water is forced along the pipe or passage *a a*, *b b*, *c c*, and ultimately to the descending pipe *d*, which passes under the floor, rising again under the centre of the press or cylinder, entering it as shown at *d*, fig. 3, Plate VII.

This description seems all that is required to explain the operation of the pumps when they are all in action; but when great strains are called for, it is necessary to shut off, first, one pair of pumps at A, and then the other pair B, so that the whole force of the men is then employed on the pair C only. The means of effecting this is very simple and ingenious: *s s* are two standards which serve as supports for the ends of the axles *g g*, each carrying a bevelled wheel *w w*, and these again work the horizontal bevelled wheels with which they are in connection; and thus the vertical axle, carrying also

the wheel *h*, is made to revolve by turning the handle *m*, and this again turns the wheel *h′* and its axle, but in an opposite direction. The lower part of each of the axles is a screw working in a nut, so that the axle rises and falls by turning the handle *m*, which is indeed the whole object of this part of the apparatus; the end of these axles being terminated conically, and each applying to a like formed hole in the water channel. When all the pumps are at work, the wheel *h′* is turned till its plug stops a hole entering the channel *a*, and the water is forced forward; the plug on the axle *h* being at this time raised, which opens a passage towards *b b*, and the same from *b b* to *c c*.

Suppose, now, it were required to shut off the pumps A, the handle *m* is turned, the axle *h′* is raised, and the other *h* depressed, till its plug shuts off the channel *a a* from *b b*; at the same time, the plug on the axle *h′* being raised, a passage is opened to a lateral pipe communicating with the cistern, so that the pumps A, although they continue in action as before, only raise the water from and return it to the cistern; and precisely the same applies to the pumps B.

The pair of pumps C have not this apparatus, and can only be opened to the cistern by means of a similar apparatus working horizontally by means of the handle *n n*, which communicates with a plug opening the main channel *c b a*, either to the pipe *d*, to communicate the pressure to the press, or to the

waste pipe *e*, thereby permitting the water to return to the cistern. This, therefore, is done at the end of each experiment, and the water remains, to be propelled as before when required.

We come now to a description of the press and its mode of operation. It has been stated that its whole length consists of eleven frames, of which only one is shown in the drawing at A B, A, B, figs. 1 and 2, Plate VII., the former being a plan and the latter an elevation. A part of the last frame is, however, shown in both figures, as is also that which supports the press: a cable is also represented in each figure as being tested. We may however observe, that in order to show the connecting pieces, the floor or platform of the bed (fig. 1) is removed; it consists of strong oak planks resting on the cross pieces seen in fig. 1. These cross pieces are not employed in the frame carrying the cylinder, the flanches of which, *ff*, *ff*, answer this purpose: fig. 3 is a section of the press, with the solid piston passing through, having the smaller end screwed into the larger, leaving the difference of the two areas for the action of the water, as already explained; each end passes through collars of leather, to prevent leakage, and of course the action of the piston is rendered perfectly uniform and in the same line: the frame A B, adjacent to that carrying the press, is supplied with two planed iron slides *d d*, on which moves a cross piece *a a*, supporting the end of the piston, the form of which

beyond the frame will be sufficiently seen by the drawing. To this one end of the cable is attached by a shackle-bolt, and in like manner to the other end of the frame in connection with the system of levers at *k*, the operation of which is still to be explained. The cable, prior to the experiment, rests on the platform of the bed, as above stated; *g h* is a heavy bent lever, as represented in fig. 2, turning on a knife edge seen at *g*; to this lever above, as at *k*, are two cheek pieces, held in their places by a strong bolt, and to the other end of these cheek pieces the last link of the cable is secured in the same manner; *l m* is a transverse lever under the floor of the room, turning on a knife edge at *l*, and passing beyond *m* into an adjacent room, where it is connected at *n* (fig. 4) with the hanging rod *o n*; this is again connected at *o* with the lever *o p*, having its fulcrum at *q*, the top of the fixed standard *q r*, which is securely fixed to the floor of the room *r s*; *t t* is a stone platform for supporting the scale and weights *w*. The first lever *g h* acts on the lever *l m* as follows: *h v* is a descending bolt, furnished at its end with a ball, and *w* is a socket on a piece rising from the lever *l m*, and which, of course, presses the lever *l m* downwards. It is now obvious, that when a strain is brought on the cable it pulls the bent end *k* of the lever *k g h* forward; this depresses the end *h*, and the ball *v*; this presses on the socket *w* of the lever *l m*; at *n* this pulls on the hanging rod *o n*, and this on the lever *o p*, which

raises the scale till such weight is introduced as balances the strain.

The arms of the several levers are so proportioned that a pound at w' shall balance a ton at k, but as this is difficult to be exactly brought out where all the parts are so large, there is an adjusting screw p which moves the point of support of the scale to and fro till this exact proportion is attained; and a detached apparatus, which belongs to the machine, enables the engineer in charge to ascertain at any time the accuracy of its action against actual weights up to four tons. The machine has lately been tested in this way, and its action found to be perfectly satisfactory, the scale exhibiting very perceptibly a change of strain of $\frac{1}{8}$th of a ton. It may be observed, that the axle of the large lever g turns in holes somewhat elliptical, and its end is borne up so as to carry the knife edge g to its proper position by a heavy weight x at the end of a long lever $x\,y$, which turns on an axle at z, attached to the side of the frame. This lever is seen dotted in fig. 5, which is the side of the frame, fig. 2, removed to show the large lever $g\,h$ more distinctly; precisely a similar lever acts on the other side frame against the axle g, to bear up its other extremity.

It only remains to say a few words more in reference to the press. The experiment being performed, the ram will be run out towards E, and will require to be pushed in towards A, prior to another trial. This is effected by means of the

rack *q r*, attached or detached at pleasure from the ram or piston at E; it is acted upon by the pinion on the axle Z Z, which is turned by hand by means of the wheel W W; but when the experiment commences, this pinion is thrown out of gear with the rack by pushing the axle a little endwise, so as to clear the teeth of each from the other.

Let us now describe one experiment: 1st, The length of cable to be tested is laid upon the bed of the press. The ram or piston of the press is run forwards as far as its shoulder towards A, by means of the wheel W W and rack *r q*. The handles *m m* (fig. 1, Plate VI.) are turned so as to open entirely the water channel to the press; a screw plug is also opened on the top of the cylinder to allow the air to escape, and when water shows itself at this aperture by the working of the pumps, this is screwed in. Every thing is now ready; the pumping commences, and all six pumps being in action, the piston retreats fast, till the cable begins to strain; the process is then slower, and after a time, when the strain is considerable, the pumps A are shut off, and afterwards, if necessary, the pumps B: the operation then continues on the pumps C only, till the proper strain is obtained, which is ascertained by a person at the scale, who continues to add pound after pound in the scale till the pounds' weight are equal to the tons' strain required. As soon as the scale rises with this weight, he pulls a handle which rings a bell in the pump-room, and

the operation ceases; the wheel *n n* (fig. 1, Plate VI.) is then turned so as to open the compressed water to the waste or return pipe *e*, through which the water returns to the cistern ready for performing the next experiment.

The experiments on bars, bolts, &c., described in the following pages, were made in the same manner, by employing two short lengths of cables, and making the trials by means of attachments to their two ends towards the centre of the bed or platform.

142. *Table showing the different kinds of best Bower Cables at present employed in the British Navy, with the corresponding Iron Cables, and the Proof Strain for each.*

Rates of Ships.	Best bower hempen cables, 100 fathoms. Sizes, circum.	Weight.			Number of threads in each.	Breaking strain by experiment.			Diameter and weight of the bolt of the iron cable substituted for the preceding.	Strain for the proof.
	in.	cwt.	qr.	lbs.		tons.	cwt.	qr.		tons.
First-rate, large	25	114	2	7	3240	..			2⅛ inches. 218 cwt.	81
middle	24	105	2	17	2988	..				
small	23	96	2	27	2736	114	0	0		
Second-rate . .	23	96	2	27	2736					
Third, large . .	23	96	2	27	2736					
small . .	22	89	0	12	2520	89	0	0	2 inches. 186 cwt. 2 qrs.	72
Fourth, 60 guns.	21	80	0	22	2268					
58 do. .	19	66	0	21	1872	..			1⅞ inch. 170 cwt. 2 qrs.	63
50 do. .	18½	62	1	14	1764	..				
Fifth, 48 do. .	18	58	2	6	1656	63	0	0	1¾ inch. 145 cwt. 3 qrs.	55
46 do. / 42 do.	17½	56	0	1	1584	..				
Sixth, 28 do. .	14½	38	0	21	1080	40	0	0	1⅝ inch. 87 cwt. 2 qrs.	34
Ship, sloop . .	13½	33	0	10	936	..			1½ inch. 74 cwt. 3 qrs.	28
Brig, large . .	13½	33	0	10	936	..				
Ditto, small . .	11	21	2	15	612	..			1⅜ inch. 61 cwt. 1 qr.	23

From the above Table the immense advantage of iron cables will be distinctly seen, and particularly when we consider that a hempen cable, on a rocky bottom, is destroyed in a few months, while the other will sustain no perceptible injury.

143. *Actual experimental Strength of Chain, made of various Descriptions of re-manufactured Foreign and English Iron, performed 2nd September,* 1816, *at Captain Brown's Manufactory.*

		tons.	cwt.
1½ inch	Old sable, 1½ inch square bars, cut into pieces 2 feet long, piled and rolled into bolts of 1½ inch	73	10
1½ inch	Old sable, ditto, ditto	80	0
1½ inch	Gurcoft new sable, ditto, ditto	71	0
1½ inch	Keiolsken, Archangel, inch square bars, cut into lengths of 2 feet, piled and rolled into bolts	71	0
1½ inch	Old bolts, found promiscuously, piled and fagoted by hand-hammers at my works	71	10
1½ inch full . .	English bars, piled and rolled	86	0
1½ inch bare . .	Ditto . . . . . . do	80	0

Further Experiments, made 13*th September,* 1816.

		tons.	cwt.
1½ inch	Old Dutch bolts, fagoted by hand-hammers at my works	71	0
1½ inch	No. 1, ⅝ square, (Welsh iron,) hammered into blooms, and rolled into bolts, at the King and Queen works	78	10
1½ inch	No. 2, ¾ inch square, (Welsh,) manufactured as above	73	5
1½ inch	No. 4, Welsh iron, fagoted by hand-hammers at my works	88	10
1½ inch	No. 6, ⅝ in. square ditto, rolled, but not hammered, at the King and Queen works	76	0
1½ inch	King and Queen scrap iron	80	5

The links of these chains were of an oval-like form, 6 inches in the clear.

S. BROWN.

The mean of these experiments gives 76 tons for the strength of a double bolt of 1½ inch diameter, *in the cable form*, which corresponds to about 21½ tons per square inch. Now by the same machine, the mean strength of wrought iron, per square inch, is 25 tons (see the following experiments); therefore, the strength of iron in the cable form is to that of the simple bolt in about the ratio of 43 to 50. But in these cables the links were without stays: when these are introduced, as in Brunton's patent cable, the strength is very nearly equal to that of the iron in the simple bar form; so that the stay may be said to increase the strength by about one-sixth part; at the same time, however, it must be considered that the weight is also increased, although perhaps in a somewhat less ratio.

Experiments on Direct Cohesion of Malleable Iron.

144. The next important application of malleable iron was in the construction of suspension bridges, also the invention of Capt. Brown. Subsequently, viz. in 1814, it was proposed by the late distinguished engineer, Thomas Telford, Esq., to suspend a bridge of this kind over the River Mersey at Runcorn, of 1000 feet span. In an undertaking of this magnitude, it became essentially necessary to know very exactly what strength could be depended upon in the material to be employed; and Mr. Telford

accordingly undertook an extensive series of experiments, both on the strength of malleable iron bolts, and on iron wire, with which he obligingly supplied me for the first edition of my 'Essay on the Strength of Materials.' These are given below, in the form in which they were recorded at the time of making them, at Messrs. Brunton's iron cable manufactory. The other experiments were in like manner supplied to me by Capt. Brown. It is only necessary to observe, that Messrs. Brunton's machine, being a hydrostatic press, registering by means of a valve, has a tendency to overrate its power, while Capt. Brown's, perhaps, slightly underrates its power; but his results certainly agree best with subsequent experiments made by myself on the machine in the Dockyard at Woolwich.

145. *Experiments on the direct Strength of Cohesion of Malleable Iron, made at Messrs. Brunton and Co.'s Patent Chain Cable Manufactory, with a Hydrostatic Machine, or Bramah Press, constructed by Mr. Fuller. By Thomas Telford, Esq.*

BAR No. 1.

Cylindrical Bar of South Wales Iron, manufactured by S. Homfrey, Esq.

April 5th, 1814.	Length of bar when put in	2 feet	$2\frac{3}{4}$ inches.
	Ditto when taken out	2	$6\frac{7}{8}$
	Diameter when put in	0	$1\frac{3}{8}$
	Ditto when taken out	0	$1\frac{1}{8}$

Torn asunder by 43 tons 11 cwt.

BAR No. 2.

Cylindrical Bar of South Wales Iron, manufactured by S. Homfrey, Esq.

April 15th, 1814.	Length of bar when put in . .	2 feet	$3\frac{3}{8}$ inches.
	Ditto when taken out . . .	2	$6\frac{5}{8}$
	Diameter when put in . . .	0	$1\frac{1}{2}$
	Ditto when taken out . . .	0	$1\frac{1}{4}$

Torn asunder by 52 tons 15 cwt. 1 qr. 10 lbs.
Time, 34 minutes.

BAR No. 3.

Square Bar of Staffordshire Iron.

May 17th, 1814.	Length of bar when put in . .	1 foot	$5\frac{1}{8}$ inches.
	Ditto when taken out . . .	1	$11\frac{1}{4}$
	Side of square when put in . .	0	$0\frac{3}{4}$
	Ditto when taken out . . .	0	$0\frac{6}{10}$

Began to stretch with 12 tons; broke with 15 tons 5 cwt. 3 qrs. 4 lbs. Time, $9\frac{1}{4}$ minutes.

BAR No. 4.

Square Bar of Staffordshire Iron.

May 17th, 1814.	Length of bar when put in . .	1 foot	$7\frac{1}{4}$ inches.
	Ditto when taken out . . .	1	$9\frac{1}{4}$
	Side of square when put in . .	0	$1\frac{1}{12}$
	Ditto when taken out . . .	0	$0\frac{5}{8}$

Began stretching with 32 tons; broke with 32 tons 6 cwt. 4 lbs. Time, 16 minutes.

BAR No. 5.

Square Bar of Welsh Iron, 1 *inch square.*

May 5th, 1817.	With 18 tons stretched . . .	$0\frac{1}{4}$ inch.	
	Ditto 21 tons ditto . . .	$0\frac{1}{2}$	
	Ditto 23 tons ditto . . .	$0\frac{3}{4}$	
	Ditto 25 tons ditto . . .	1	
	Ditto 27 tons ditto . . .	$2\frac{1}{4}$	
	Ditto 29 tons ditto . . .	$2\frac{3}{8}$	Broke with this weigbt.

BAR No. 6.

Bar of Swedish Iron, 1 *inch square.*

May 5th, 1817.	Began to stretch with 17 tons.		
	Stretched[2] with . .	20 tons, $\frac{1}{10}$th inch.	
	Ditto with	27 tons, $\frac{3}{8}$ths.	
	Ditto with	29 tons.	Broke at a flaw.

BAR No. 7.

Bar of Fagoted Iron, from Scrap Iron. By Mr. Howard, of Rotherhithe. 1 *inch square.*[3]

May 5th, 1817.	Began to stretch with 16 tons.		
	Stretched with . .	20 tons, $0\frac{3}{8}$ inch.	
	Ditto with	25 tons, $0\frac{3}{4}$	
	Ditto with	28 tons, $2\frac{3}{8}$	
	Ditto with	29 tons.	Broke with this weight.

BAR No. 8.

Bar of common Staffordshire Iron, 1 *inch square.*

May 5th, 1817.	Began to stretch with 19 tons.		
	Stretched with . .	24 tons, $0\frac{1}{4}$ inch.	
	Ditto with . . .	28 tons, $0\frac{5}{8}$	
	Ditto with . . .	29 tons, $0\frac{5}{8}$	
	Ditto with . . .	30 tons, 1	
	Ditto with . . .	31 tons.	Broke with this weight.

[2] The stretchings were measured on 12 inches in the middle of the bar.

[3] A similar bar began to stretch with 18 tons, and broke with the same weight as above; viz. 29 tons.

BAR No. 9.

Cylindrical Bar of common Iron, 2 *inches diameter.*[4]

May 21st, 1817.	With 45 tons.	Began to stretch; about $\frac{1}{10}$th of an inch on 12 inches in the middle. The machine being relieved, the bar shortened $\frac{1}{40}$th of an inch.
	With 50 do.	Stretched ·125 inch; relieved, and shortened as before.
	With 55 do.	Do. ·25; do. do.
	With 60 do.	Do. ·26
	With 70 do.	Do. ·375 inch; recover[d] very little when the machine was relieved.
	With 75 do.	Do. ·544; do. do.
	With $80\frac{1}{10}$ do.	Do. ·75; reduced in diameter to $1\frac{1}{16}$ inch.
	With 85 do.	Do. ·86; no perceptible change.
	With 90 do.	Do. 1·00; do. do.
	With 95 do.	Do. 1·35; reduced in diameter to $1\frac{7}{8}$ inch.
	With 100 do.	Do. 2·2; do. do. to $1\frac{1}{2}$, nearly.

With the last weight the bar gave evident signs of fracture; and, in a few minutes, gradually gave way.

[4] The whole length of the above bar was 2 feet; and it stretched in its whole length $2\frac{7}{8}$ inches; of which $2\frac{1}{8}$ inches were in 12 inches in the middle part. The whole time of making this experiment was three hours; and it was performed with the utmost care.

The machine was frequently relieved; and, when re-applied, constantly brought up the weight to what it was before, but never exceeded it; which is evidence of its accuracy.

Note.—It is a curious fact, and deserving the attention of philosophers, that frequently, at the moment of rupture, the bar acquires such a degree of heat in the fractured part, as scarcely to allow a person to hold it grasped in his hand without a painful sensation of burning.

Reduction of the above to 1 *inch square.*

	tons.	cwt.	
No. 1, reduced to 1 inch square, gives	29	6	Welsh.
No. 2, ...	29	16	Ditto.
No. 3, ...	27	3	Staffordshire.
No. 4, ...	27	10	Ditto.
No. 5, ...	29	0	Welsh.
No. 6, ...	29	0	Swedish.
No. 7, ...	29	0	Fagoted.
No. 8, ...	31	0	Staffordshire.
No. 9, ...	31	16	
	9)263	11	
Mean strength of an inch square bar	29	$5\frac{2}{3}$	

146. *Experiments on Iron Bars and Cables, made at the Patent Iron Cable Manufactory of Captain S. Brown, Mill Wall, Poplar, with a Machine which acts on the Principle of the Weigh-Bridges. From a Report presented to the Author by the above Gentleman.*

(COPY.)

Mill Wall, Poplar, 28th May, 1817.

Experiments on different Descriptions of Iron.

BAR No. 1.

A bar of Swedish iron, 3 feet 6 inches long, $1\frac{5}{16}$ inch square, required a strain of 40 tons 19 cwt. to tear it asunder in a straight line. It stretched during the operation $\frac{3}{16}$ths of an inch. No perceptible alteration in the general appearance of the bar, except at the place of rupture, where it was reduced to $1\frac{1}{16}$th of an inch.

The particles remarkably small and close, of a whitish grey colour; not the least heated in the operation.

BAR No. 2.

Another piece, 3 feet 6 inches long, same mark, required a strain of 39 tons 15 cwt. to tear it asunder in a straight line. It stretched $\frac{1}{4}$th of an inch, the bar being torn into cracks in various places. It reduced to $1\frac{1}{16}$th of an inch at the place of rupture. The particles remarkably close and small, as before, intermixed with a few fibrous specks.

Colour, whitish grey; not heated at the time of rupture.

BAR No. 3.

A Swedish bar, 3 feet 6 inches long, (different mark,) $1\frac{3}{16}$ inch square, required 33 tons 10 cwt. to tear it asunder in a straight line. This bar was exceedingly soft and ductile, having stretched 3 inches in the operation, and reduced at the place of rupture to $\frac{7}{8}$ths of an inch. It broke extremely fibrous, exhibiting no particles. The complexion silvery; very much heated at the place of rupture.

BAR No. 4.

A bolt of Russia old sable, marked C C N, 3 feet 6 inches long, $1\frac{5}{16}$ inch diameter, required a strain of 36 tons 2 cwt. to tear it asunder in a straight line. This iron, very soft and ductile, stretched $2\frac{1}{4}$ inches, and reduced at the place of rupture to 1 inch in diameter. This iron appeared at the place of rupture in the form of a scarf, as if it had been cut with a pair of shears: the surface so smooth, that there was no appearance of fibres or particles: its fibrous quality was, however, sufficiently indicated by the whole appearance of the bolt.

BAR No. 5.

A bar of Welsh iron, denominated No. 3; 3 feet 6 inches long, $1\frac{1}{4}$ inch square, required a strain of 38 tons 1 cwt. to

tear it asunder. This iron possessed considerable ductility, but reduced in diameter more gradually than in the two preceding experiments. It stretched 2 inches, and was reduced at the place of rupture to $1\frac{1}{16}$th inch. The complexion of this iron, when looking directly down upon the place of rupture, was a dingy blue; and when held horizontally to the light, and viewed obliquely, bright and fibrous, though not so white or silvery as the foreign iron. Very much heated at the place of rupture.

BAR No. 6.

A bar of common Welsh iron, 3 feet 6 inches long, $1\frac{1}{8}$th inch square. It required a strain of 31 tons. This bar had little ductility, and suffered no general derangement in the operation. It broke directly across the bar, and measured at the place of rupture $1\frac{1}{16}$th inch. The particles of this iron were fine, and exceedingly condensed, resembling steel; and there appeared nothing of a fibrous nature in it: indeed, its complexion and texture seemed to be at variance with the general rules for judging of the quality of iron. Its measure of strength, however, was most accurately ascertained.

BAR No. 7.

A highly interesting one. A bolt of Welsh iron denominated No. 3; 12 feet 6 inches long, 2 inches in diameter; required a strain of 82 tons 15 cwt. to tear it asunder. When subject to a strain of 68 tons, it stretched 3 inches, and was reduced to $1\frac{15}{16}$ths inch in diameter. When the strain was increased to 74 tons 15 cwt., it had stretched 6 inches, and was reduced $\frac{1}{8}$th of an inch gradually in the diameter. With 82 tons it stretched 14 inches. With 82 tons 15 cwt., the bolt broke about 5 feet from the end, the levers being exactly balanced. It had stretched during the whole process $18\frac{1}{2}$ inches; and measured at the place of rupture $1\frac{5}{8}$ inch in diameter.

SAMUEL BROWN.

BAR No. 8.

A bolt of Welsh iron, $1\frac{3}{8}$ inch diameter, 5 feet in length, was torn asunder by a force of $43\frac{1}{2}$ tons.

With 28 tons its diameter was reduced to 1·4 inch.

With 35 tons .... 1·35 inch.

With 40 tons .... 1·30 inch.

With 43 tons the bolt broke, having lengthened during the experiment 7 inches. Considerable heat about the section of fracture.

This is the only one of the above experiments at which I was present.

Reducing the above to inch square.

			Tons.
No. 1	Swedish iron	1 square inch,	23·77
No. 2	Ditto	ditto	23·19
No. 3	Ditto	ditto	23·75
No. 4	Russia	ditto	26·55
No. 5	Welsh	ditto	24·35
No. 6	Ditto	ditto	24·90
No. 7	Ditto	ditto	26·33
No. 11	Ditto	ditto	26·34

Mean . . . . 25 tons.

The mean of Mr. Telford's experiments is $29\frac{1}{2}$ tons.

Mean of the two . . . . 27 tons.

147. *Experiments made on Malleable Iron Bolts with the Testing Machine in Woolwich Dockyard. By the Author.*

The machine on which these experiments were made having been already described, it only remains to explain the manner in which the bars were held in order to be submitted to the strains requisite to produce rupture. In the experiments described in

the preceding pages, the ends of the bars were upset, as it is termed; that is, they had their ends hammered up into a conical lump, and were inserted into conical sockets made in two parts, which were placed over the ends and united by hoops. But there seems in this way a danger of injuring the texture of the iron; and it is, moreover, inapplicable in other metal without actual damage. The machine which I employed was invented by Mr. John Kingston, assistant-engineer in the above establishment; it is very simple and effectual, as will be seen by the following description:

Mr. Kingston's Nippers for Testing Metal Bars.

These are represented in Plate VI., figs. 6, 7, 8, 9. Here *a*, *a*, *a*, *a*, figs. 6, 7, are heavy wrought-iron sockets; *b*, *b*, strong double hooks of the same metal; *c*, *c*, *c*, *c*, links of the same metal, which connect the sockets *a*, *a*, &c., with the hooks *b*, *b*, the latter of which have eyes seen in fig. 7, by which they are connected with the cable proceeding from the ram of the press at one end, and with the levers at the other; and the ends of the sockets at *a*, *a*, have grooves to receive the links, to prevent them from slipping. Figs. 8 and 9 are the nuts, on an enlarged scale; they are formed in two parts with a cylindrical hole in each, which, being placed together, have a coarse screw cut in them; fig. 8 shows their form on the side, which is slightly bevelled, as is the socket. *B B* is the bar; this is placed in the nuts

as represented in figs. 6 and 7, and slightly driven with a hammer: the strain being now applied, the nuts are drawn into the sockets, which nip the bars more and more strongly as the strain increases, without any danger of slipping.

148. *Experiments on Malleable Iron Bars with the above Machine, in his Majesty's Dockyard, Woolwich. By the Author.*

BAR No. 1.—Solly's Patent Iron.

Round bar 1 inch in diameter, broke with a strain of 21 tons. It stretched before the fracture 10¼ inches in 8 feet in the middle; its whole length was 10 feet 2 inches.

Strength per square inch 26·7 tons.

The bar broke at a part where it had been nicked with a chisel. It was therefore tried again, the marked part being inserted in the nippers, and the breaking weight was now 23 tons, or strength per square inch 29¼, and stretch 2¼ inches more.

BAR No. 2.—Solly's Patent Iron.

Square bar 1 inch in diameter, broke with 23¼ tons at the place where it had been nicked with a chisel to mark it. It stretched 13⅝ inches in 8 feet. In consequence of this defect, the broken parts were again tried, and one of these, after being broken, was again tried; the following are the results:

Second trial, breaking weight	. . . .	26¼ tons.
Third do.	do.	26½
Fourth do.	do.	25¾

The following are results of other experiments on iron of good medium quality:

1. Bar 1 inch square, breaking weight	. .	24 tons.
2. Ditto	ditto . . .	25½
3. Round bar reduced to inch square	. .	25½
4. Ditto	ditto . . .	26
Mean strength	. . . .	25¼

In the preceding experiments the mean of Messrs. Brunton's and Brown's experiments gives 27 tons; but from these experiments I consider that we ought not to assume the strength of good medium iron at more than 25 tons per square inch. It will be seen by subsequent experiments that the elasticity is destroyed with about 10 tons, and that iron ought not to be strained beyond its elastic power.

149. Experiments on the strength of Yorkshire Iron, by M. I. Brunel, Esq. These were made on bars reduced in the centre part (per hammer) to $\frac{3}{8}$ths and $\frac{4}{8}$ths, or $\frac{1}{2}$ inch square; but the results are all reduced to rods of 1 inch square.

Experiments on the Direct Cohesive Power of Hammered Iron. By M. I. Brunel, Esq.

Iron denoted *best* $\frac{3}{8}$ths in the middle.			Iron denoted *best best* $\frac{3}{8}$ths in the middle.			Iron denoted *best* $\frac{1}{2}$ in the middle.		
No.	Began to stretch.	Breaking weight.	No.	Began to stretch.	Breaking weight.	No.	Began to stretch.	Breaking weight.
	Tons per inch.	Tons per inch.		Tons per inch.	Tons per inch.			Tons.
1	21	29·8	1	28·16	35·12	1		27
2	24	32	2	27·4	36·4	2		31·12
3	18·15*	25*	3	24·16	32·16	3		31·62
4	22	34·19	4	27·16	33·10	4		32·25
5	20	34·6	5	22·15	31·14	5		32·75
6	20	28·2	6	25·18	31·15	6		30·00
7	23·2	28·2	7	22·3	31·9			
8	24	31·6	8	21·9	29·6			
9	26·9	32·11	9	23·9	31·7			
10	23·1	28·12	10	21·9	30·7			
Mean	22·2	30·4		24·4	32·3			30·8

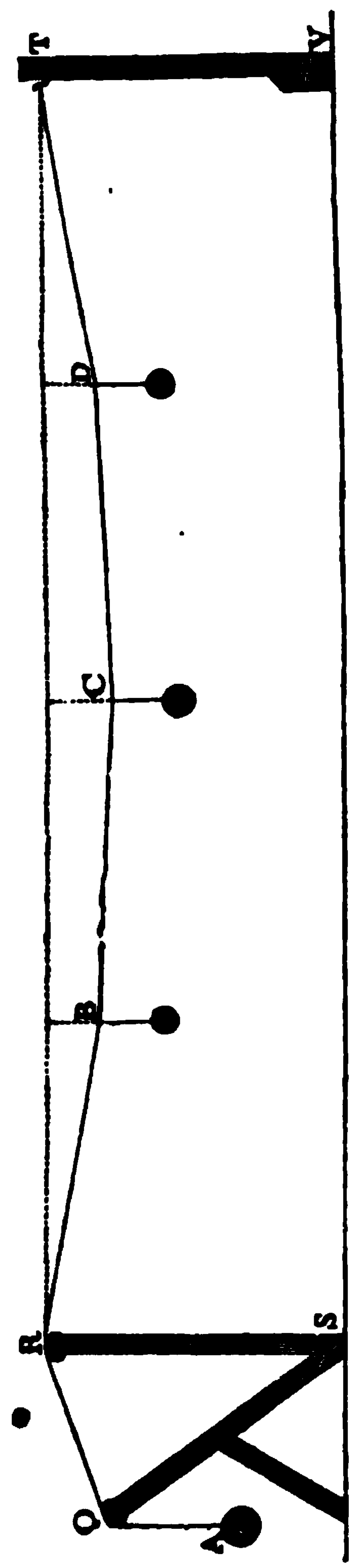
* The Experiment No. 3 of the first series was obviously defective.

The mean strength of these bars considerably exceeds that drawn from the preceding Articles; a circumstance which may, it is presumed, be explained from the fact of their having been reduced per hammer.

Experiments on the Strength of Iron Wire.

150. Amongst other propositions for suspension bridges, that of iron wire for the purpose has been included, and bridges of this kind have been executed; and as far as actual strength and facility of joining are concerned, it would appear to have a preference, but it is not thought applicable to the larger constructions of this kind. Mr. Telford, however, in the infancy of the practice, thought it desirable to try its strength under various circumstances, by submitting it to strains as nearly resembling those of the bridge itself as possible, with a statement of which he kindly furnished me in the form given in the following Tables.

In order to comprehend the tabulated results, it will be necessary to explain the apparatus with which the experiments were made: these are presented in the annexed figure.



Here R S, T V, represent the supporting pillars upon which the wire was extended; Q S, another prop over which the wire passed; being placed at such an angle as made it coincide with the direction of the resultant of the vertical and horizontal tensions, in order to prevent any strains upon the other support, R S.

A, B, C, D, represent the places of the several weights with which the wire was loaded; C being in the centre of the length, and B and D at ¼th of the length from each end; and the deflections from the horizontal line R T were measured at these points, as the different weights were applied.

EXPERIMENT No. 1.

Distance of the Props, 100 *feet; Weight of* 100 *feet of Wire,* 29½ *ounces; Diameter, rather more than* $\frac{6}{10}$*ths of an inch; and it broke when suspended vertically, at a medium of different trials, with* 531 lbs.

Weight at A, including the Wire QA.		Weight at B.		Weight at C.		Weight at D.		Deflection at B.		Deflection at C.		Deflection at D.		REMARKS.
lbs.	oz.	lbs.	oz.	lbs.	oz.	lbs.	oz.	ft.	in.	ft.	in.	ft.	in.	
5	6½	0	0	0	0	0	0	..		4	10	..		Deflections at B and D not taken.
10	5	0	0	0	0	0	0	..		2	11½	..		
30	5⅛	0	0	0	0	0	0	..		0	10½	..		
do.		0	0	1	0½	0	0	..		1	8	..		
do.		0	0	2	0½	0	0	..		2	7	..		
do.		0	0	5	0½	0	0	..		4	11	..		The weight at C being taken off, the deflection became 11 inches.
176	0	5	0	30	4	5	0	2	1	4	6½	2	1	
do.		9	0	30	4	5	0	2	5½	4	10¼	2	2½	Raised weight A 1 in.
226	0	9	0	56	0	5	0	3	11	7	10½	3	7½	
286	0	9	0	56	0	5	0	2	8¾	5	11½	2	6½	
342	0	9	0	56	0	5	0	2	3½	5	0¾	2	1¾	
do.		9	0	66	0	5	0	2	5	5	4½	2	3¼	
do.		9	0	72	0	5	0	2	7	5	9½	2	5¼	
do.		9	0	77	0	5	0	2	7	5	10	2	5¼	
do.		9	0	81	0	5	0	2	9¾	6	4¾	2	8	
do.		9	0	87	0	5	0	2	10¼	6	6¼	2	8½	
do.		15	0	71	0	15	0	2	11¾	6	3¾	2	11¾	
402	0	15	0	71	0	15	0	2	8¼	5	8¾	2	8¼	
402	0	30	0	56	0	30	0	..		..		..		Broke after sustaining these weights for a short time.

EXPERIMENT No. 2.

Distance of the Props, 31 *feet* 6 *inches; the same specimen of Wire as in Experiment No.* 1, *but had not been before used: the two Ends of the Wire, in this Experiment, were fixed, after drawing it as tight as possible; viz. to within less than* $\frac{1}{8}$*th of an inch of a horizontal line; and the Weights applied only in the centre.*

End at R and T fixed.	Weight at B.	Weight at C.	Weight at D.	Deflection at B.	Deflection at C.	Deflection at D.	Remarks.
		lbs.			ft. in.		
Fixed.	0	$10\frac{1}{4}$	0	..	0 2·83	..	
do.	0	$20\frac{1}{4}$	0	..	0 · 5·5	..	
do.	0	$30\frac{1}{4}$	0	..	0 7·75	..	
do.	0	$40\frac{1}{4}$	0	..	0 10	..	
do.	0	$50\frac{1}{4}$	0	..	1 0	..	
do.	0	$60\frac{1}{4}$	0	..	1 1·75	..	
do.	0	$70\frac{1}{4}$	0	..	1 3·5	..	
do.	0	$80\frac{1}{4}$	0	..	1 5	..	
do.	0	$90\frac{1}{4}$	0	..	1 6·5	..	
do.	0	$100\frac{1}{4}$	0	..	1 8	..	
do.	0	$110\frac{1}{4}$	0	..	1 9·75	..	
do.	0	$120\frac{1}{4}$	0	..	1 10·75	..	
do.	0	$130\frac{1}{4}$	0	..	...	..	

Just bore the last weight, and then broke.

EXPERIMENT No. 3.

Distance of Props, 100 *feet; Diameter,* $\frac{1}{10}$*th of an inch; Weight of* 100 *feet,* 2 lbs. 9 oz.: *bore vertically* 736 lbs., *but broke with* 738 lbs.

Weight at A.	Weight at B.	Weight at C.	Weight at D.	Deflection at B.	Deflection at C.	Deflection at D.	Remarks.
lbs.	lbs.	lbs.	lbs.	ft. in.	ft. in.	ft. in.	
362	0	0	0	..	0 5	..	
362	30	15	30	2 2	2 $11\frac{3}{4}$	2 $1\frac{1}{4}$	
362	35	30	35	2 8	3 $10\frac{5}{8}$	2 $7\frac{1}{4}$	
362	40	35	40	2 $11\frac{1}{10}$	4 $3\frac{1}{2}$	2 $10\frac{1}{2}$	
362	40	41	40	3 3	4 11	3 $2\frac{1}{4}$	
468	56	41	56	3 $4\frac{6}{10}$	4 $9\frac{4}{10}$	3 $4\frac{7}{10}$	
498	56	41	56	3 $0\frac{4}{10}$	4 $3\frac{6}{10}$	3 $0\frac{6}{10}$	
558	61	41	61	3 $1\frac{1}{2}$	4 $4\frac{1}{4}$	3 $1\frac{1}{4}$	
608	76	76	76	3 $5\frac{9}{10}$	5 $3\frac{2}{10}$	3 $6\frac{1}{2}$	Fixed the wire at A.
Fixed	56	56	56	3 0	4 $6\frac{7}{10}$	2 $11\frac{1}{2}$	.
do.	71	68	71	3 $3\frac{8}{10}$	5 0	3 4	Refixed the wire.
do.	do.	do.	do.	3 $4\frac{7}{10}$	5 $1\frac{8}{10}$	3 $4\frac{7}{10}$	
do.	77	74	77	3 $6\frac{8}{10}$	5 $4\frac{8}{10}$	3 $6\frac{8}{10}$	Refixed the wire.
do.	77	74	77	3 $3\frac{7}{10}$	4 $11\frac{8}{10}$	3 $3\frac{8}{10}$	

Bore this weight; but in attempting to add 4 lbs. more to the weights at B and D, the wire broke.

EXPERIMENT No. 4.

The same Wire as in last Experiment. Distance of the Props, 31 *feet* 6 *inches.*

Weight at A.	Weight at B.	Weight at C.	Weight at D.	Deflection at B.		Deflection at C.		Deflection at D.		Remarks.
	lbs.	lbs.	lbs.	ft.	in.	ft.	in.	ft.	in.	
Fixed	0	0	0	.	.	0	0⅛	.	.	Both ends fixed.
do.	40	41	40	0	7⅝	0	10⅞	0	7½	
do.	44	47	44	0	8⅛	1	0⅛	0	8⅜	
do.	50	47	50	0	9	1	0⅝	0	9	
do.	56	47	56	0	9¾	1	1¼	0	9½	
do.	56	53	56	0	10⅛	1	2	0	9¾	
do.	61	53	61	0	10⅛	1	2⅝	0	10¼	
do.	61	59	61	0	10¾	1	3⅛	0	10¾	
do.	67	68	67	1	0	1	4⅝	0	11⅝	
do.	71	68	71	1	0	1	4⅞	1	0	
do.	71	76	71	1	0⅛	1	5⅛	1	0¼	

With the last weights suspended a few minutes, the wire broke.

EXPERIMENT No. 5.

Distance of the Props, 100 *feet; Diameter,* $\frac{6}{100}$*ths of an inch; Weight of* 100 *feet,* 16½ *ounces. Vertically, the Wire bore* 277 lbs. *a few minutes, and then broke.*

Weight at A.	Weight at B.	Weight at C.	Weight at D.	Deflection at B.		Deflection at C.		Deflection at D.		Remarks.
lbs.	lbs.	lbs.	lbs.	ft.	in.	ft.	in.	ft.	in.	
180	0	0	0	0	1¾	0	1¼	0	1¾	
180	6	5	6	1	0⅞	1	5½	0	11¾	
180	12	10	12	1	10⅛	2	7¾	1	9½	
210	16	14	16	2	3½	3	2⅛	2	2	
248	16	14	16	2	2⅝	3	2⅛	2	2¼	Took off the weight A, and tightened the wire.
Fixed	16	14	16	1	9⅝	2	7¼	1	9¼	Broke the wire in attempting to draw it tighter.
ANOTHER PIECE OF THE SAME WIRE.										
Fixed	0	0	0	0	2¾	0	4	0	3¼	
do.	16	15	16	2	4	3	5	2	4⅞	
do.	22	19	22	2	7½	3	10	2	8⅒	

In attempting to increase these weights to 25, 26, and 27 lbs., the wire broke at a defective place.

EXPERIMENT No. 6.

Same Wire as in the preceding Experiment. Distance of the Props, 31 *feet* 6 *inches.*

Weight at A.	Weight at B.	Weight at C.	Weight at D.	Deflection at B.	Deflection at C.	Deflection at D.	Remarks.
	lbs.	lbs.	lbs.	ft. in.	ft. in.	ft. in.	
Fixed	22	30	22	0 11⅛	1 6	0 10⅞	
do.	28	30	28	1 1¼	1 6½	1 0⅝	
do.	30	30	30	1 1⅝	1 6⅜	1 1⅛	
do.	30	35	30	1 1⅜	1 7⅝	1 1⅛	

Broke in attempting to add 4 lbs. more at B and D.

EXPERIMENT No. 7.

Distance of the Props, 140 *feet; Diameter,* $\frac{1}{81}$ *of an inch; Weight of* 140 *feet,* 14 *ounces. Broke, vertically, with* 157 lbs.

Weight at A.	Weight at B.	Weight at C.	Weight at D.	Deflection at B.	Deflection at C.	Deflection at D.	Remarks.
lbs.	lbs.	lbs.	lbs.	ft. in.	ft. in.	ft. in.	
120	0	0	0	0 1½	0 1⅝	0 1⅜	
120	6	5	6	2 8	3 5$\frac{3}{10}$	2 7¾	
120	12	10	12	4 8$\frac{3}{10}$	6 4½	4 7$\frac{7}{10}$	
120	15	20	15	7 1½	10 0	7 0¾	
132	15	20	15	6 3¼	8 9⅛	6 4½	
132	21	25	21	8 8⅛	11 11	8 7	
150	21	25	21	7 11⅛	10 10	7 0	
150	25	25	25	8 3	10 11	8 2	Broke.

EXPERIMENT No. 8.

Same Wire as in the last Experiment. Distance of the Props, 31 *feet* 6 *inches.*

Weight at A.	Weight at B.	Weight at C.	Weight at D.	Deflection at B.		Deflection at C.		Deflection at D.		REMARKS.
	lbs.	lbs.	lbs.	ft.	in.	ft.	in.	ft.	in.	
Fixed	0	0	0	0	5⅛	0	5¼	0	4⅛	
do.	6	5	6	1	1¾	1	4¼	1	1⅛	
do.	12	10	12	1	4¾	1	8	1	3¼	
do.	16	15	16	1	6¼	1	10⅛	1	4⅞	
do.	20	20	20	1	7⅛	2	1	1	6⅝	

Broke in attempting to add 2 lbs. at B, 4 lbs. at C, and 2 lbs. at D.

EXPERIMENT No. 9.

The same Wire as last Experiment, and the Props the same distance; viz. 31 *feet* 6 *inches.*

Weight at A.	Weight at B.	Weight at C.	Weight at D.	Deflection at B.		Deflection at C.		Deflection at D.		REMARKS.
lbs.	lbs.	lbs.	lbs.	ft.	in.	ft.	in.	ft.	in.	
120	20	30	20	2	6	3	3⅛	2	2¼	
120	25	30	20	2	9¼	3	7	2	5	
120	31	34	31	3	$5\frac{4}{10}$	4	4⅛	2	11⅛	
120	34	34	34	3	6¾	4	5⅛	3	1¼	
120	34	42	34	3	9¾	4	11¼	3	2¾	
120	34	50	34	4	0	5	3¼	3	4	
150	34	50	34	3	$3\frac{6}{10}$	4	4⅛	2	$9\frac{8}{10}$	
150	34	55	34	3	6¼	4	8⅛	3	0	
150	37	55	37	3	$9\frac{4}{10}$	5	0	3	2⅛	
150	37	56	37	3	9¼	5	0	3	2⅛	
156	37	56	37	3	9¼	5	0	3	2⅛	
160	39	57	39	3	$9\frac{2}{10}$	5	$0\frac{2}{10}$	3	$2\frac{2}{10}$	Broke in attempting to add 6 lbs. more.

Note.—The above experiments were made at the Patent Iron Cable Manufactory of Messrs. Brunton & Co.

EXPERIMENT No. 10.

Distances of the Props, 900 *feet; Diameter of Wire,* $\frac{1}{10}$*th inch; Weight of* 900 *feet,* 28 lbs. *by the steelyard; Weight of* 100 *feet,* 3 lbs. $3\frac{1}{2}$ oz. *by the scales. Mean vertical Strength, from* 9 *Experiments,* 630 lbs.

Weight at A.	Weight at B.	Weight at C.	Weight at D.	Distance of C from the ground.		Remarks.
	lbs.	lbs.	lbs.	ft.	in.	
Fixed	0	0	0	15	6	On account of the length of the wire the curvature was measured from the ground; which latter was about 22 feet from the horizontal line, between the props or points of suspension.
do.	28	14	28	4	$0\frac{1}{4}$	
do.	28	17	28	3	4	
do.	28	19	28	3	0	
do.	28	20	28	2	10	
do.	28	21	28	2	$5\frac{1}{4}$	
do.	28	22	28	2	4	Removed the weights and re-tightened the wire.
do.	0	0	0	16	8	
do.	28	0	28	9	1	
do.	28	14	28	4	8	
do.	28	17	28	..		Broke the wire; not at a joint.

This experiment was made at Ellesmere; the points of suspension were, at one end a building, at the other a tree.

151. The nine experiments from which the mean vertical strength of 630 lbs. was deduced, are as follow:

	lbs.
1st broke with	616
2nd "	616
3rd "	620
4th "	652
5th "	616
6th "	637
7th "	616
8th "	646
9th "	651
	9)5670
Mean of 9 experiments . .	630 lbs.

The wire broke in these experiments at joints or unsound places; it may therefore be considered the minimum of strength.

The mean of twelve other experiments, on wires of the same diameter, but of different specimens, was 634 lbs.

Strength per square inch, 36 tons.

The following Table shows the strength of the different specimens reduced to square inches:

	Diam.		Tons.
Experiment 1 . . .	$\frac{6}{70}$	Strength per square inch	35·7
,, 2 . . .	$\frac{1}{8}$	,, ,,	42·0
,, 3, 4 . .	$\frac{1}{10}$	,, ,,	42·9
,, 5, 6 . .	$\frac{6}{100}$	,, ,,	38·1
,, 7, 8, 9 .	$\frac{1}{21}$	,, ,,	35·8
,, 10 . .	$\frac{1}{10}$	,, ,,	36·1
		Mean	38·4

Considerable discrepance will be observed between the strength of the wire in experiments 3, 4, and 10, which are of the same diameter. Perhaps a mean strength of 36 tons for a wire of less than, or not exceeding, $\frac{1}{10}$th inch diameter, is all that can be depended upon.

EXPERIMENTS

(152.) *On the Momentum which Wires stretched, as in the preceding Experiments, will bear before breaking.*

Experiment 1. A piece of wire, which bore vertically 277 lbs., was stretched between two props,

140 feet distant from each other, till the versed sine, or deflection in the centre, was only 4¾ inches.

A 5 lb. weight was then tied to a cord, and the other end fastened to the middle of the wire; the length of the cord between the weight and the wire was 10 feet 6 inches. The weight being now lifted up to the level of the wire, it was let fall and struck the ground, but without injuring the wire.

Shortened the cord to 7 feet 7 inches, and proceeded as above: it did not strike the ground, nor did it injure the wire.

With the same length of cord, and a 10 lb. weight instead of the 5 lb., proceeding in the same manner: struck the ground, but did not break the wire.

But the same weight hung by a string 6 feet 7 inches, let fall as above, broke the wire at a joint.

Note.—The distance of the middle of the wire from the ground was 13 feet 6 inches.

By the laws of falling bodies, we have for the

1st momentum		$(8 \times \sqrt{10{\cdot}5}) \times 5 = 129$
2nd	„	$(8 \times \sqrt{7{\cdot}58}) \times 5 = 110$
3rd	„	$(8 \times \sqrt{7{\cdot}58}) \times 10 = 220$
4th	„	$(8 \times \sqrt{6{\cdot}58}) \times 10 = 204$

The third momentum is greater than the fourth; but, as the weight strikes the ground, it is not all expended upon the wire.*

* In the preceding editions the talented author has inadvertently attributed the breaking with a diminished momentum to an injury of the wire at the third trial.—*Ed.*

Experiment 2. Distance of the props, 31 feet 6 inches. Diameter of the wire, $\frac{1}{10}$th inch. Stretched to within $\frac{1}{8}$th of an inch of a straight line.

A 10 ℔. weight was tied to the middle of the wire by a cord 7 feet 9 inches long: it was lifted up to the level of the wire, as in the last experiment, and then let fall; but it did not break the wire.

A 15 ℔. weight was tied, and let fall in the same manner, without breaking the wire.

A 20 ℔. weight was then tried. It did not break the wire.

A 25 ℔. weight, being let fall from the same height, *broke the wire*.

Here our four momenta are,

1st momentum	$(8 \times \sqrt{7 \cdot 75}) \times 10 = 222 \cdot 6$	
2nd	„	$(8 \times \sqrt{7 \cdot 75}) \times 15 = 333 \cdot 9$
3rd	„	$(8 \times \sqrt{7 \cdot 75}) \times 20 = 445 \cdot 2$
4th	„	$(8 \times \sqrt{7 \cdot 75}) \times 25 = 556 \cdot 5$

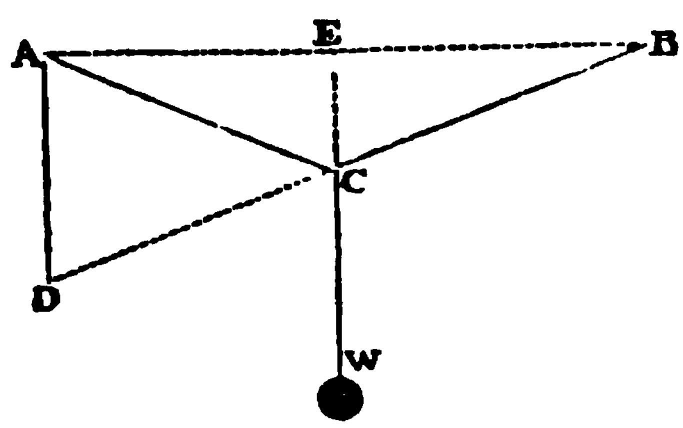
Comparing these momenta with the direct vertical strength, we have

1st vertical strength . .	277 ℔s.	momentum	220,
2nd ditto for wire of $\frac{1}{10}$ inch,	630 ℔s.	ditto	556·5;

that is, in the 1st experiment, the number expressing the momentum is less by $\frac{1}{8}$th than the vertical strength; and in the 2nd by $\frac{1}{8}$th: but it is probable that in the latter the wire would have been broken with a less weight than 25 ℔s.

153. *Comparison of the preceding Experiments on extended Wires, with their Strengths computed theoretically.*

In experiment No. 2, page 265, it appears that a piece of wire, whose vertical strength was 531 lbs., being stretched on props 31·5 feet apart, and having a weight of 120·25 lbs. hung at its middle point, had that point deflected 1 foot 10¾ inches, and that it afterwards broke with the addition of 10 lbs. Let us endeavour to compute how much this 10 lbs. exceeded what was absolutely necessary to break the wire; or, which is the same, let there be given the distance of the props, the deflection, and the tension of the wire, to find the weight which, suspended from its middle point, will produce the rupture.



Let A, B, in the preceding figure, represent the two fixed points; C E, the deflection; A C B, the wire: then it is obvious that the point C is kept in equilibrio by three forces; viz. the tension of A C, the equal tension of C B, and the unknown weight, W, *plus* the weight of the wire, *w*.* Now, when

* The tensions are those at the points A and B, where they are greatest. To obtain the actual tension at the point C of the wire, *w* must be omitted.—*Ed.*

three forces, acting on a material point, preserve that point in equilibrio, each of the three forces is equal and directly opposed to the resultant of the other two. If, therefore, C B be produced to meet the vertical A D, D E will be the direction of the resultant of the two forces, T, and $(W+w)$, representing by T the tension of A C; and A D C will be the triangle of forces which keeps the point C in equilibrio: the side A D, then, will denote the vertical force or weight, $W+w$; and by the nature of the construction $AD = 2\,CE$: we have, therefore,

$$AC : AD \text{ or } 2\,CE :: T : W+w,$$

$$\text{or } W = \frac{2\,CE \times T}{AC} - w.$$

Now, $CE = 1\cdot8958$ feet, or $2\,CE = 3\cdot7916$.
Also, $AC = \sqrt{(AE^2+EC^2)} = 15\cdot86$.
And by the data of Experiment 1, $w = \cdot29$ lb.

$$\text{Whence } W = \frac{3\cdot7916 \times 581}{15\cdot86} - \cdot29 = 126\cdot65 \text{ lbs.}$$

This is about 4 lbs. less than the weight found by the experiment.

We may arrive at the same conclusion on principles a little different from the above, and somewhat more general; viz., since the weight W is kept in equilibrio by the tensions of A C and C B; and since this weight, W *plus* w, the weight of the wire, is the only vertical force in the system, if we denote the tension of the wires A C and C B by T and T′, and

the angles E A C, E B C, by a and a', and resolve these two forces each into its component horizontal and vertical force; we must have the two former equal to each other, and the sum of the other two equal to the sum of the vertical weights, $W+w$; that is, we shall have

$$T \cos a = T' \cos a',$$
$$T \sin a + T' \sin a' = W + w;$$

from which equations the two tensions, T and T', may be determined, whatever may be the ratio of the two parts A C, C B; but in our case, as these are equal, the first equation disappears, and the second becomes

$$2 T \sin a = W + w, \text{ or}$$

$$T = \frac{W + w}{2 \sin a}.$$

Or if T be given, and W required,

$$W = 2 T \sin a - w.$$

In the experiment above referred to,

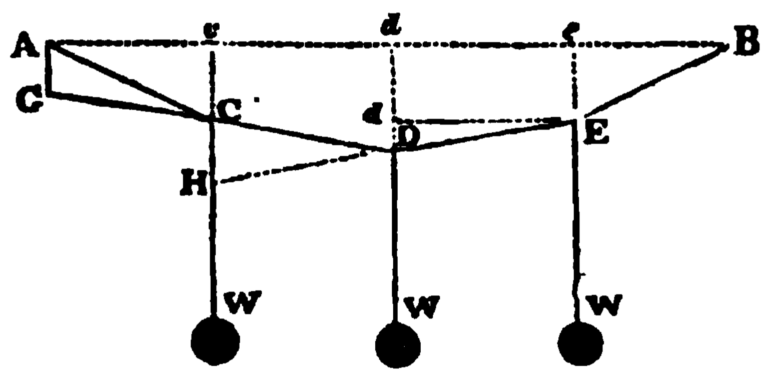
$$T = 531, \sin a = \cdot 1195593, \text{ and } w = \cdot 29.$$

Whence

$$W = 2 \times 531 \times \cdot 1195593 - \cdot 29 = 126 \cdot 65 \text{ lbs., as before.}$$

In a similar manner might be computed the tensions of the extreme points, when there are more than one weight, as in the third and subsequent experiments: but it will be, perhaps, more simple to begin here by computing the tensions of the two adjacent sides, C D and D E; which may be effected

precisely in the same manner as in the preceding case. For it is a principle in mechanics, that if a



system of forces be in equilibrio, no alteration will take place in that state, by supposing any two or more of its points to become fixed: we may, therefore, suppose the points C and E fixed, and compute the tension of C D, or D E, exactly as above; viz. calling the angle D C E $= a'$, and the centre weight W′, and the tension t, we shall have

$$t \sin a = \tfrac{1}{2} W' + \tfrac{1}{4} w,$$

$$t = \frac{W' + \frac{1}{2} w}{2 \sin a'},$$

where w is the whole weight of the wire: then, having the tension t, the weight W, and the angle D C W, compute the value of the resultant of these two forces, which will obviously be the tension of A C; that is, if we denote this tension by T, we shall have

$$T = \sqrt{\{t^2 + W^2 + 2 \sin a' \, t W\}}.$$

In experiment 3, $W' = 74$, $w = 2{\cdot}5625$, and $\sin a = {\cdot}06685$, whence

$$t = \frac{75{\cdot}2812}{{\cdot}1337} = 563 \text{ lbs.}$$

$$\text{And } T = \sqrt{\{563^2 + 77^2 + \cdot 1337 \times 563 \times 77\}} = 573.$$

This gives the tension too little: let us therefore compute the same from the 1st deflection; that is, by resolving T into two forces, the one horizontal, and the other vertical, and equating the latter with half the sum of the weights, *plus* half the weight of the wire; for as the whole system is retained in equilibrio by the two extreme tensions, the vertical component of each ought to be equal to half the entire vertical force, or half the whole weight. This consideration gives,

$$T \sin a = \tfrac{1}{2}(W + W' + W'' + w),$$

where a denotes the angle C A c.

In the 3rd experiment,

$$a = 7^\circ\ 32', \text{ and } \sin a = \cdot 1311.$$

$$\text{Whence } T = \frac{230 \cdot 56}{\cdot 2622} = 879.$$

If now we take the mean of our two results, we shall have

$$\frac{879 + 573}{2} = 726 \text{ lbs.}$$

Whereas the vertical strength, as determined from experiment, was 736 lbs.

The two different results given by the two methods show that the system had assumed a form inconsistent with a perfect state of equilibrium, supposing the several lengths, or distances, A a, c d, &c., to be equal: but it is obvious, that, besides the probable

unequal extensibility of the wire, the point C, as the wire stretches, will approach towards A, and recede from the perpendicular; for D being exposed to equal actions on each side, will continue in the same vertical: this will obviously have a tendency to increase the angle a, and decrease the angle a'; and, consequently, to increase the value of the tension computed according to the former method, and to diminish the same according to the latter, and therefore approximate them towards that medium result we have obtained above, which differs only 10 lbs. from what was found experimentally; viz. about 1 lb. out of 73 lbs.

In the 4th experiment,

$$a' = 2^\circ\ 51'\ \sin a' = \cdot 04893, \text{ and } \tfrac{1}{2} w = \cdot 39 \text{ lb.}$$

$$t = \frac{76 \cdot 39}{\cdot 09786} = 790 \text{ lbs.} \quad \text{And}$$

$$T = \surd\{790^2 + 71^2 + \cdot 0979 \times 790 \times 71\} = 797 \text{ lbs.}$$

According to the second principle, viz.

$$T \sin a = \tfrac{1}{2}(W + W' + W'' + w),$$

we have

$$W + W' + W'' + w = 218 \cdot 79, \text{ and } \sin a = \cdot 12648;$$

$$\text{whence } \frac{218 \cdot 79}{\cdot 25296} = 864 \text{ lbs.};$$

the mean of which is 831 lbs. instead of 736 lbs., which is in excess by about $\frac{1}{8}$th part.

154. It will be observed, however, that these methods are only approximative; but they are per-

haps more intelligible to many readers than if we had entered upon the problem with all the generality that belongs to the doctrine of equilibrium of flexible bodies: but it may not be amiss to give a sketch of this general method, at least as applied to the action of vertical weights upon a perfectly flexible line.

Here we may suppose any number of weights W, w, w, &c., W′; and a corresponding number of distances, L, l, l', l'', &c., L′, which may be equal or unequal: the tensions of these lines we may denote by

$$T, t, t', t'', \&c.\ T',$$

and their several angles, with reference to a horizontal axis A x, passing through A, by

$$a, \alpha, \alpha', \alpha'', \&c.\ a',$$

and their angles with reference to the other axis A y,

$$b, \beta, \beta', \beta'', \&c.\ b'.$$

Also, let n be the co-ordinate of the point B with reference to A y, and m its co-ordinate as referred to B y.

Then if we resolve each of the tensions into its corresponding horizontal and vertical components, we shall have from the theory of equilibrium,

$$T \cos a + t \cos \alpha + t' \cos \alpha' + \&c.\ T' \cos a' = 0,$$
$$T \cos b + t \cos \beta + t' \cos \beta' + \&c.\ T' \cos b' = 0.$$

And by means of the co-ordinates,

$$L \cos a + l \cos \alpha + l' \cos \alpha' + \&c.\ L' \cos a' = n,$$
$$L \cos b + l \cos \beta + l' \cos \beta' + \&c.\ L' \cos b' = m,$$

and by the known property of cosines,

$$\cos^2 a + \cos^2 b = 1,$$
$$\cos^2 a' + \cos^2 b' = 1.$$

From which six equations the six unknown quantities, viz. T, T', cos a, cos b, cos a', cos b', may be determined; after having first computed t, t', &c., and cos a, cos a', &c., in functions of T, cos a, and W, w, w', &c., which may, in all cases, be effected on the general principle of the composition of forces; that is, taking t as the resultant of T and W, t' as the resultant of t and w, and so on.

The computations, however, if the number of weights be considerable, become extremely laborious, and difficult to execute: but if, as in the experiment, we limit the weights to three, and consider the two extreme ones equal to each other, and the points A and B as being situated in the same horizontal line; then, as the several tensions and angles from each extreme are equal, we may reduce the above equations to three; in which, however, we have still to compute cos a in functions of T, cos a, and W; on which account we prefer, in this case, retaining the six equations under the form

$$\mathrm{T} \cos a = t \cos \alpha,$$
$$\mathrm{T} \cos b = t \cos \beta + \mathrm{W},$$
$$l \cos a + l \cos \alpha = \tfrac{1}{2} n,$$
$$\mathrm{T} \cos b = \tfrac{1}{2}(\mathrm{W} + w + \mathrm{W}),$$
$$\cos^2 a + \cos b^2 = 1,$$
$$\cos^2 \alpha + \cos^2 \beta = 1.$$

From which these several quantities may be determined, in functions of each other.

If we denote the less deflection, c C, by d, and the greater, D d, by $d + d'$, we shall have

$$\frac{d}{l} = \cos b, \text{ and } \frac{d'}{l} = \cos \beta;$$

and substituting these in the first four equations, and denoting the entire weight of the system by π, we shall have, after reduction,

$$\begin{aligned} &T \sqrt{(l^2 - d^2)} = t \sqrt{(l^2 - d'^2)} \\ &T d = t d' + W \\ &\sqrt{(l^2 - d^2)} + \sqrt{(l^2 - d'^2)} = \tfrac{1}{2} s \\ &T d = \tfrac{1}{2} l \pi. \end{aligned}$$

From which we may determine any one of these quantities in terms of the others: but it will be observed here, as in our partial solution, that if we suppose both deflections d and d' as known quantities, there will be a superfluity of data; viz. we shall have more equations than unknown quantities; and by assuming values for both these, we may give such as are inconsistent with the other data, and therefore also inconsistent with a state of perfect equilibrium: it is proper, therefore, in the solution of these equations, to include one of these quantities with the data, and one with the *quæsiti* of the problem: in which case a rational solution will be obtained.

We shall not attempt the numerical solution of these equations; but the reader who is desirous of doing so will find no other difficulty than what belongs to the algebraical operations: we shall content ourselves with the approximative numbers as

above determined, considering it useless to expect a nearer approximation between theory (which is founded on a supposition of a perfect uniformity of matter, and the most accurate mode of action) and experiments, in which every kind of irregularity with regard to the composition of the material, and all the errors of fixing, observing, &c., are presented: indeed the agreement between the two deductions may be considered a confirmation of the correctness of the theory, and of the accuracy with which the experiments were performed; and on the basis of the two combined every confidence may be placed, as to computation, relative to works which from their magnitude bid defiance to any experiment, except that of their actual construction.

Calculation of the Strength of a Suspension Bridge, on the supposition of its forming a perfect Catenary Curve.

155. The foregoing experiments and computations, although they would probably have constituted the only data on which Mr. Telford would have proceeded in his proposed construction of the Runcorn Bridge, yet they can only be considered as roughly approximative to the real case. And it must perhaps be admitted, that by assuming the bars to form a perfect catenary, we still only approximate. The approximation is, however, much more close in this case than in the former, and sufficiently so for all practical purposes.

The properties of the catenary are investigated in most treatises on mechanics; we shall not, therefore, retrace steps which have been so often taken; but merely bring under one point of view these several relations; referring such of our readers as may be desirous of actual investigations to the several works in which they may be found, particularly to Poisson, 'Traité de Mécanique,' whence the following have been selected:

Let l denote the length of the catenary;
- l' the distance of its points of suspension;
- c the angle between the tangent at the point of suspension, and the above horizontal line of distance;
- A the tension of the chain at the same point;
- T the tension at any other point;
- x any variable absciss;
- y the corresponding ordinate;
- s the corresponding arc;
- h the weight of an inch, or a foot, &c., of the chain x, y, s, &c., being taken in the same unit of measure.

This notation being established, the following are the principal properties of this curve; viz.

1. $\frac{l'}{l} = \frac{\cos c}{\sin c} \text{ hyp log } \frac{\cos c}{1 - \sin c}.$

2. $\frac{A \sin c}{h} = \frac{1}{2} l$, or $A = \frac{h l}{2 \sin c}.$

3. $T = \sqrt{\{A^2 - 2 A h s . \sin c + h^2 s^2\}}.$

Which at the lowest point becomes

$$T = A \cos c.$$

4. $\frac{1}{2} l = \frac{A \sin c}{h}$, and $\frac{1}{2} l' = \frac{A \cos c}{h} \text{ hyp log } \frac{\cos c}{1 - \sin c}$.

5. $x = \frac{A \cos c}{h} \text{ hyp log } \left\{ \frac{A - hy \mp \sqrt{\{(A - hy)^2 - A^2 \cos^2 c\}}}{A(1 - \sin c)} \right.$.

6. $y' = \frac{A(1 - \cos c)}{h}$.

Where y' is the ordinate to the middle or lowest point of the curve:

7. $s = \frac{A \sin c}{h} \pm \sqrt{\frac{\{(A - hy)^2 - A^2 \cos^2 c\}}{h}}$.

These formulæ are not all necessary for the solution of our problem, but are given as embracing the principal properties of this curve.

156. Let us now suppose a bar of iron, which we must consider as flexible, to be fixed to two points of suspension, 1000 feet distant, the lowest point of the curve being $\frac{1}{20}$th of the whole distance, or 50 feet; and let it be required to find the length of the bar and its action on the points of suspension, the weight h of one foot of it being given. In the present question, assuming the specific gravity of iron 7788, and the diameter of the bar $\sqrt{18}$ inches, we find $h = 48$ lbs.; also $l' = 1000$ feet; whence, by formula 6,

$$y' = \frac{A(1 - \cos c)}{h}, \text{ or}$$

$$A(1 - \cos c) = y'h = 48 \times 50 = 2400.$$

And formula 4 gives

$$\tfrac{1}{2} l = \frac{A \cos c}{h} \text{ hyp log} \frac{\cos c}{1-\sin c} = 500.$$

$$\frac{2400 \cos c}{48\,(1-\cos c)} \text{ hyp log} \frac{\cos c}{1-\sin c} = 500.$$

$$\text{Whence } \frac{\cos c}{10\,(1-\cos c)} \text{ hyp log} \frac{\cos c}{1-\sin c} = 1;$$

which, by approximation, gives angle $c = 11^\circ\ 15'$, nearly.

And hence, by formula 1, we find

$$l = 1008 \text{ feet} = \text{length of the catenary}.$$

Again, by formula 2,

$$A = \frac{h\,l}{2 \sin c} = \frac{1008 \times 48}{\cdot 39018} = 124005 \text{ lbs., or}$$

about 55 tons, the tension at the point of support.

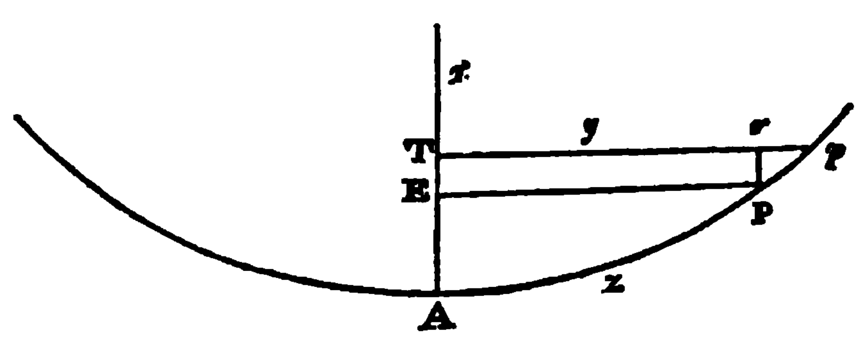
In a similar way, the tension and length being given, the depth of the curve may be computed.

It is not necessary, however, to have recourse to this mode of calculation, Mr. Davies Gilbert having, in an extremely ingenious paper, in the Phil. Trans. for 1826, supplied two Tables, by means of which every circumstance connected with these kinds of calculation becomes merely a matter of tabular inspection, as explained in the following article.

Tables for computing all the circumstances of Strain, Strength, &c. of Suspension Bridges. By Davies Gilbert, Esq., F.R.S. (Philosophical Transactions for 1826.)

157. It may be proper to premise that in our preceding experiments we have seen that the mean ultimate strength of malleable iron is 25 tons per square inch, which is equal in weight to a bar of the same dimensions whose length is 16,500 feet, which is sometimes called the modulus of the strength of iron, and is constant for bars of all dimensions. In like manner the strain or tension on a bar may be expressed by the number of feet in length of a bar of the same dimension.

In the following Table I., Mr. Gilbert uses the same method, but the unit, instead of being a foot, is $\frac{1}{100}$th of the half distance of the points of support or of the ordinate of the semi-catenary; and the values of x, z, a and T, have also for their unit $\frac{1}{100}$th



of the same length, x being the greatest depth of the curve, and z the length of the semi-catenary, a the modulus of tension at the lowest point of the curve, and T the tension at the point of support. In Table II. the unit is $\frac{1}{100}$th of the modulus of tension a (in *feet*) at the lowest point of the curve, and x, y, z and T, have also the same unit; x and z in this Table

being the length of each absciss with its corresponding arc, for every unit of y, reckoning from the lowest point of the curve.

Suppose, for example, the span of a proposed bridge were 800 feet, and its greatest deflection 50 feet. Here the unit or $\frac{1}{100}$th of the half-span is 4, and consequently the value of x in the Table is 12·5. Now, opposite 12·565, (which is the nearest tabular number,) we have the tension T at the point of support, 412·56, and this, as the unit is 4 feet, is equal to 1650·24 feet of a bar of the same section, whence the tension in lbs. or tons becomes known. We find also a, the tension, at the lowest point 400, whence

$$400 \times 4 = 1600, \text{ the tension in feet.}$$

Since the tension is thus found to be 1600, the hundredth part of this is 16, which is therefore the unit of Table II., and the several values of x and z multiplied by 16, will be their corresponding values in feet for each unit of y. Thus the maximum and minimum tension of the bar, and the lengths of the several suspending bars, are determined, or rather perhaps their difference of length; for their absolute length will of course depend upon the depth of the platform below the lowest point of the curve. It has been seen that in the case here assumed the greatest tension is 1650 feet; whereas the ultimate strength is 16,500 feet; the bar is therefore stretched with only $\frac{1}{10}$th of strain that would destroy it; and supposing the weight of the suspending bars, roadway, &c., to be $\frac{2}{3}$rds of the weight of the bar, the strain would still be only 2750, or $\frac{1}{6}$th of the full

power of the iron, that is, about $4\frac{1}{2}$ tons per inch, whereas we have seen that iron will bear 9 or 10 tons per inch without destroying its elasticity or power of restoration.

In the Menai Bridge the span is about 580 feet, and the unit, therefore, of our first Table 2·90; the greatest deflection is 43 feet, therefore $\frac{43}{2{\cdot}9}=14{\cdot}8=x$, nearly, whence $T=354{\cdot}8$, which, multiplied by 2·9, gives for the modulus of maximum tension 1028·9 feet from the weight of the bars alone. This weight, according to Mr. Provis's statement, is 394 tons, and the whole weight, including platform, &c., 643 tons: hence 394 : 643 : : 1028·9 : 1680, the whole strain.

The strength therefore here is nearly 10 times greater than the strain, independently of a passing load. Again, the tabular value of $a = 340$, and $340\times2{\cdot}9=986$; then 394 : 643 : : 986 : 1610, value of a, Table II.

Therefore, to find now the several abscisses or the length of the suspending bars for every division or hundredth part of the ordinate y, since the whole value of a is 1610, our unit (one hundredth of this) is 16·1; we have therefore only to multiply the several numbers in the column x by 16·1 for the lengths required.

For more on the subject the reader is referred to an excellent Memoir on Suspension Bridges, by Mr. Eaton Hodgkinson, vol. v. of the Manchester Memoirs. See also, Drewry on Suspension Bridges.

TABLE I.

Table for the Computation of Suspension Bridges.

a	x	z	T	ANGLE.
2000	2·500	100·0	2002	87° 8′
1950	2·564	100·0	1952	87 3
1900	2·632	100·0	1902	86 59
1850	2·703	100·0	1852	86 54
1800	2·778	100·0	1802	86 49
1750	2·857	100·0	1752	86 43
1700	2·942	100·0	1702	86 37
1650	3·031	100·0	1653	86 31
1600	3·125	100·0	1603	86 25
1550	3·226	100·0	1553	86 18
1500	3·334	100·0	1503	86 10
1450	3·449	100·0	1453	86 3
1400	3·572	100·0	1403	85 54
1350	3·705	100·0	1353	85 45
1300	3·847	100·0	1303	85 35
1250	4·002	100·1	1254	85 25
1200	4·168	100·1	1204	85 13
1150	4·350	100·1	1154	85 1
1100	4·548	100·1	1104	84 47
1050	4·765	100·1	1054	84 33
1000	5·004	100·1	1005	84 16
980	5·106	100·1	985·1	84 9
960	5·213	100·1	965·2	84 2
940	5·324	100·1	945·3	83 54
920	5·440	100·1	925·4	83 47
900	5·561	100·2	905·5	83 38
880	5·687	100·2	885·6	83 30
860	5·820	100·2	865·8	83 21
840	5·959	100·2	845·9	83 11
820	6·105	100·2	826·1	83 1
800	6·258	100·2	806·2	82 51
780	6·418	100·2	786·4	82 40
760	6·588	100·2	766·5	82 28
740	6·767	100·3	746·7	82 16
720	6·955	100·3	726·9	82 4
700	7·154	100·3	707·1	81 50
680	7·366	100·3	687·3	81 36
660	7·590	100·3	667·5	81 21
640	7·828	100·4	647·8	81 5
620	8·081	100·4	628·0	80 47
600	8·352	100·4	608·3	80 29
580	8·642	100·4	588·6	80 10
560	8·952	100·5	568·9	79 49
540	9·283	100·5	549·2	79 27
520	9·645	100·6	529·6	79 2

TABLE I.—(CONTINUED.)

a	*x*	*z*	T	ANGLE.
				° ′
500	10·03	100·6	510·0	78 36
480	10·45	100·7	490·4	78 8
460	10·91	100·7	470·9	77 38
440	11·41	100·8	451·4	77 5
420	11·96	100·9	431·9	76 29
400	12·56	101·0	412·5	75 49
380	13·23	101·1	393·2	75 5
360	13·97	101·2	373·9	74 17
340	14·81	101·4	354·8	73 32
320	15·75	101·6	335·7	72 22
300	16·82	101·8	316·8	71 14
280	18·04	102·1	298·0	69 57
260	19·46	102·4	279·4	68 29
240	21·12	102·8	261·1	66 47
220	23·11	103·4	243·1	64 48
200	25·52	104·2	225·5	62 28
180	28·55	105·3	208·5	59 39
160	32·28	106·6	192·2	56 19
140	37·25	108·7	177·2	52 10
120	44·13	111·9	164·1	46 58
100	54·30	117·5	154·3	40 23
95	57·67	119·5	152·6	38 28
90	61·51	121·8	151·5	36 26
85	65·85	124·6	150 8	34 17
80	71·07	128·1	151·0	31 58
75	77·14	132·3	152·1	29 32
70	84·43	137·6	154·4	26 57

TABLE II.

Table for the Computation of Suspension Bridges.

y	*x*	*z*	T	ANGLE.
				° ′
1	·0049	1·000	100·0	89 25
2	·0200	2·000	100·0	88 51
3	·0450	3·000	100·0	88 16
4	·0800	4·000	100·0	87 42
5	·1250	5·002	100·1	87 8
6	·1800	6·003	100·1	86 33
7	·2450	7·005	100·2	85 59

TABLE II.—(CONTINUED.)

y	x	z	T	ANGLE.
8	·3201	8·008	100·3	85° 25′
9	·4052	9·012	100·4	84 51
10	·5004	10·01	100·5	84 16
11	·6056	11·02	100·6	83 42
12	·7208	12·02	100·7	83 8
13	·8461	13·03	100·8	82 34
14	·9815	14·04	100·9	82 0
15	1·127	15·05	101·1	81 26
16	1·282	16·06	101·2	80 52
17	1·448	17·08	101·4	80 18
18	1·624	18·09	101·6	79 44
19	1·810	19·11	101·8	79 10
20	2·006	20·13	102·0	78 36
21	2·213	21·15	102·2	78 3
22	2·429	22·17	102·4	77 29
23	2·656	23·20	102·6	76 56
24	2·893	24·23	102·8	76 22
25	3·141	25·26	103·1	75 49
26	3·399	26·29	103·3	75 16
27	3·667	27·32	103·6	74 42
28	3·945	28·36	103·9	74 9
29	4·234	29·40	104·2	73 36
30	4·533	30·45	104·5	73 3
31	4·843	31·49	104·8	72 30
32	5·163	32·54	105·1	71 58
33	5·494	33·60	105·4	71 25
34	5·835	34·65	105·8	70 53
35	6·187	35·71	106·1	70 20
36	6·550	36·78	106·5	69 48
37	6·923	37·84	106·9	69 16
38	7·307	38·92	107·3	68 44
39	7·701	39·99	107·7	68 12
40	8·107	41·07	108·1	67 40
41	8·523	42·15	108·5	67 8
42	8·950	43·24	108·9	66 36
43	9·388	43·33	109·3	66 5
44	9·837	45·43	109·8	65 33
45	10·29	46·53	110·2	65 2
46	10·76	47·63	110·7	64 31
47	11·24	48·74	111·2	64 0
48	11·74	49·86	111·7	63 29
49	12·24	50·98	112·2	62 59
50	12·76	52·10	112·7	62 28
51	13·28	53·23	113·2	61 58
52	13·82	54·37	113·8	61 27
53	14·37	55·51	114·3	60 57
54	14·93	56·66	114·9	60 27
55	15·51	57·81	115·5	59 57

TABLE II.—(CONTINUED.)

y	x	z	T	ANGLE.
56	16·09	58·97	116·0	59° 28′
57	16·68	60·13	116·6	58 58
58	17·29	61·30	117·2	58 29
59	17·91	62·48	117·9	58 0
60	18·54	63·66	118·5	57 31
61	19·18	64·85	119·1	57 2
62	19·84	66·04	119·8	56 33
63	20·51	67·25	120·5	56 4
64	21·18	68·45	121·1	55 36
65	21·87	69·67	121·8	55 7
66	22·58	70·89	122·5	54 39
67	23·29	72·12	123·2	54 11
68	24·02	73·36	124·0	53 44
69	24·76	74·60	124·7	53 16
70	25·51	75·85	125·5	52 48
71	26·28	77·11	126·2	52 21
72	27·05	78·38	127·0	51 54
73	27·84	79·65	127·8	51 27
74	28·65	80·94	128·6	51 0
75	29·46	82·23	129·4	50 34
76	30·29	83·53	130·2	50 7
77	31·13	84·83	131·1	49 41
78	31·99	86·15	131·9	49 15
79	32·86	87·47	132·8	48 49
80	33·74	88·81	133·7	48 23
81	34·63	90·15	134·6	47 57
82	35·54	91·50	135·5	47 32
83	36·46	92·86	136·4	47 7
84	37·40	94·23	137·4	46 42
85	38·35	95·61	138·3	46 17
86	39·31	96·99	139·3	45 52
87	40·29	98·39	140·2	45 27
88	41·28	99·80	141·2	45 3
89	42·28	101·2	142·2	44 39
90	43·30	102·6	143·3	44 15
91	44·34	104·0	144·3	43 51
92	45·39	105·5	145·3	43 27
93	46·43	106·9	146·4	43 4
94	47·53	108·4	147·5	42 40
95	48·62	109·9	148·6	42 17
96	49·72	111·4	149·7	41 54
97	50·85	112·9	150·8	41 31
98	51·98	114·4	151·9	41 8
99	53·14	115·9	153·1	40 46
100	54·30	117·5	154·3	40 23

On the Transverse Strength, &c. of Malleable Iron.

158. It has been already observed, that iron has been only lately employed to any extent to resist a transverse strain; and writers, therefore, who have undertaken experiments to investigate the strength of materials, have hitherto passed over those inquiries which relate to the transverse strength of this metal.[5] The extraordinary extent, however, to which malleable iron is now applied to resist transversely a passing load, renders it highly essential that this resistance, and its other properties, should be fully investigated; for it is obvious that every additional weight of metal, beyond that which is requisite for perfect safety and durability, is not only uselessly, but injuriously employed,—it being generally admitted that bars beyond a certain weight cannot be so well or so cheaply manufactured as those of less dimensions; and it is no less certain, that by a proper disposition of the metal in the sectional area

[5] Some few experiments on the transverse strength of malleable iron have certainly been made. I have given three in my 'Essay on the Strength of Materials.' Mr. Hodgkinson has also glanced at this subject in his valuable paper of 'Experiments on Cast Iron,' published in the 'Memoirs of the Manchester Philosophical Society,' and M. Duleau has treated of the subject in his 'Essai Théorique et Experimental,' &c.; but those points of greatest importance connected with the application of this metal to the purposes of railways have never formed the subject of inquiry.

of the bar, (which depends on the data in question,) a greater strength may be obtained with a given weight of iron, than with a greater weight injudiciously disposed. Under these impressions, the following experiments have been undertaken, and to these inquiries they have been principally directed; but as there will be found references to some other matters connected with the practical application of malleable iron, &c. to railways, it may be well to state the circumstances under which the experiments were undertaken, in order to render some remarks and observations the more intelligible to the reader. These were as follow:

The Board of Directors of the London and Birmingham Railway Company, desirous of carrying on the great work in which they were engaged on the most scientific principles, and, if possible, to avoid the enormous cost of repairs which had attended some large works of a similar description, offered, by public advertisement, a prize of one hundred guineas "for the most improved construction of railway bars, chairs, and pedestals, and for the best manner of affixing and connecting the rail, chair, and block to each other, so as to avoid the defects which had been felt more or less on all railways hitherto constructed;" stating, that their object was to obtain, with reference to the great momentum of the masses to be moved by locomotive steam engines on the railway,—

1. "The strongest and most economical form of rail.

2. "The best construction of chair.

3. "The best mode of connecting the rail and chair; and also the latter to the stone blocks or wooden sleepers. And that the railway bars were not to weigh less than fifty pounds per single lineal yard."

In consequence of this advertisement, a number of plans, models, and descriptions were deposited with the Company within the time limited; and others were received afterwards, which, although not entitled to the prize, were still eligible to be considered with reference to their adoption for trial. On the 24th of December, 1834, a resolution was passed at a meeting of the Directors, appointing J. U. Rastrick, Esq., of Birmingham, N. Wood, Esq., of Newcastle, civil engineers, and myself, to examine and report upon the same, with a view to awarding the prize; and, at the same time, we were requested to recommend to the Directors such plans, whether entitled to the prize or not, as might be considered deserving of a trial. We met accordingly in London; and, after a long and careful examination of the several plans, drawings, and written descriptions, recommended those we thought entitled to the prize, which was awarded by the Directors accordingly. But that part of our instructions which required us to recommend one or

more rails for trial, we were unable to fulfil to our satisfaction, principally for want of data to determine which of the proposed rails would be strongest and stiffest under the passing load, and whether permanently fixing the rail to the chair, for which there were several plans, would be safe in practice; and as no experiments on malleable iron had ever been made, bearing on these points, it was considered better to leave the question unanswered, than to recommend, on no better ground than mere opinion, an expensive trial, which might ultimately prove a failure.

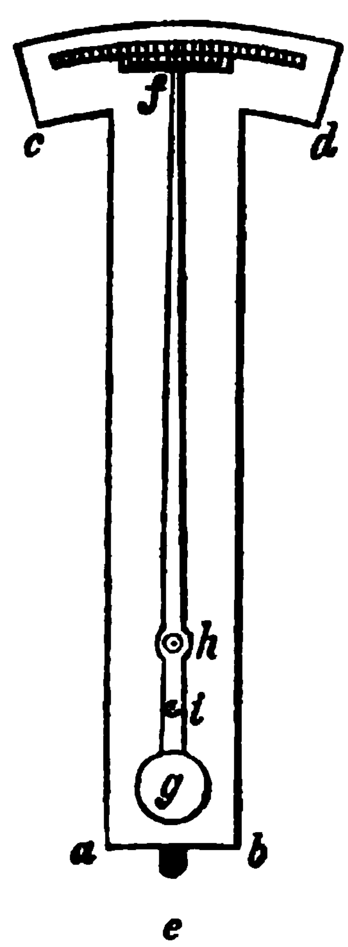
Seeing, however, how desirable it was that such data should be obtained, I proposed to the Directors to undertake a course of experiments, which should be conducted on a scale adequate to the importance of the subject, provided my Lords Commissioners of the Admiralty would allow me the conveniences His Majesty's Dockyard at Woolwich afforded, (which I had every reason to hope they would do, from the liberality I had so frequently experienced from that Board on similar occasions,) and that the Directors would supply such instruments, material, and workmanship, as might be required for the purpose.

The Admiralty, as I had anticipated, immediately granted my request; and, at a public meeting of the proprietors, held at Birmingham, a resolution was passed, embodying my proposition. I accordingly commenced and continued my experiments

till I had elicited such facts as I thought necessary; and having arranged them, I delivered the results, with a Report founded upon them, to the Secretary of the London Committee, to lay them before the Board; which being done, the Directors were pleased to express their approbation of my labours, and their wish that their results should be made public. They were accordingly printed and very generally circulated, nearly in the form in which they are given in the following pages; such experiments, however, being added, as I have since made for other railway companies, and such remarks and observations as have arisen out of a more extended examination of the subject.

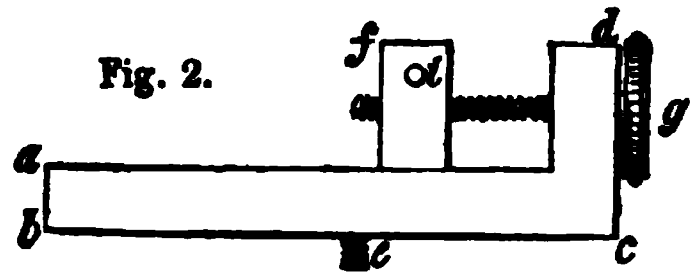
Experiments to determine the Quantity which Iron extends under different Degrees of Tension.

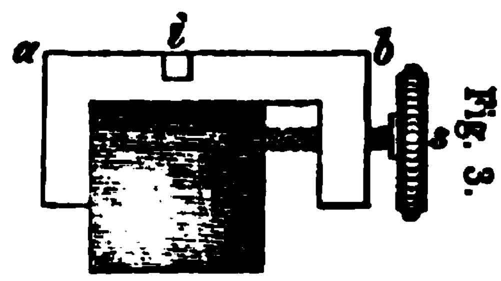
Fig. 1.



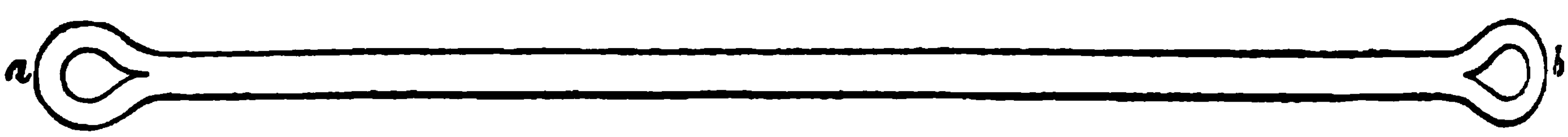
159. With a view to this inquiry, an instrument was made as in the annexed sketch.—*abcd*, fig. 1, is a piece of brass, about one-fifth of an inch thick, having an arc at top, divided into tenths of inches; *ghf* is a hand, with a vernier, turning freely on a centre *h*; and *i* is a steel pin, about half an inch long, projecting perpendicularly forward; the distances *fh* to *hi* being as 10

to 1. *e* is a small end with a screw, for the purpose described below; *abcd*, fig. 2, is another piece of brass, having a screw *e*; *f* is a piece working in a dovetail, adjustable for position by the screw *g*, and *i* is another steel pin projecting forward. *a b*, fig. 3, is an iron saddle-piece, with a set-screw *s*; and at *i* a hole is tapped to receive the screw *e*, fig. 2; and another saddle-piece, exactly like this, is made to receive the screw *e*, of fig. 1.





The iron bars intended to be experimented on were made of the annexed form,[6] about 10 feet in length;



these, by proper bolts and shackles, were fixed at *a* and *b* in the proving machine already described (Art. 141); the two saddle-pieces were then fixed on at the exact distance of 100 inches; the instruments, figs. 1 and 2, screwed into their respective saddle-pieces, and a light deal rod hung, by means of two small holes formed in it, (also at the distance of 100 inches,) upon the two pins *i i*; and then by means of the set-screw, fig. 2, the vernier of fig. 1 was adjusted exactly to zero. The pump of the press was now put in action, and after one, two, or more tons' pressure was on, according to the size of

[6] Mr. Kingston's nippers (described in Art. 147) were not made when these first experiments were being carried on.

the bar, and every thing brought well to its bearing, the hand was again adjusted to zero, after which the index was read for every additional ton. Here it will be seen, that whatever the bar stretched between the two instruments, the lower pin of fig. 1 was drawn forward, and the index end thrown back ten times that amount, consequently to ten times the actual amount of the quantity stretched.

It has been observed, that after one, two, or more tons' strain was applied, to bring every thing well to its bearing, the index was adjusted to zero, and its reading afterwards carefully registered as each additional ton was added. The strain during the experiment was repeatedly let off, and the index was found to return to zero, till the strain amounted to about nine or ten tons per inch, when the stretching became greater for each ton, and the bar did not any longer retain its original length when the strain was removed, its elasticity with this tension being obviously injured.

These experiments required more attendance than it was possible for one person to give; the adjustment of the weights, the reading and registering the index, required each the undivided attention of one individual; the pumping also required to be watched with care. And I have great pleasure in acknowledging the ready assistance I received from Messrs. Lloyd and Kingston, the engineers of the yard; from Mr. P. W. Barlow, civil engineer; as also from Lieut. Lecount, who came from Birmingham to witness and assist in the experiments.

EXPERIMENTS

(160.) *On the Longitudinal Extension of Malleable Iron Bars, under different Degrees of direct Tension.*

TABLE I.

Bar No. 1, 1 inch square. February 21st.			Bar No. 2, 1 inch square. February 21st.		
Weight in tons.	Index readings.	Parts of the whole bar extended by each ton.	Weight in tons.	Index readings.	Parts of the whole bar extended by each ton.
2	zero		2	zero	
3	·0625	·0000625	3½	·11	·0000733
4	·156	·0000935	4	·15	·0000800
5	·265	·0001090	5	·24	·0000900
6	·375	·0001100	6	·35	·0001100
7	not observed.	mean.	7	·44	·0000900
8	·562	·0000935	8	·52	·0000800
9	not observed.	mean.	9	·62	·0001000
10	·750	·0000940	10	·70	·0000800
11	·875	·0001250	11	·81	·0001100
			12	1·13	Elasticity injured.

Bar No. 3, 1 inch diameter. February 23rd.			Bar No. 4, 1 inch diameter. February 23rd.		
Weight in tons.	Index readings.	Parts of the whole bar extended by each ton.	Weight in tons.	Index readings.	Parts of the whole bar extended by each ton.
1	zero		1	zero	
2	·16	·0001600	2	·15	·0001500
3	·31	·0001500	3	·28	·0001300
4	·44	·0001300	4	·42	·0001400
5	·56	·0001200	5	·56	·0001400
6	·67	·0001100	6	·69	·0001300
7	·79	·0001200	7	·79	·0001000
8	·91	·0001200	8	·97	·0000800
9	·103	·0001200	9	·116	Elasticity destroyed.

Mean extension per ton, per square inch.

Bar No. 1.	·0000982
No. 2.	·0000903
No. 3.	·0001010
No. 4.	·0000976
Mean of the four . .	·0000967

TABLE II.

Bar No. 5, 2 inches square. February 28th.			Bar No. 6, 2 inches square. February 28th.			Bar No. 7, 2 inches square. March 7th.		
Weight in tons.	Index readings.	Parts of the whole bar extended by each 4 tons.	Weight in tons.	Index readings.	Parts of the whole bar extended by each 4 tons.	Weight in tons.	Index readings.	Parts of the whole bar extended by each 4 tons.
4	zero		4	zero		4	zero	
6	·100		6	·090		6	·065	
8	·180	·000180	8	·150	·000150	8	·125	·000125
10	·240	·000140	10	·210	·000120	10	·175	·000110
12	·290	·000110	12	·250	·000100	12	·230	·000050
14	·350	·000110	14	·290	·000080	14	·280	·000050
16	·400	·000110	16	·335	·000085	16	·335	·000050
18	·450	·000110	18	·375	·000080	18	·385	·000105
20	·500	·000100	20	·410	·000075	20	·435	·000100
22	·550	·000100	22	·445	·000070	22	·480	·000095
24	·600	·000100	24	·485	·000075	24	·530	·000095
26	·650	·000100	26	·525	·000080	26	·575	·000095
28	·695	·000095	28	·565	·000080	28	·625	·000095
30	·740	·000090	30	·620	·000095	30	·670	·000095
32	·790	·000095	32	·660	·000095	32	·715	·000090
34	·825	·000085	34	·730	·000110	34	·755	·000085
36	·860	·000075	36		{ Full elasticity. }	36	·805	·000090
38	·920	·000095	38			38	·850	·000095
40	1·05	·000145	40			40	·900	·000095
		{ Elasticity exceeded. }						{ Elasticity perfect. }

Mean extension per ton, per square inch.

Bar No. 5.	·0001082
No. 6.	·0000957
No. 7.	·0000841
Mean	·0000946
Mean of preceding Table	·0000967

Collecting the results of these seven experiments, and reducing them all to square inch sections, we find that the strain which was just sufficient to balance the elasticity of the iron, was in—

Bar No.	1, re-manufactured iron, . . .	10 tons.
„	2, ditto, . . .	11 „
„	3, New bolt, ...	11 „
„	4, Ditto, ...	10 „
„	5, re-manufactured, ...	9·5 „
„	6, ditto, from old furnace bars, .	8·25 „
„	7, New bar, by Messrs. Gordon,	10 „

We may consider, therefore, that the elastic power of good medium iron is equal to about ten tons per inch, and that this force varies from ten to eight tons in indifferent and bad iron. It appears, also, (considering ·000096 as representing in round numbers $\frac{1}{10000}$th,) that a bar of iron is extended one ten-thousandth part of its length by every ton of direct strain per square inch of its section; and, consequently, that its elasticity will be fully excited when stretched to the amount of one-thousandth part of its length.

Remarks on the foregoing Experiments.

161. These results have an important bearing on the question of railway bars. We shall see, in the following section, how they become applicable to the investigation of the transverse strain; but, at

present, I shall only speak of them as they apply to the fixing of the rail to the chair. Amongst the numerous models which the Directors did Messrs. Rastrick, Wood, and myself the honour to submit to our inspection, for the purpose of awarding their prize, there were several in which it was intended to fix the rail permanently to the chair,—a very desirable object, if it could have been safely adopted; and it was the want of data to enable us to decide on this point, which first led me to propose this course of experiments. The question is now satisfactorily answered. We have seen that, with about ten tons per inch, a bar of iron is stretched $\frac{1}{10000}$th part of its length, and its elasticity wholly excited or surpassed. Again, admitting 76° to be the extreme range of the thermometer in this country, between summer and winter, it appears, from the very accurate experiments of Professor Daniell,[7] that a bar of malleable iron will contract with this change $\frac{1}{2000}$th part of its length. And hence it follows, that if the rails were permanently fixed to the chair in the summer, the contraction in the winter would bring a strain of five tons per inch upon the bar, and a strain of twenty-five tons upon the chair, (the bar being supposed of five-inch section,) thereby deducting from the iron more than, or full, half its strength, and submitting the chair to a strain very likely to destroy it. Every

[7] See 'Philosophical Transactions,' 1831.

proposition, therefore, for permanently attaching the rail to the chair is wholly inadmissible.

These remarks may also be carried farther, for if it be dangerous to attach the rail *directly* to the chair, it must be bad in practice to affix it *indirectly* by wedges, cotters, or otherwise, beyond what is absolutely essential to give it steadiness under the passing load; for it is evident, that if by these means we could prevent any motion taking place, we should fall into the same evil as by the permanent attachment; and if, as most probably will happen, we fail of entirely accomplishing this, still all the friction which is produced must be overcome by the contracting force of the iron, and be so much strength deducted from its natural resisting power.

The problem, therefore, which engineers have to solve is, "To find a mode of fixing the rail to the chair, which shall give sufficient steadiness to the former; but which, at the same time, shall produce the least possible resistance to the natural expansion and contraction of the bar."

The quantity of motion which thus takes place is certainly but small, viz. about $\frac{1}{11}$th of an inch between summer and winter, with a fifteen-feet bar; but the force of contraction is great, amounting to five tons per sectional inch for the annual extremes, and frequently to not less than two and a half tons, between the noon and night of our summer season, while the whole power of iron within the limits of its elasticity does not exceed nine or ten tons.

This is an important consideration, and for want of attention to it, or rather in consequence of its amount not having been ascertained, a practice of wedging or fixing the rails has prevailed, which must necessarily have been the cause of great destruction to the bars.

I would also state here a suggestion by Mr. Woodhouse, one of the candidates for the prize, as a matter deserving the attention of practical men,— that as the bar must necessarily contract, it will draw from that side which is least firmly fixed, and hence all the shortening will most probably be exhibited at one end, however slight the hold on either may be; and when it happens that the adjacent ends of two bars both yield, the space between the two is rendered double that which is necessary. To avoid this evil, one of the two middle chairs in each bar might be permanently attached to the rail, in which case the contraction must necessarily be made from each end, and the space occasioned by the shortening of the bars would then be uniform throughout, and much unnecessary and injurious concussion would thus be saved both to the rail and to the carriage.

Experiments to determine the comparative Resistance of Malleable Iron to Extension and Compression, and the Position of the Neutral Axis in Bars submitted to a Transverse Strain.

162. It has been already demonstrated, (Art. 46,) that if the length of a bar of any kind, supported at both ends and loaded in the middle, be denoted by l, the depth by d, the depth of tension and compression by d' and d'', the tension per square inch by t, and the weight by w, then will

$$\tfrac{1}{3}(d'' + d')\,d'\,t = \tfrac{1}{4}\,l\,w, \text{ or}$$
$$\tfrac{1}{3}\,d\,.\,d'\,t = \tfrac{1}{4}\,l\,w\,;$$

d being the whole depth, and d' the depth of tension: whence, for any given breadth a,

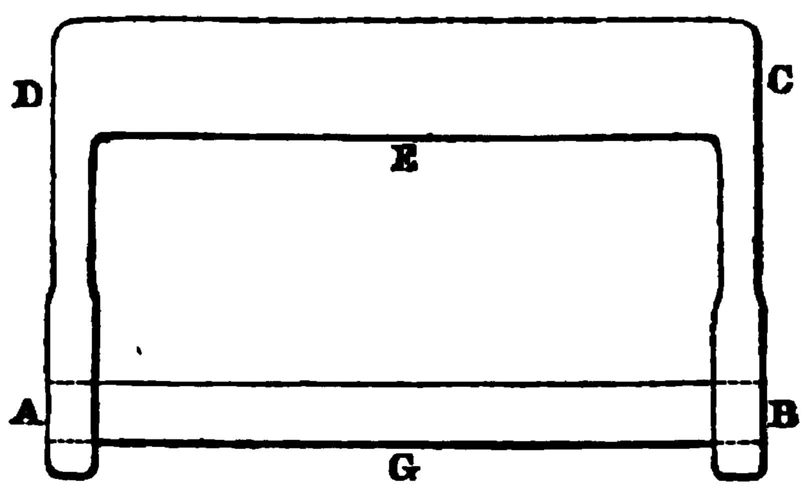
$$d' = \frac{3\,l\,w}{4\,d\,a\,t} = \text{depth of tension, and}$$

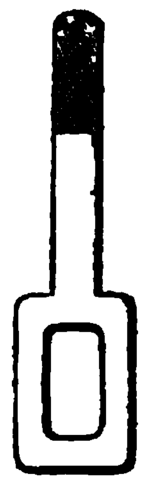
$$d'' = d - d' \text{ the depth of compression;}$$

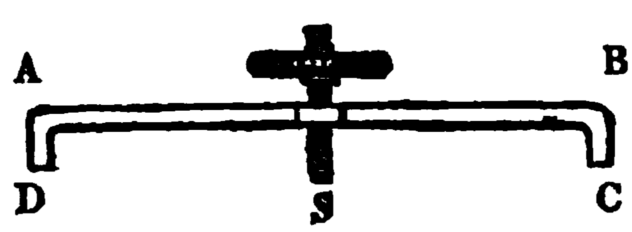
$\frac{d'}{d - d'}$ = the ratio in which the neutral axis divides the sectional area in rectangular bars.

163. In order to submit these formulæ to practical results, a strong iron frame was forged, of the form shown in the figure, p. 307: D C is 36 inches long, 6 inches broad, by 2 deep; the two arms 2 inches square, and the ends of proportional dimensions to

those represented. The other view of the arms is shown in the side figure, with an opening 6 inches by 3, in which the bars for experiment were placed, as represented by A G B; the space between is 33 inches. The shackles were applied at E and G, and connected by strong iron cables to the press; the strain was then brought on, and the results recorded.







In order to measure with every requisite accuracy the deflections which the bar sustained, as different weights were applied, an instrument of the form shown in the annexed figure was neatly and accurately made in iron, having two feet, A D, B C; the centre was tapped to receive the brass screw, H S, of twenty threads to the inch, and the head was divided into five equal parts, and by again subdividing these divisions into ten, a deflection of $\frac{1}{1000}$ of an inch might be measured with great ease.

The method of applying it was to rest its feet on the bar, and then to retain it in its place by cramps and screws. The micrometer screw was then run down till it was in contact with the bar, and the divisions read and registered, either before any strain was on, or when the first slightest strain could be estimated, as stated in the following Table.[8]

The first six experiments were made on different parts of the bars, Nos. 5, 6, and 7, without cutting them, by introducing them into the iron frame above described (having 33 inches clear bearing), and straining them till the successive deflections showed a tendency to increase in amount, which was taken as a sign of the elasticity being injured; and the amount of this strain having been previously ascertained by the former experiments, they furnish the best possible data to apply to the formula for determining the position of the neutral axis.

[8] As the numbers in the second column of the following Table have been misunderstood by a reviewer of my Report, it may be well to observe, that the reader must not understand them to be actual deflections, as it was quite accidental what the index read at the commencement. The actual deflections are given in the adjacent column.

Experiments made to ascertain the Deflections due to different Transverse Strains, and the Weight which first produces a Strain equal to the Elastic Power, and thence the Position of the Neutral Axis.

TABLE III.

Part 1. Bar No. 5. Bearing 33 inches. 2 inches square.			Part 2. Bar No. 5. Bearing 33 inches. 2 inches square.		
Weight in tons.	Readings by scale.*	Deflections for each half ton.	Weight in tons.	Readings by scale.	Deflections for each half ton.
No weight.	1·96		No weight.	1·95	
·875	1·92	·023	·750	1·92	·020
1·00	1·90		1·00	1·91	·020
1·50	1·90	·016	1·50	1·89	·020
2·00	1·88	·020	2·00	1·86	·030
2·50	1·86	·020	2·50	1·84	·020
Weight removed.	returned to 1·96		Weight removed.	returned to 1·95	
3·00	1·80		3·00	1·67	Elasticity injured.
Weight removed.	1·88	Elasticity injured.	Weight removed.	1·81	

Part 1. Bar No. 6.			Part 2. Bar No. 6.		
Weight in tons.	Readings by scale.	Deflections for each half ton.	Weight in tons.	Readings by micro. screw.	Deflections for each half ton.
No weight.			No weight.	·025	
·50	1·56 ?		·50	·043	·018
1·0	1·50		1·0	·068	·025
1·5	1·48	·020	1·5	·091	·023
2·0	1·45	·030	2·0	·128	·037 inj^{d}.
2·5	1·24	·210 Elas. inj^{d}.	2·25	·178	·100
3·0			2·50	·313	·185

* In the first of these experiments the deflections were measured by a scale in front of the bar, the micrometer screw not being ready.

TABLE III.—(CONTINUED.)

Part 1. Bar No. 7.

Weight in tons.	Readings by micro. screw.	Deflections for each half ton.
No weight.	·031	
·50	·053	·022
1·0	·077	·024
1·5	·096	·019
2·0	·126	·030
2·5	·147	·021
3·0	·211	·064 inj[d].

Part 2. Bar No. 7.

Weight in tons.	Readings by micro. screw.	Deflections for each half ton.
No weight.	·025 ?	
·50	·056	·031
1·0	·077	·021
1·5	·098	·021
2·0	·109	·011
2·5	·137	·028 inj[d].
3·0	·180	

Part 3. Bar No. 7.

Weight in tons.	Readings by micro. screw.	Deflections for each half ton.
No weight.	·075	
·50	·130	
1·0	·153	·023
1·5		·023
2·0	·199	·023
2·5	·220	·021
3·0	·290	·070 inj[d].

Part 2. Bar No. 7. Reversed.

Weight in tons.	Readings by micro. screw.	Deflections for each half ton.
No weight.	·025	
·50	·054	·029
1·0	·092	·038
1·5	·153	·061
2·0	·235	·082

Elasticity clearly injured by the former experiment.

It appears from these experiments, that both parts of the bar No. 5 (whose direct elasticity was 9·5 tons) had their restoring power just preserved with a transverse strain of two and a half tons on a bearing length of 33 inches. Hence in the formula $d' = \frac{3\,l\,w}{4\,d\,a\,t}$ we have $l = 33$, $w = 2\frac{1}{2}$, $d = 2$, $a = 2$, $t = 9{\cdot}5$, and $d' = 1{\cdot}62$ inch, depth of tension.

Consequently $d'' = {\cdot}38$ inch, depth of compression, and the ratio of the area of compression to tension	1 : 4·3
In the first part of bar No. 6, w is not quite 2 tons, and $t = 8{\cdot}5$ tons; and hence the ratio	1 : 2·7
In the second part of the same bar, ditto	1 : 2·7
In the first, second, and third parts of bar No. 7, $w = 2\frac{1}{2}$ tons, and $t = 10$ tons .	1 : 3·4

As far as these experiments are authority, therefore, the neutral axis divides the sectional area of a rectangular bar in about the ratio of 1 to $3\frac{1}{2}$.

In the following experiments the iron was all supplied by Messrs. Gordon, and was of the same quality as the bar No. 7,—its elasticity may therefore be taken as 10 tons, but it was not determined by testing, as in the preceding experiments.

TABLE IV.

Bar No. 8.

Distance of bearing.	Breadth.	Depth.	Weights.	Deflections.	Deflections each half ton.	Remarks.
inches.	inch.	inches.	tons.			
33	1·9	2	·125	·034		
			·250	·046		
			·500	·060		
			1·00	missed.	·019	
			1·50	·098	·019	Mean ·024
			2·00	·120	·022	
			2·25	·134	·028	w = 2·25. Neutral axis 1 : 3·4
			2·50	·151	·034	
			2·75	·173	·044	Elasticity injured with 2·50 T.

Bar No. 9.

Distance of bearing.	Breadth.	Depth.	Weights.	Deflections.	Deflections each half ton.	Remarks.
33	1·9	2	·250	·047		
			·500	·055	·016	
			1·00	·077	·022	
			1·50	·097	·020	Mean ·021
			2·00	·123	·026	
			2·25	·132	·018	
			2·50	·145	·026	w = 2·25. Neutral axis 1 : 3·4
			2·75	·164	·038	Elasticity injured with 2·50
			3·00	·210	·092	Ditto destroyed with 3·00

Bar No. 10.

Distance of bearing.	Breadth.	Depth.	Weights.	Deflections.	Deflections each half ton.	Remarks.
33	1·9	2	·500	·056		
			1·00	·076	·020	
			1·50	·095	·019	
			2·00	·124	·029	Mean ·024
			2·50	·151	·027	w = 2·5. Neutral axis 1 : 4·2
			3·00			

Deductions from the last three Experiments, confirmed by direct Observation of the place of the Neutral Axis.

164. These experiments, like the former, imply, according to the formula, that the neutral axis lies at about one-fourth or one-fifth of the depth of the bar from its upper surface; but a method was adopted in these to discover, if possible, its position mechanically. With this view, a key-way, or groove, was cut in the side of the bar, 1 inch broad and 1-10th of an inch deep,—thus reducing the breadth to 1·9 inch. To this key-way, or groove, was fitted a steel key, which might be moved easily; and when the strain was on, the key was introduced, which it was expected would be stopped at the point where the compression commenced, and this was accordingly found to be the case in two out of the three bars, but not in the third, the fitting not being sufficiently accurate. The other two, however, showed obviously a contraction of the groove, at about half an inch from the top, agreeing with the preceding computations. To make the results more certain, three other bars, exactly like the former, had deeper grooves cut, and the key more exactly fitted, and with these the results were as definite as could be desired. The key, as above stated, moved smoothly and easily before the experiment; but when two tons' strain was on, and the

key applied, it was stopped, and stuck at a definite point. The strain being then relieved, the key fell out by its own weight: the strain was again put on, the key sticking as before: the strain being relieved, the key again fell, and so on, as often as repeated. Precisely the same happened with all the three bars. One of them was then reversed, so that the part which had been compressed was now extended, and exactly the same result followed, showing, most satisfactorily, that our former computed situation of the neutral axis was very approximative; the measurements obtained in these experiments being tension 1·6, compression ·4, giving exactly the ratio of 1 to 4 in rectangular bars. These results seem the most positive of any hitherto obtained: still there can be little doubt this ratio varies in iron of different qualities; but looking to the preceding experiments, it is probably always between 1 to 3, and 1 to 5 in rectangular bars.

On the Stiffness of Rectangular Iron Bars, and their Deflections under different Weights.

165. Although it is necessary to know the actual resisting power of bars in their ultimate state of strain, in order to determine the relative strengths of differently shaped bars, yet the question of most practical importance is, the stiffness they exhibit when loaded with smaller weights; for we ought

never to strain a bar so nearly to its full power of bearing as to make the ultimate strength the immediate subject of inquiry.

The experiments recorded in the last section are applicable to this purpose; but as these were all made on bars of the same depth, it was thought more satisfactory to make a few other experiments on bars of different breadths and depths. These are given in the following page. They were performed precisely like the last, and therefore require no particular description.

EXPERIMENTS

On the Deflection of Malleable Iron Bars, under different Strains.

Bar No. 11.

Distance of bearing.	Breadth.	Depth.	Weight.	Deflections.	Deflections for each half ton.	Remarks.
inches.	inch.	inches.	tons.			
33	1·5	3	·125	·043		
			·500	·059		
			1·00	·074	·015	
			1·50	·083	·009	
			2·00	·095	·012	
			2·50	·101	·006	Mean ·0103
			3·00	·109	·008	
			3·50	·120	·011	
			4·00	·131	·011	$w = 4\frac{1}{2}$. Neutral axis 1 : 4·9
			4·50	·148	·017	Elasticity preserved at 4½ tons.

Bar No. 12.

Distance of bearing.	Breadth.	Depth.	Weight.	Deflections.	Deflections for each half ton.	Remarks.
33	1·5	3	0	0		
			·50	·017		
			1·00	·037 ?		
			1·50	·052	·015	
			2·00	·061	·009	
			2·50	·064	·003	Mean ·0108
			3·00	·078	·014	
			3·50	·089	·011	
			4·00	·102	·013	$w = 4\frac{1}{2}$. Neutral axis 1 : 4·9
			4·50	·124	·022	Elasticity injured.

Bar No. 13.

Distance of bearing.	Breadth.	Depth.	Weight.	Deflections.	Deflections for each half ton.	Remarks.
33	1·5	2·5	0	·006		
			·50	·030	·024	
			1·00	·050	·020	
			1·50	·060	·010	Mean ·0173
			2·00	·074	·014	
			2·50	·093	·019	
			3·00	·110	·017	$w = 3$. Neutral axis 1 : 4·9
			3·50	·149		Elasticity preserved, 3 tons.
			7·5	Bent 8 inches.		

To obtain the law of deflection from these results, we may have recourse to two well-known and well-established formulæ (Arts. 28 and 66), viz.

$$\frac{l w}{4 a d^2} = \text{S}, \text{ and } \frac{l^3 w}{16 a d^3 \delta} = \text{E},$$

which are both constant quantities for the same material, w being the greatest weight the bar will bear without injuring the elasticity; consequently, when l is also the same in both, $d\,\delta$ will be also constant, a being the breadth, d the depth, and δ the deflection. That is, all rectangular bars having the same bearing length, and loaded in their centre to the full extent of their elastic power, will be so deflected, that their deflection (δ) being multiplied by their depth (d), the product will be a constant quantity, whatever may be their breadths or other dimensions, provided their lengths are the same.

Let us see how nearly our several results agree with this condition.

In the several bars, Nos. 8, 9, 10, 11, 12, 13, multiplying the mean deflection for each half ton by the number of half tons which excited its whole elasticity, and this again by the depth of the bar, we find

No. 8, ultimate deflection	·108 ×	depth	2 =	·2160
No. 9	·094 ×	„	2 =	·1880
No. 10	·120 ×	„	2 =	·2400
No. 11	·0876 ×	„	3 =	·2628
No. 12	·0918 ×	„	3 =	·2754
No. 13	·1038 ×	„	$2\frac{1}{2}$=	·2595
				6)1·4417
		Mean . . .		·2403

There is rather a large discrepance in bar No. 9; the others are as approximative to the mean as can be expected in such cases.

If we make the same trial on the three parts of bar No. 7, we have

1st part	·116	×	2	=	·2320
2nd part	·105	×	2	=	·2100
3rd part	·115	×	2	=	·2300
					3)·6720
	Mean				·2240
	Former mean . .				·2403
					2)·4643
	General mean . .				·2321

We may therefore say, that any malleable iron bar, of 33 inches bearing, being strained to its full elasticity, will be so deflected, that its depth, multiplied by the deflection due to 30 inches, will produce the decimal ·23; consequently $\frac{\cdot 23}{d}$ = the deflection, d being the whole depth in inches.

In this form, however, it applies only to rectangular bars. To make it general, we must estimate it from the neutral axis, which, in rectangular bars, being $\frac{1}{5}$th of the depth below the upper surface, the above constant, when thus referred, becomes $\cdot 2321 \times \frac{4}{5} = \cdot 1857$. But, on the other hand, our instrument for measuring the deflection was but 30 inches long; it has therefore to be increased again in the ratio $30^2 : 33^2$, or as $10^2 : 11^2$ on this account; so that, ultimately, the formula is $d'' \delta = \cdot 22$, d'' denoting the depth of the bar below the neutral axis; and in this form it is general for parallel rails of any section whatever.

A curious circumstance was observed in these experiments, which, although it has no immediate bearing on the subject in question, it may be well to notice, and which is, I apprehend, characteristic of good malleable iron, viz. that the resistance to compression, although so much greater than the resistance to extension, is the first of the two which loses its restoring power; for if we so far increased the strain as to overcome the elastic power, the point of compression descended to nearly the middle of the depth, proving that the tensile force, although so much less, is the most tenacious.

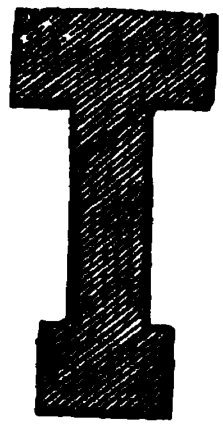
Comparative Strength of Parallel Rails of equal Sections but of different Figures.

166. Various figures have been proposed for the transverse section of railway bars, some with a view to a more convenient and efficient mode of fixing the rail to the chair, and others with a view to greater strength: at present the question is the comparative strength of different sections; the best form for fixing will be considered afterwards.

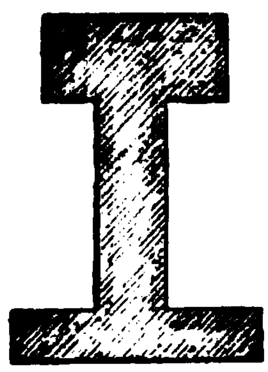
It would be useless in this inquiry to go generally into the section of greatest strength, as we must have regard to certain conditions and limitations; we must, for example, necessarily have a head to the rail of a certain breadth and not less than a certain depth. The depth of the whole rail must also be confined within certain limits. We shall not, therefore, treat this question generally, but only as applicable to practical sections, which may be stated to be comprised under the four following forms:

1. The plain T shaped rail, fig. 1.

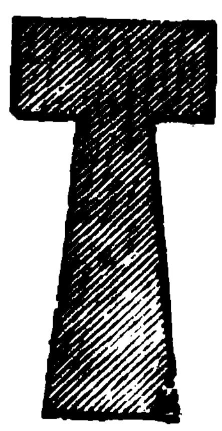
2. The I or double T formed rail, with a lower table, as fig. 2.



3. The inverted T rail, having a broad lower table, as fig. 3.



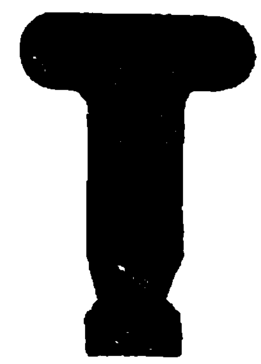
4. The trapezoidal rail, as fig. 4.



Each of these will admit of various changes of proportions, without altering the general character of the section; indeed, the second figure may be considered as comprehending also fig. 1 and fig. 3. We have only to suppose the lower flanch as having no projection to give fig. 1, and to have it more extended to give fig. 3.

The upper and the lower tables are here repre-

sented as rectangular, with sharp edges. In practice these are rounded off, the metal thus displaced furnishing a sort of bracket between the table and stem, or rib, as shown in the annexed figure; but to treat of them in this form would introduce great intricacy into the calculation, without much affecting the results. It will therefore be sufficient to consider them as rectilinear, but preserving the same area.



167. I would here observe, also, that some projectors have made the upper and lower tables of equal figure, upon the distant contingency, that when the upper table has been worn down, the rail may be turned, and the lower table made the upper. But this is certainly providing without foresight; for the bottom table is the most efficient for strength, and it would be a very dangerous experiment, after one side of a bar has been submitted for many years to a high compressing force, and its substance (by the hypothesis) greatly worn, to turn the rail, and expose this worn part to a still greater strain, but tensile instead of compressive, which could not fail very soon to destroy it. Instead of this, therefore, I should certainly recommend to work whatever metal is introduced into the lower table or web, into that form which is most efficient for present purposes, without regard to the contingency alluded to above.

That the rail is deteriorated by exposure and wear is undoubtedly true, although, perhaps, the amount is not yet well ascertained. Amongst the papers submitted to Messrs. Rastrick and Wood, with whom I was associated, we found it estimated at the rate of $\frac{1}{8}$th of a pound per yard per annum; but I have since seen it stated, in a letter from Mr. Dixon to Mr. Bidder, at $\frac{1}{10}$th of a pound per yard per annum. This was determined by taking up three rails, having them well cleaned and weighed, and then putting them in their places, and afterwards washing and re-weighing them at the end of a twelvemonth, when two of them were found to have lost $\frac{1}{2}$ lb. in weight for the 5 yards length, and the third $\frac{3}{4}$ lb., which last was taken up from a particular situation where it was more exposed to friction; but even this does not prove that the whole loss of weight is in the upper face of the rail; and if it did, it would be, as I have before observed, a stronger reason for not turning the rail: and, on the other hand, should the waste not be on the upper surface, the provision alluded to is unnecessary. Mr. Rastrick informs me, that even the small fins left at the meeting of the rolls are still quite distinctly seen on the face of the upper table. And Mr. Stephenson states, that the marks of the tools left in turning the flanches of the wheels are seldom obliterated; which proves, at all events, that there is no side wear.

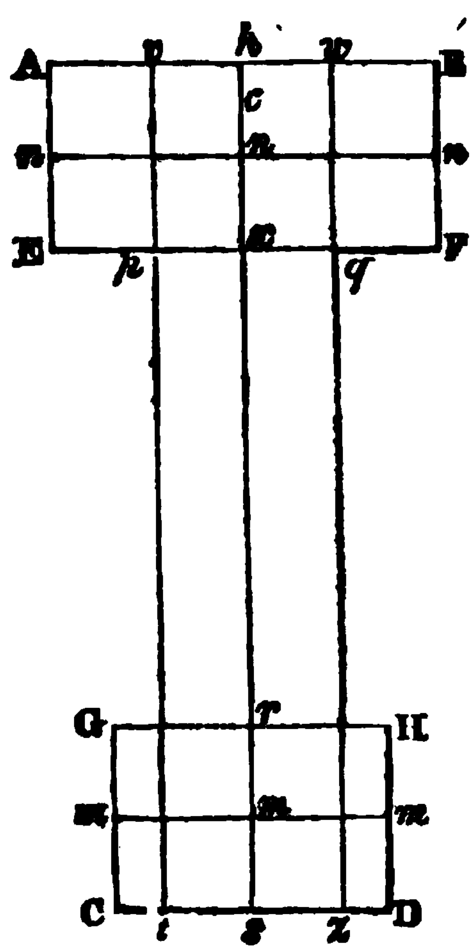
Mr. George Bidder, who attributes all the waste

to the wear on the upper surface, estimates the annual reduction at $\frac{1}{90}$th part of an inch; in which case the rails would not last more than thirty years before they would require to be replaced. And it then becomes a question, whether, in point of economy, it would not be better to lay an additional third of an inch upon the upper table, which would, by this reckoning, make the rail last sixty years. This increase of $\frac{1}{3}$rd of an inch would call for an additional expense, to the amount of about $7\frac{1}{2}$ per cent. on the present cost; and this $7\frac{1}{2}$ per cent., at compound interest, would amount to about 30 per cent. in thirty years. If, therefore, a charge of 30 per cent. at the end of thirty years, would meet the amount of re-manufacture, and supply the waste, the two accounts would be about balanced. In this case, I must consider the latter as preferable. 1st. Because the other plan would increase the weight of the bar, and the difficulty of the manufacture, and probably diminish its soundness. 2nd. Because thirty years' experience may introduce improvements, of which, at the end of that period, it would be desirable to take advantage. And, lastly, because I do not (judging from the opinions of different practical men) think that it has yet been clearly determined what part of the waste is due to wear on the upper face.

To return again to the subject of the best-formed section, I beg to repeat, that whatever figure the above, or other considerations, may lead practical

men to adopt in the upper or lower table and rib, it will be fully sufficient for the purposes of calculation to consider them as rectilinear, which will greatly facilitate the investigation, without sensibly affecting the results.

168. Let ABCD, in the annexed figure, represent any rectangular rail with a bottom table; *n n* its neutral axis; *c* the centre of compression, *c n* being $\frac{2}{3}$rds of *h n*. Now, the tension of each fibre being as its distance from the neutral axis, and that of the lower fibre being given equal to *t*, the tension at any variable distance *x*, will be $\frac{tx}{d'}$ (d' being taken to denote the whole depth *n s*), and therefore the sum of all the tensions will be



$$\frac{t}{d'}\int x \,.\, dx, \qquad (1)$$

which, therefore, becomes known, *x* being taken within its proper limits, according to the figure of the section.

But as the moment of the resistance of each fibre is also as its depth below the line *n n*, the moment of all the resistances will be

$$\frac{t}{d'}\int x^2 \,.\, dx, \qquad (2)$$

x being taken here also within its proper limits. And then, to find the centre of tension, or that point into which, if all the tensions were collected, the moment of the whole resistance would be the same as in the actual case, this would be given by the formula

$$\frac{\int x^2 \,.\, dx}{\int x \,.\, dx}, \qquad (3)$$

which is precisely the expression for the centre of oscillation of a disc of the same figure.

We have hence the following general rule for finding the moment of the resistance when any given bar or rail is strained by a weight at its middle point within the limits of its elastic power.

Calling the integral of formula (1) = A,
Ditto ditto ditto (2) = B,
Ditto ditto ditto (3) = D,
And the distance cn = C,

then, taking the moments of all the resistances B about the centre of compression, we have

$$D : : D + C : : B : \frac{B\,(D + C)}{D},$$

which measures the whole effect.

169. For those who understand the integral calculus, this solution is sufficient; but as this work will probably be consulted principally by practical men, it may be convenient to show the origin of these formulæ, particularly the third, which is not investigated in the preceding pages, except that it has been shown generally, that, if d' denote the

depth of the lower fibre below $n\,n$, and its tension be made $= t$, and any variable distance $= x$,

$\frac{t}{d'}\int x\,d\,x =$ sum of all the tensions to a unit of breadth:

$\frac{t}{d'}\int x^2\,d\,x =$ moment of all the resistances referred to the axis n; and

$$\frac{\frac{t}{d'}\int x^2\,d\,x}{\frac{t}{d'}\int x\,d\,x}, \text{ or } \frac{\int x^2\,d\,x}{\int x\,d\,x} = \delta', \text{ distance of centre of tension.}$$

From which it follows, that $\frac{t\,\delta'}{d'}\int x\,.\,d\,x =$ moment of all the resistances for a unit of breadth, x being taken in its ultimate state.

Now, in the rib, when

$$x = d',\ \delta' = \tfrac{2}{3}\,d', \text{ and } \int x\,d\,x = \tfrac{1}{2}\,d'^2,$$

whence the above becomes $\frac{1}{3}\,d'^2\,t$; but to refer this to the centre of compression c, we have (calling the whole depth d)

$$\tfrac{2}{3}\,d' : \tfrac{2}{3}\,d :: \tfrac{1}{3}\,d'^2\,t : \tfrac{1}{3}\,d\,d'\,t;$$

and introducing the breadth $p\,q$, it becomes

$$\tfrac{1}{3}\,h\,s\,.\,n\,s\,.\,p\,q\,.\,t = \text{moment of resistance of the rib.}$$

In the same way, calling the tension at $x = t'$, and the breadth $(n\,n - p\,q)$, we have for the moment of resistance of the head $\frac{1}{3}\,h\,x\,.\,n\,x\,.\,(n\,n - p\,q)\,t'$; but the tension at $x = \frac{n\,x}{n\,s}\,t$; therefore, substituting this for t', we have

$$\tfrac{1}{3}\,h\,x\,.\,\overline{n\,x}|^2\,\frac{(n\,n - p\,q)}{n\,s}\,t, \text{ or}$$

$$\tfrac{1}{3}\,h\,x\,.\,n\,x\,.\,(n\,n - p\,q)\,\frac{n\,x}{n\,s}\,t = \text{moment of resistance of the head.}$$

For the lower flanch,

$$\frac{\int x^2 dx}{\int x dx} = \delta'.$$

Calling $n r = d''$, and x any variable distance below r, it becomes

$$\frac{\int (d'' + x)^2 dx}{\int (d'' + x) dx} = \delta';$$

which, when $x = r s$, gives

$$\delta' = n m + \frac{r s^2}{12 m n},$$

$$\text{and } \frac{t}{d'} \int (d'' + x)\, dx = \frac{t \delta'}{d'}\, n m \,.\, r s,$$

whence the moment of resistance referred to $n n$ is, for the breadth $(m m - p q)$,

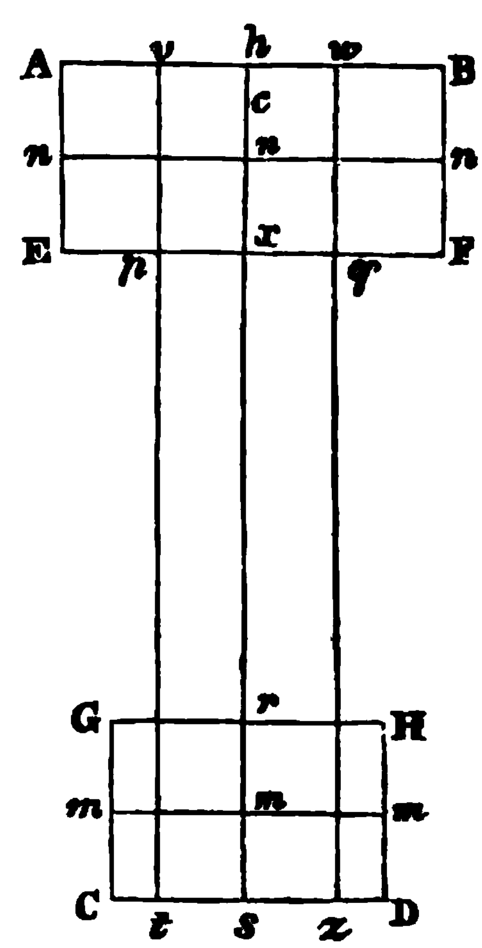
$$n m \,.\, r s\, (m m - p q) \frac{t \delta'}{d'};$$

and calling $\delta' + n c = \delta''$, it is, when referred to c,

$$n m \,.\, r s\, (m m - p q) \frac{\delta'' t}{d'} = \text{moment of resistance of lower flanch,}$$

which is the formula in question.

Let now ABCD, in the annexed figure, represent a section of which all the dimensions are given, as also the position of $n n$, the neutral axis, the point c, which is the centre of compression, $c n$ being $\frac{2}{3}$rds of $n h$, and the point m, which is in the centre of $r s$. The breadths $n n$ and $m m$ are also known. Then the moment of resistance of the whole section referred to the common centre of compression c, may be considered to be made up of the moments of the three resistances.



1st. Of the middle rib, continued through the head and foot tables, $v\,t\,z\,w$.

2nd. Of the head A E F B, minus the breadth of the centre rib.

3rd. Of the lower web, G C D H, also minus the continuation of the centre rib.

Now, t being taken to represent the tension of iron per square inch, just within its limits of elasticity, we shall have

1. Moment of resistance of $v\,t\,z\,w = \frac{1}{3}\,h\,s\,.\,n\,s\,.\,p\,q\,.\,t$.

2. Moment of resistance of A E F B $= \frac{1}{3}\,h\,x\,.\,n\,x\,.\,(n\,n - p\,q)\,\frac{n\,x}{n\,s}\,t$.

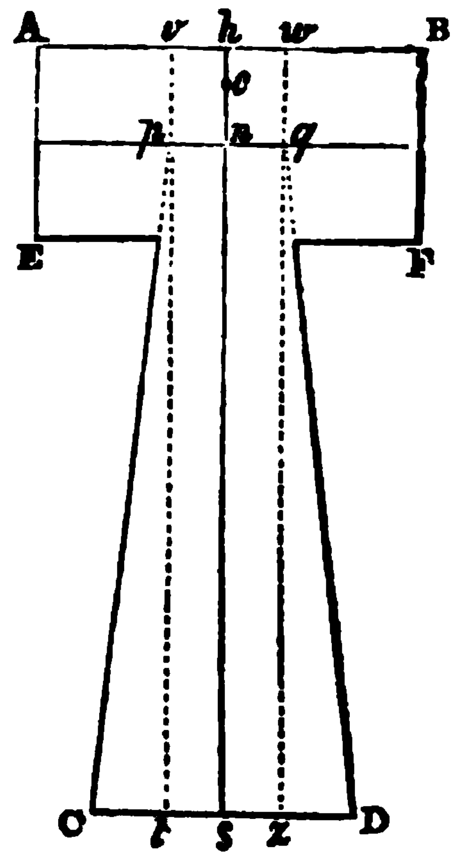
3. Moment of resistance of G C D H $= n\,m\,.\,r\,s\,.\,(m\,m - p\,q)\,\frac{\delta''}{d''}\,t$.

These three moments being computed, let their sum be called s, and the clear bearing l; then $\frac{4\,s}{l} = w$, the load the bar ought to sustain at its middle point, for an indefinite time, without injury to its elasticity.

Note.—In the case of the plain T rail, the formula No. 3 vanishes, the flanch having no thickness.

Trapezoidal Rail.

170. Produce the sloping sides till they intersect the neutral axis in p, q. Then the rule for the head, A E F B, and middle rib, $v\,t\,z\,w$, will be the same as given above; and for the two sides, $p\,\mathrm{C}\,t$, $q\,\mathrm{D}\,z$, the formula is



$$\tfrac{1}{3}\left(\tfrac{3}{4}ns + cn\right) \times (\mathrm{CD} - pq)\,nst.$$ [9]

The sum of this, with that of the head and middle rib, multiplied by 4, and divided by l, as before, will be the weight required.

Mechanical Solution.

171. Another general and very curious mechanical method of finding the resistance of a railway bar, is suggested by the remark in p. 326, viz., that the centre of tension is the same as the centre of oscillation of a disc of the form of the section, cut off at its neutral axis, which in words may be given as follows:

[9] This includes the small dotted part of the triangular sides in the head and in the sides, but the amount is so very inconsiderably in error, as to be nearly or wholly insensible in the result.

Find the centre of oscillation, and the centre of gravity of the area below the axis, by the usual mechanical methods, and call the distance of the former below the neutral axis o, that of the latter g, the area a, the depth d', and the distance $cn = c$, the tension t, and bearing l, as usual, then the weight the bar will support will be

$$w = 4\frac{(o + c)\, a g t}{l d'}.$$

The following numerical rules, however, will be generally more convenient, particularly when some of the dimensions become fixed, as necessarily happens in such cases as we are considering. For instance, whatever figure may be given to the transverse section, the head may generally be supposed to occupy $\frac{2}{5}$ths of it, and therefore, in the larger rails, to have about 2 inches section, and to be 1 inch deep; the lower web, when there is one, to be the same depth as the head, and the neutral line to bisect the head, or upper table.[10] With these, as fixed dimensions, the preceding formulæ, Art. 169, are reducible to words at length. They apply, however, only to the larger rails; for other cases, it will be best to have recourse to the general formulæ.

[10] The correct rule is, that the area of compression into the distance of its centre of gravity from the neutral axis, is to the area of tension into the distance of its centre of gravity from the same line, as $1^2 : 4^2$, or as 1 to 16.

172. *Moment of Resistance of the Head or Upper Table.*

1. Subtract the thickness of the middle rib from 2 inches, and multiply the remainder by 10.

2. Subtract ½ an inch from the whole depth, and multiply the remainder by 12.

Then the former product divided by the latter will be the moment of resistance due to the head, not including the continuation of the middle rib.

Moment of Resistance of the Centre Rib.

Multiply the whole depth of the rail by the whole depth, minus ½ an inch, and that product by 10 times the thickness of the rib; and the last product, divided by 3, will be the moment of resistance of the middle rib continued through the whole depth, *i. e.* through the upper and lower tables.

Moment of Resistance of the Lower Web.[11]

1. Multiply the whole depth of the rail, minus 1 inch, by the breadth of the bottom web, minus the thickness of the rib, and that product by 10.

[11] This rule only applies when the depth of the head and lower web are each 1 inch. In other cases, recourse must be had to the general formulæ.

2. From the whole depth of the rail subtract 1 inch, and to 12 times the square of the remainder add 6 times the remainder, and call this the first number. From this subtract twice the remainder, and add 1, and call this the second number. Then say, as the first number is to the second, so is the product obtained in the former part of the rule to the moment of resistance of the lower web, not including the continuation of the middle rib.

Lastly, the sum of these three moments, multiplied by 4, and divided by the clear bearing length in inches, will be the weight in tons the rail will sustain without injury. A few examples worked at length are given below, to illustrate the rules.

173. *Examples.*

(1.) Let the depth of the rail be 5 inches, with a plain rib whose thickness is ·9 of an inch. Here

$$\text{Moment of resistance of head} \left\{ \begin{array}{l} (2-\cdot 9)\times 10 = 11 \\ (5-\tfrac{1}{2})\times 12 = 54 \end{array} \right\} \frac{11}{54} = 0\cdot 20$$

$$\text{Ditto of rib} \;.\;.\;.\;.\;.\;.\; \frac{4\tfrac{1}{2}\times 5\times \cdot 9\times 10}{3} \;.\;.\; = 67\cdot 50$$

$$\overline{67\cdot 7}$$

$$\text{And } \frac{4\times 67\cdot 7}{33} = 8\cdot 21 \text{ tons, the greatest weight.}$$

$$\text{Deflection with this weight } \frac{\cdot 22}{4\cdot 5} = \cdot 05, \text{ nearly.}$$

(2.) Parallel rail of the same depth, the thickness of the centre rib being $= \cdot 78$. Here

Moment of resistance of head . . . . $\left\{\begin{array}{l}(2-{\cdot}78)\times 10 = 12{\cdot}2 \\ (5-\frac{1}{2})\times 12 = 54\end{array}\right\} \frac{12{\cdot}2}{54} = 0{\cdot}225$

Ditto of rib . . . $\frac{4\frac{1}{2}\times 5\times{\cdot}78\times 10}{3}$ . . . . $= 58{\cdot}5$

58·725

$$\text{And } \frac{4\times 58{\cdot}725}{33} = 7{\cdot}11 \text{ tons, the greatest weight.}$$

$$\text{Deflection with this weight } \frac{{\cdot}22}{4{\cdot}5} = {\cdot}049.$$

(3.) Parallel rail with bottom web, the depth being still 5 inches, the thickness of rib ·6 of an inch, thickness or breadth of section of lower web 1·32, the weight being 50 ℔s.

Moment of resistance of head . $\left\{\begin{array}{l}(2-{\cdot}6)\times 10 = 14 \\ (5-\frac{1}{2})\times 12 = 54\end{array}\right\} \frac{14}{54} = 0{\cdot}26$

Ditto of rib . . $\frac{4\frac{1}{2}\times 5\times{\cdot}6\times 10}{3}$ . . . $= 45{\cdot}00$

Lower web $\left\{\begin{array}{l}(5-1)\times{\cdot}72\times 10 = 28{\cdot}8 \\ 12\,(5-1)^2+24 = 216 = \text{1st No.} \\ 216-7 = 209 = \text{2d No.}\end{array}\right.$

As 216 : 209 :: 28·8 : 27·94 . . . . 27·94

73·20

$$\text{And } \frac{73{\cdot}20\times 4}{33} = 8\tfrac{3}{4} \text{ tons, the greatest weight.}$$

$$\text{Deflection with this weight } \frac{{\cdot}22}{4{\cdot}5} = {\cdot}048.$$

(4.) As another example, let us take a parallel rail of 50 ℔s. per yard, depth $4\frac{1}{2}$ inches, thickness of rib $\frac{7}{10}$ths of an inch, and of the bottom web 1·39.

$$\text{Moment of resistance of head} \left\{\begin{array}{l}(2\ -\cdot 7)\times 10=13\\ (4\tfrac{1}{2}-\tfrac{1}{2})\times 12=48\end{array}\right\}\frac{13}{48} = 0\cdot 27$$

$$\text{Ditto of rib} \quad \frac{4\times 4\tfrac{1}{2}\times\cdot 7\times 10}{3} \quad = 42\cdot 00$$

$$\text{Do. of lower web} \left\{\begin{array}{l}3\tfrac{1}{2}\times(1\cdot 39-\cdot 7)\times 10=24\cdot 15\\ 12\ (3\tfrac{1}{2})^2+21=168=\text{1st No.}\\ \quad 168-6=162=\text{2d No.}\end{array}\right.$$

$$\text{As } 168 : 162 :: 24\cdot 15 : 23\cdot 28 = 23\cdot 28$$

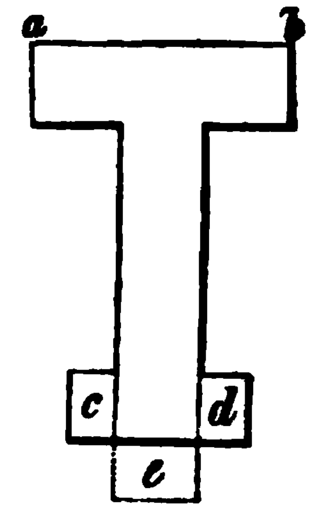
$$65\cdot 55$$

$$\frac{4\times 65\cdot 55}{33} = 8 \text{ tons, nearly, the greatest weight.}$$

$$\text{Deflections with this weight } \frac{\cdot 22}{4} = \cdot 055.$$

Rail of Maximum Strength.

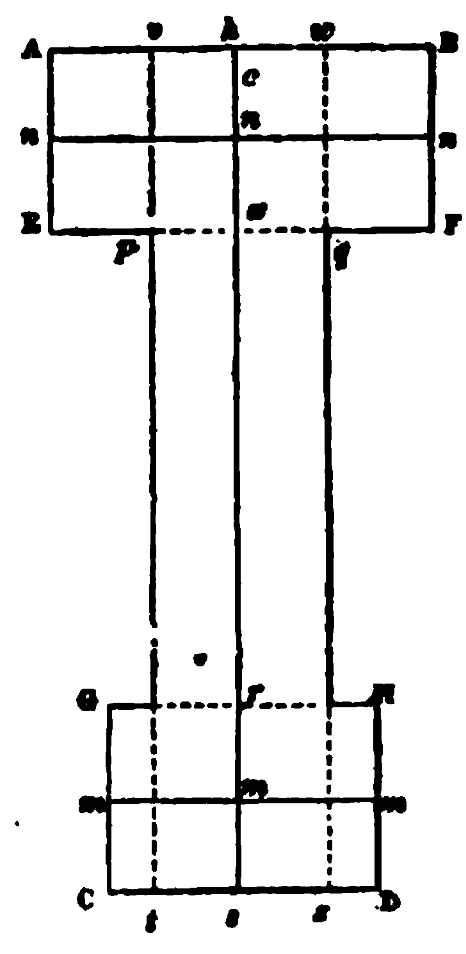
174. The preceding rules and examples will enable any one to estimate the strength of any proposed rail; but the question of the strongest rail with a given quantity of metal, remains still to be decided. In this, of course, we must limit ourselves to practical forms; but even under this limitation, considerable difference of opinion exists. Thus, while one party contend that the strongest form is that which has the lowest and broadest flanch, others maintain, that if the flanch were wholly removed, and the metal placed so as to continue the centre rib to a greater depth, a considerable additional strength would be obtained. The argument advanced in support of the latter doctrine is this:—Suppose *a b c d* to denote a rail with a double flanch, the lower one being marked *c* and *d*, it is maintained, if these parts were removed, and placed in



continuation of the centre rib, as shown at *e*, that these fibres being now placed further from the neutral axis than in their former position, they would become more effective. So far this is true, but then the part of the centre rib between *c* and *d* would become less effective; because, in the first form, this part is amongst the lower fibres, which are all exerting their full tension of 10 tons, while, in the second form, the parts between *c* and *d* are no longer found amongst the lower fibres, and it is the lower ones only in which this full tension can be exerted.

It is clear, therefore, that this is a question which comes immediately under the class of *maxima* and *minima*, the solution of which is as follows; viz.

Given the area of section of a railway bar below the neutral axis, to find the dimensions of the lower flanch, so that the strength shall be a maximum; the breadth of the middle rib, and the depth of the lower flanch, being also given. Referring to the annexed figure; let the whole of the sectional area below



$n n$ . . . . $= a$
the breadth of rib $p q$. $= b$
the depth $n s$ . . . . . $= d$
the depth of lower flanch $r s = e$
the tension of the lower fibre $= t$

1. To find first the expression for the strength of the middle rib: take any variable distance x, then

$$d' : t :: x : \frac{t\,x}{d'} = \text{tension of fibre at } x.$$

Multiply by the distance x, and breadth b, and we have for the sum of the moments of all the resistances

$$\int \frac{t}{d'} b\,x^2\,d\,x = \frac{t\,b}{3\,d'} x^3;$$

this, when $x = d'$, becomes $\frac{1}{3}\,d'^2\,b\,t$.

2. To find an expression for the strength of the lower flanch: let the breadth $= b'$; any distance from $n\,n = x$,

$$\text{then } d' : t :: x : \frac{t\,x}{d'} = \text{tension of fibre at } x.$$

Multiply by the distance x, and breadth b'.

We have for the sum of the moments of all the resistances

$$\int \frac{t}{d'} b'\,x^2\,d\,x.$$

This, taken between the values

$$x = d', \text{ and } x = d' - e,$$

gives: Moment of resistance,

$$\frac{3\,d'^2\,e - 3\,d'\,e^2 + e^3}{3\,d'} b'\,t = \left(d'\,e - e^2 + \frac{e^3}{3\,d'}\right) b'\,t.$$

Therefore, the moment of the resistance of the rib and lower flanch is

$$\tfrac{1}{3}\,d'^2\,b\,t + \left(d'\,e - e^2 + \frac{e^3}{3\,d'}\right) b'\,t.$$

And the question is, to determine what value must be given to d', that this expression may be a maximum.

To effect this it is only necessary to consider d' as variable, to denote it by x, to find the value of the dependent quantity b' in terms of x, to substitute these quantities in the preceding expression, and to make its differential equal to zero.

Now, since the depth of the middle rib is x and breadth b, the area is $b\,x$, and consequently the area of the lower flanch $= a - b\,x$, and its depth being e, its breadth

$$= \frac{a - b\,x}{e}, \text{ that is, } b' = \frac{a - b\,x}{e}.$$

Substituting, therefore, $\frac{a}{e} - \frac{b}{e}x$ for b', and x for d', in the preceding expression, it becomes, rejecting t, which is common,

$$\tfrac{1}{3}\,b\,x^2 + \left(e\,x - e^2 + \frac{e^3}{3\,x}\right)\left(\frac{a}{e} - \frac{b}{e}x\right) = \text{a max. Or,}$$

$$d\left(\tfrac{1}{3}\,b\,x^2\right) + d\left(e\,x - e^2 + \frac{e^3}{3\,x}\right) \cdot \left(\frac{a}{e} - \frac{b}{e}x\right) + d\left(\frac{a}{e} - \frac{b}{e}x\right)$$

$$\left(e\,x - e^2 + \frac{e^3}{3\,x}\right) = 0. \text{ Or, } \tfrac{2}{3}\,b\,x\,d\,x$$

$$+\left(e - \frac{e^3}{3\,x^2}\right) \cdot \left(\frac{a}{e} - \frac{b}{e}x\right) d\,x$$

$$-\frac{b}{e}\left(e\,x - e^2 + \frac{e^3}{3\,x}\right) d\,x. = 0. \text{ Or,}$$

$$\tfrac{2}{3}\,b\,x + \left(e - \frac{e^3}{3\,x^2}\right) \cdot \left(\frac{a}{e} - \frac{b}{e}x\right) - \frac{b}{e}\left(e\,x - e^2 + \frac{e^3}{3\,x}\right) = 0.$$

$$\text{Or, } \tfrac{2}{3}\,b\,x + a - b\,x - \frac{e^2\,a}{3\,x^2} + \frac{e^2\,b}{3\,x} - b\,x + b\,e - \frac{b\,e^2}{3\,x} = 0.$$

Reducing every term to the denominator $3\,x^2$, and rejecting it, this becomes

$$2bx^3 + 3ax^2 - 3bx^3 - e^2a + be^2x - 3bx^3 +$$
$$3ebx^2 - be^2x = 0. \quad \text{Or,}$$
$$-4bx^3 + (3a + 3eb)x^2 - e^2a = 0. \quad \text{Or,}$$
$$x^3 - \frac{3(a + eb)x^2}{4b} = \frac{-e^2a}{4b}$$
$$x^3 - \tfrac{3}{4}\left(\frac{a + eb}{b}\right)x^2 = \frac{-e^2a}{4b}.$$

From which x may be determined for any given values of a, b, and e.

As an example, take a rail in which the middle rib is ·78 inch, or $b = {\cdot}78$, as given in Example 2, p. 333, to find what flanch must be given, and the corresponding depth of rail to produce the maximum strength.

The rail being $4\frac{1}{2}$ inches below the neutral axis, and its breadth $b = {\cdot}78$, its area is $\cdot 78 \times 4\frac{1}{2} = 3{\cdot}51 = a$; and it is required to distribute this area so as to produce a rail of maximum strength, the depth of the proposed flanch being 1 inch. Substituting

$$a = 3{\cdot}51,\ b = {\cdot}78,\ e = 1,$$

the foregoing equation becomes

$$x^3 - 4{\cdot}11\,x^2 = -1{\cdot}12.$$

Whence $x = 4{\cdot}04$, the depth of the rail required.

Now, $4{\cdot}04 \times {\cdot}78 = 3{\cdot}15$ area middle rib,

$$a - bx = 3{\cdot}51 - 3{\cdot}15 = {\cdot}36 = b'.$$

Or, ·36 is the area of the lower flanch, which is also its breadth, its depth being 1.

The strongest rail, therefore, of this weight, whose

breadth is ·78, is that whose depth is 4·04 inches, and the breadth of the lower flanch, including the middle rib, is ·78 + ·36 = 1·14 inch.

It will be observed, that in this solution, for the sake of simplification, the strength or resistance of the head, which is very little, has been neglected. Nor is the resistance transferred to the centre of compression; but it is obvious, that when the strength is a maximum estimated from the neutral axis, it must be so also when referred to the centre of compression, or very nearly so.

Strength of the Rail computed.

The whole depth 4·54 = hs, ns = 4·04, pq = ·78.

$$\text{Head} \left\{ \begin{array}{l} (2 - \cdot78) \times 10 = 12\cdot2 \\ (4\cdot54 - \tfrac{1}{2}) \times 12 = 48\cdot48 \end{array} \right\} \frac{12\cdot2}{48\cdot48} \quad . = \quad \cdot25$$

$$\text{Middle rib} \frac{4\cdot54 \times 4\cdot04 \times \cdot78 \times 10}{3} \quad . \;. \;. \;. \;. = 47\cdot69$$

$$\begin{array}{l} \text{Lower} \\ \text{flanch} \end{array} \left\{ \begin{array}{ll} (4\cdot54 - 1) \times \cdot36 \times 10 & = 12\cdot7 \\ 12\,(4\cdot54 - 1)^2 + 4 \times 3\cdot54 & = 171\cdot6 \\ 171\cdot6 - 2 \times 3\cdot54 + 1 & = 165\cdot6 \end{array} \right\}$$

$$171\cdot6 : 165\cdot6 : 12\cdot7 : \;. \;. \;. \;. \;. \;. \;. = 12\cdot25$$

$$\begin{array}{r} 60\cdot19 \\ 4 \\ \hline 33)240\cdot76 \\ \hline \end{array}$$

nearly 7·3 tons.

If we take the area the same, 3·51, with the flanch 1 inch deep, and the rib ·6 thick, the general equation becomes

$$x^3 - 5{\cdot}137\,x^2 = -1{\cdot}46.$$

This gives $x = 5{\cdot}08$, very nearly.

So that the depth assumed in Example 3, p. 334, is for the thickness ·6 of an inch, the rail of maximum strength.

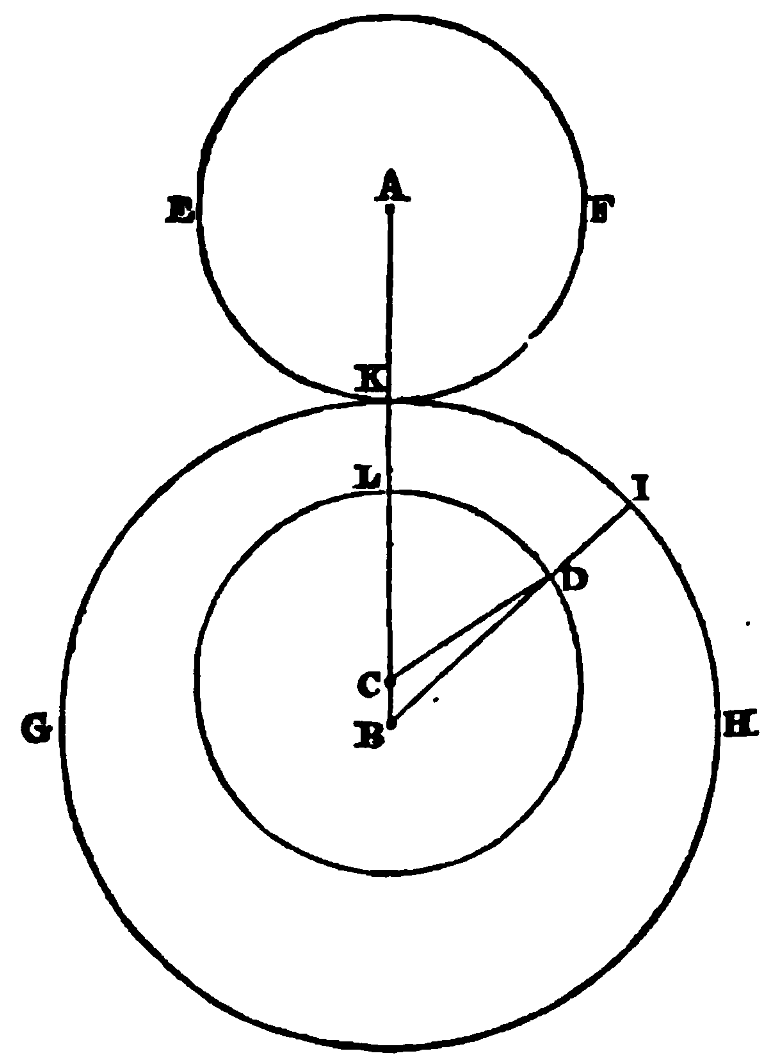
On the Longitudinal Figure of Rails.

175. To produce the lightest kind of rail, having a given strength, was in the infancy of railway practice considered an object of importance; and as the strain upon a rail is greater in the middle than elsewhere, it was considered that it would be economical to have the rail of such a figure longitudinally, that the depth should be every where proportional to the strain; and we have seen (Art. 21) that the strain is proportional to the rectangle of the two parts into which the length is divided by the load, and the resistance is as the square of the depth. What is required, therefore, for a rail to be of equal strength throughout, is to have one in which the depth is every where as the square root of the rectangle of the two parts, which is the property of an ellipse. The bars, therefore, according to this view of the subject, ought to be

elliptical, the length forming the transverse axis and the depth the semi-conjugate axis. In cast iron such a form is commonly given in buildings, &c., with great advantage, and it was thought that it would be equally advantageous here also. Great ingenuity was accordingly displayed by Mr. Birkenshaw, in contriving a pair of rollers that would produce this form, if not exactly, at all events pretty nearly; and before we enter upon an examination of the comparative advantages of this and the parallel rail, it may be interesting to some readers to be informed of the means by which this figure is obtained in rolled iron.

176. This is done by a pair of finishing rollers, of the kind shown in Plate VI., fig. 3. The iron is first drawn down to a square bar of a proper size; it is then passed successively through the rollers, as numbered in the figure. First passing through the grooves No. 1, it takes the form shown in that figure, its sides being parallel; then the form No. 2, still parallel; it then passes through the edging groove No. 3, in which, it will be observed, the lower cylinder is turned eccentric to the axis of the rollers, so that as the iron passes on it is rendered of different depths, as shown in fig. 4; but it has not yet received its finished form; this is obtained by passing it successively through the grooves No. 4 and No. 5, by which it obtains its final thickness and shape, fig. 4. This is not, however, as we have

said, strictly elliptical, as will be seen by examining what takes place in the edging rollers, an enlarged section of which is represented in the annexed diagram.



EF is the section of the upper roller; GH, the section of the other. This latter being hung on a false centre C, is turned down, leaving a groove of varying depth as represented in the figure. The cylinder GH being now again placed on its proper centre B, the bars, as already stated, are introduced between the two rollers at KL; and as the iron passes through, it acquires the variable depth intended. The inner circle, or bottom of the groove, is generally 1 foot in diameter, and the upper 3 feet in circumference; consequently, the figure is completed in a length of 3 feet, and there are commonly five such lengths in a bar.

177. The computation of the ordinates to the curve thus formed is by no means difficult; for, calling the radius of the cylinder $CD = r$, and the

distance of the centres B C = d, and x any angle L C D, we find the ordinate,

$$\mathrm{ID} = \mathrm{BI} - \sqrt{(r^2 + d^2 - 2rd \cos x)}.$$

And by this formula the ordinates of the curves have been computed for two different fish-bellied rails; the extreme depth in both being 5 inches, but the lesser depth in one 3 inches, and in the other 3¾ inches, the latter being that proposed by Mr. Stephenson for the London and Birmingham Railway. The ordinates are taken for each 10°, or for every inch of the half-length, and in the last column are given the ordinates of the true ellipse.

TABLE OF ORDINATES.

Abscissæ.			Ordinates in fish-bellied rail. Greatest depth 5 in. Least do. 3 in.	Ordinates in Mr. Stephenson's rail.	Ordinates in the ellipse.
Deg.		Inch.			
0	=	0	3·00	3·75	0
10	or	1	3·01	3·76	1·64
20	..	2	3·05	3·78	2·29
30	..	3	3·12	3·82	2·76
40	..	4	3·21	3·88	3·14
50	..	5	3·31	3·95	3·46
60	..	6	3·44	4·04	3·72
70	..	7	3·59	4·14	3·96
80	..	8	3·75	4·23	4·16
90	..	9	3·92	4·34	4·33
100	..	10	4·09	4·45	4·48
110	..	11	4·27	4·55	4·61
120	..	12	4·43	4·66	4·71
130	..	13	4·59	4·75	4·80
140	..	14	4·72	4·84	4·87
150	..	15	4·84	4·91	4·93
160	..	16	4·93	4·95	4·97
170	..	17	4·98	4·99	4·99
180	..	18	5·00	5·00	5·00

We see by this Table, (although it is impossible, with any proportions or degrees of eccentricity, to work out a true elliptic figure by this method,) that we may approximate towards it sufficiently near for any practical purpose, as Mr. Stephenson has done; while, on the other hand, without due precaution, we may so far deviate from it as to render the bar dangerously weak in the middle of its half-length.

As far as relates to ultimate strength, there can be no doubt Mr. Stephenson's rail is equal to that of an elliptic rail, and consequently to that of a rectangular rail of the same depth; but there is still an important defect in all elliptical bars, viz., that although this form gives a uniform strength throughout, it is by no means so stiff as a rectangular bar of a uniform depth, equal to that of the middle of the curved bar, and it is the stiffness rather than the strength that is of importance; for the dimensions of the rail must so far exceed those which are barely *strong enough*, as to put the consideration of ultimate strength quite out of the question. The object, therefore, with a given quantity of metal, is to obtain the form least affected by deflection; and unfortunately the elliptical bar, although equally as strong as the rectangular bar of the same depth, as far as regards its ultimate resistance, is much less stiff.

178. This will appear from the following investigation:

Fig. 1.

Fig. 2.

The deflections which beams sustain when supported at the ends and loaded in the middle, are the same as the deflections of the ends would be if the beams were sustained in the middle, and equally loaded at the ends, each with half the weight; and the deflection is the same in the latter case as when the beam is fixed in a wall and loaded at its end. It is quite sufficient, therefore, to consider the corresponding effects on two half-beams, each fixed in an immoveable mass, as represented in the preceding figures.

Now, in the first place, the elementary deflection at C is the same in both beams, because the lengths and loads are the same, and the depths at CA equal; but the whole deflection at any other point P, will be directly as MB^2, and inversely as MP^3. If,

therefore, we call $MB = x$, and $MP = y$, the sum of all the deflections in the two beams will be $\int \frac{x^2}{y^3} . dx \, \Delta$, Δ being the sine of deflection at C. But in fig. 1, y is constant and equal to d (the depth), while in the latter,

$$y = \frac{d}{l} \sqrt{(2\,l\,x - x^2)},$$

l being the semi-transverse or length, and x any variable distance.

The whole deflections, therefore, in the two cases are, fig. 1,

$$\text{Deflection} = \int \frac{x^2\,dx}{d^3} \Delta = (\text{when } x = l) \tfrac{1}{3} \frac{l^3}{d^3} \Delta\,;$$

and in fig. 2,

$$\text{Deflection} = \int \frac{x^2\,dx}{\frac{d^3}{l^3} \; (2\,l\,x - x^2)^{\frac{3}{2}}} \Delta.$$

This is best integrated in parts; thus in the expression,[12]

$$\int \frac{x^2\,dx}{(2\,l\,x - x^2)^{\frac{3}{2}}},$$

Let $x = l - y \;\therefore\; x^2 = l^2 - 2\,l\,y + y^2$ and $2\,l\,x - x^2 = l^2 - y^2$

$$\therefore \int \frac{x^2\,dx}{(2\,l\,x - x^2)^{\frac{3}{2}}} = \int \frac{l^2\,dy}{(l^2 - y^2)^{\frac{3}{2}}} - \int \frac{2\,l\,y\,dy}{(l^2 - y^2)^{\frac{3}{2}}} + \int \frac{y^2\,dy}{(l^2 - y^2)^{\frac{3}{2}}}.$$

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[12] In my former Reports I had found the integration by a series. I am indebted to my colleague, S. H. Christie, Esq., for the above complete integration by parts.

$$\text{Assume} \int \frac{l^2\,dy}{(l^2-y^2)^{\frac{3}{2}}} = \frac{A\,y}{(l^2-y^2)^{\frac{1}{2}}} + B\int \frac{dy}{(l^2-y^2)^{\frac{1}{2}}}$$

$$\therefore \frac{l^2\,dy}{(l^2-y^2)^{\frac{3}{2}}} = \frac{A\,dy}{(l^2-y^2)^{\frac{1}{2}}} + \frac{A\,y^2\,dy}{(l^2-y^2)^{\frac{3}{2}}} + \frac{B\,dy}{(l^2-y^2)^{\frac{1}{2}}}.$$

$$\text{And } l^2 = A\,(l^2-y^2) + A\,y^2 + B\,(l^2-y^2)$$

$$B\,y^2 - (A+B-1)\ .\ l^2 = 0$$

$$\therefore B = 0,\ A = 1.$$

$$\text{And } \therefore \int \frac{l^2\,dy}{(l^2-y^2)^{\frac{3}{2}}} = \frac{y}{(l^2-y^2)^{\frac{1}{2}}}.$$

$$\int \frac{2\,l\,y\,dy}{(l^2-y^2)^{\frac{3}{2}}} = \frac{2\,l}{(l^2-y^2)^{\frac{1}{2}}}$$

$$\int \frac{y^2\,dy}{(l^2-y^2)^{\frac{3}{2}}} = \frac{y}{(l^2-y^2)^{\frac{1}{2}}} - \int \frac{dy}{(l^2-y^2)^{\frac{1}{2}}}$$

$$= \frac{y}{(l^2-y^2)^{\frac{1}{2}}} - \sin^{-1}\frac{y}{l}$$

$$\therefore \int \frac{x^2\,dx}{(2\,l\,x-x^2)^{\frac{3}{2}}} = -\frac{2\,(l-y)}{(l^2-y^2)^{\frac{1}{2}}} - \sin^{-1}\frac{y}{l} + C$$

$$= -\frac{2\,x}{(2\,l\,x-x^2)^{\frac{1}{2}}} - \sin^{-1}\frac{l-x}{l} + C.$$

Which, taken from $x = 0$ to $x = l$, will be

$$2 - \frac{\pi}{2} = 2 - 1{\cdot}5707963 = {\cdot}4292037.$$

That is, the deflection is here

$${\cdot}4292037\,\frac{l^3}{d^3}\,\Delta.$$

The deflections, therefore, in the two cases are, with the same weights, as 33 to 43,[13] or nearly as 3 to 4,—a result fully borne out by subsequent

[13] Experiments have been made from which it has appeared, that the fish-bellied rail was stiffer than the parallel rail, which

experiment. It is to be observed, also, that this investigation applies only to the deflection when the weight is in the middle of the bar, and that it would be much greater in comparison with the parallel rail towards the middle of its half-length.

179. This want of stiffness is, I should imagine, but badly compensated by the trifling saving of metal thus effected; for I find that an addition of little more than four pounds per yard would convert this rail into a rectangular one of the same depth, which would have one-fourth more stiffness at its middle point, and probably one-half more a little beyond the middle of the half-lengths. I am aware objections are made to rectangular bars having so much depth of bearing in their chairs, and this may be a practical defect; at all events, however, it is well to estimate properly both evils, and then to choose the least.

For my own part, I have not the least hesitation in stating, after having well considered every point that has been advanced in favour of the fish-bellied rail, that the parallel rail is the best.

First.—Because, although it is not stronger in the middle point than the former, it is both stronger

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is certainly possible, if the parallel rail be of inferior metal or of injudicious figure; but it is mechanically impossible if the parallel bar be made of the figure here assumed.

and stiffer in a very sensible degree in every other point.

Secondly. — The deflection of a parallel rail, during the passage of a load, is less every where than in the middle, which is not the case in the fish-bellied rail. The fall and rise of a carriage, therefore, after passing over a support, is more rapid in one case than in the other; and to this I am induced in part to attribute many of the fractures which have occurred in these curved rails within a short distance of their point of support; but, perhaps, more are caused by the unequal drawing of the iron through the rollers, as well as by some of the early rails of this kind deviating too much from the real elliptical figure. Mr. Stephenson, by a judicious and scientific distribution of the metal, has now avoided this latter evil, and no doubt such fractures will be with his rail less common; but the objection I offer above applies not merely to the fish-bellied rail, but to the truly elliptical form itself, if it were possible to arrive at it.

Thirdly.—The parallel rail is the best, because it enables the engineer to keep the blocks and chairs of the two rails directly opposite to each other, so that the wheels of the carriage shall pass over both supports at the same time, — a point, I believe, not hitherto much attended to, but which is, I conceive, of great importance. There can be no doubt that the motion of a locomotive carriage consists of a succession of ascents and descents; and it must

be evident how much easier and better the motion would be, to have the opposite wheels both rising and both falling together, than to have one always rising while the other is falling, and the contrary. The difference is similar to that of a vessel keeping her head to the waves, and another crossing their direction obliquely. And every one who has never been further than Margate must have experienced this difference.

It may be said, that the waves of the railway, or the deflections of the rails, are very small; but I would observe also, that the weights and velocities of the carriages are very great, and that it is very desirable every possible cause of momentum should be removed, particularly when it is as easy to do it as not to do it, as is the case with parallel rails, because they can always be cut to determinate lengths, which is not so easily done in the fish-bellied rail, in consequence of the occasional slipping of the bar in the rollers, as before mentioned.[14] At all events, their length cannot be varied at pleasure, which the former will admit of, and which is necessary, in going round sweeps, to preserve the blocks always parallel. For example, in going round a sweep of 800 feet, to keep the sup-

[14] I ought to mention that the late Mr. Blackmore, the engineer of the Carlisle and Newcastle Railway, by a judicious selection of long and short rails, preserved the parallelism of the blocks; and it is the only case, that I am aware of, in which this has been attended to, not only in the general line, but in the curves.

ports parallel, the rails of the inner course require to be about an inch shorter than the outer ones; and they are as easily cut into lengths of 14 feet 11 inches as of 15 feet, which is not practicable in the other form.

Having arrived at these conclusions, the author delivered in a Report to the Directors of the London and Birmingham Railway, which was published in the previous editions of this work, and from which the following remarks are extracted.

Extracts from Reports addressed to the Directors of the London and Birmingham Railway Company.

180. The contraction of iron between summer and winter amounts to the $\frac{1}{8000}$th part of its length, and as, when stretched the $\frac{1}{1000}$th part of its length, in which case it is strained with ten tons per inch, its elasticity is injured, it follows that the bars cannot be fixed permanently to the chairs and blocks without great danger of drawing so much upon their strength, as materially to impair their efficiency for bearing a passing load.

The parallel rail, formed according to the requisite proportions, is decidedly the best (see pages 349–351).

The whole depth of a rail should not be less than $4\frac{1}{2}$ inches; the thickness of the middle rib should not exceed that which is essential to the

perfect manufacture of the bar; and the lower web should be made of the best form for giving to it a steady bearing in its seat.

A difference of level at a joint chair between the two abutting rails of only $\frac{1}{10}$th of an inch will, when the carriage is moving from the higher to the lower level at a speed of 30 miles per hour, cause the wheel to pass the distance of a foot without pressing on the rail.

It is strongly recommended that all rails should be proved and gauged before being received as efficient.

Experiments on the actual Strength of Railway Bars of various forms and dimensions.

181. Having in the preceding pages investigated every circumstance which has a theoretical bearing on the question of the strength of malleable iron generally, and as applied to railway bars in particular, the following trials on the bars themselves will be useful as offering the best means of comparing the rules with actual experimental results.

Experiments on the Resistance and Deflection of Railway Bars, without a Lower Web.

Mr. Stephenson's Fish-bellied Rail, 50 lbs. per yard.

Greatest depth 5 inches, less depth 3¾ inches, thickness of centre rib $\frac{9}{16}$ inch.

Bar No. 1.			Bar No. 2.		
Weights.	Deflections by index.	Deflections for each ton.	Weights.	Deflections by index.	Deflections for each ton.
1	·035		1	·014	
2	·045	·010	2	·022	·008
3	·055	·010	3	·030	·008
4	·065	·010	4	·042	·012
5	·071	·006	5	·050	·008
6	·076	·005	6	·062	·012
7	·087	·011	7	·075	·013
7½	·095	·016	8	·085	·010
			9	·101	·016
			10	Elasticity injured.	
			11	·300	

Bar No. 3.			Bar No. 4.		
Weights.	Deflections by index.	Deflections for each ton.	Weights.	Deflections by index.	Deflections for each ton.
1	·018		1	·045	
2	·025	·007	2	·056	·011
3	·038	·013	3	·065	·009
4	·054	·016	4	·075	·010
5	·062	·008	5	·084	·009
6	·069	·007	6	·095	·011
7	·080	·011	7	·105	·010
8	·094	·014	8	·110	·005
8½	·100	·012	9	·116	·006
9	·112	·018	10	·125	·009
9½	·118	·018	11	·165	
10	·126	·014			
11	·160	·034			
17	Destroyed.				

Mean deflection per ton, Bar No. 1. . .	·0097
No. 2. . .	·0101
No. 3. . .	·0110
No. 4. . .	·0090
Mean . . .	·0100

TABLE—(CONTINUED).

Bar No. 5, Fish-bellied. Great depth, 5 inches. Less ditto, 3¾. Thickness of rib, $\frac{9}{16}$. Head estimated, 2 by 1. Weight, 50 lbs.			Bar No. 6, Fish-bellied. Great depth, 3½ inches. Less ditto, 2¼. Thickness of rib, $\frac{7}{16}$. Head estimated, 2 by ¾.			Bar No. 7, Fish-bellied. Great depth, 3 inches. Less ditto, 2. Thickness of rib, $\frac{6}{16}$. Head estimated, 2 by ½.		
Weight in tons.	Deflection by index.	Deflection for each ton.	Weight in tons.	Deflection by index.	Deflection for each half ton.	Weight in tons.	Deflection by index.	Deflection for each half ton.
1	·030		0·5	·120		0·5	·033	
2	·260		1·0	·140	·020	1·0	·060	·027
							re-adjusted	
3	·270	·010	1·5	·170	·030	1·5	·062	
4	·290	·020	2·0	·180	·010	2·0	·090	·028
5	·300	·010	2·5	·200	·020	2·5	·120	·030
6	·320	·020	3·0	·230	·030	3·0	·155	·035
7	·335	·015	3·5	·280	·050	3·5	·240	
8	·410	·060	4·0	·420	·140	4·0		
Mean deflection per ton to 7 tons		·015	Mean deflection per ½ ton to 3 tons		·022	Mean Deflection per ½ ton to 2 tons		·030
Do., with 7½ tons ·107			Do., with 3 tons ·066			Do., with 2 tons ·060		

Comparison of the above Results with the Formulæ,[15] p. 329.—viz.

Rib . . . . $\frac{1}{3} hs \,.\, ns \,.\, pq \,.\, t$

Head . . . . $\frac{1}{3} hx \,.\, \overline{nx}^2 \,\frac{nn-pq}{ns}\, t$

BARS NOS. 1, 2, 3, 4, 5.

Here . . $\begin{cases} hs = 5, ns = 4\cdot5, pq = \cdot9, t = 10 \\ hx = 1, nx = \cdot5, nn - pq = 1\cdot1 \end{cases}$

Hence . $\frac{1}{3} \times 5 \times 4\cdot5 \times 9 = 67\cdot5$

[15] The same formulæ of course apply to the fish-bellied as to the parallel rail, but for the deflection we must multiply the result by ⅘. See p. 349.

$$\tfrac{1}{3} \times 1 \times \overline{0{\cdot}5^2} \times \frac{11}{45} = {\cdot}20$$

$$\frac{67{\cdot}7 \times 4}{32} = 8\tfrac{5}{11} \text{ tons}$$

$$\frac{{\cdot}22}{4{\cdot}5} \times \frac{4}{3} \times {\cdot}066 \text{ deflection.}$$

BAR No. 6.

Here . . $\begin{cases} hs = 3{\cdot}25,\ ns = 2{\cdot}88,\ pq\ {\cdot}7,\ t = 10 \\ hx = {\cdot}75,\ nx = {\cdot}375,\ nn - pq = 1{\cdot}3 \end{cases}$

Hence . $\tfrac{1}{3} \times 3{\cdot}25 \times 2{\cdot}88 \times 7 = 21{\cdot}84$

$\tfrac{1}{3} \times {\cdot}75 \times \overline{{\cdot}375^2} \times \frac{13}{2{\cdot}88} = {\cdot}15$

$21{\cdot}99 = s$

$$\frac{4s}{33} = 2\tfrac{2}{3} \text{ tons}$$

$$\frac{{\cdot}22}{2{\cdot}88} \times \frac{4}{3} = {\cdot}092 \text{ deflection.}$$

BAR No. 7.

Here . . $\begin{cases} hs = 3,\ ns = 2{\cdot}75,\ pq = {\cdot}6,\ t = 10 \\ hx = {\cdot}5,\ nx = {\cdot}25,\ nn - pq = 1{\cdot}4 \end{cases}$

Hence . $\tfrac{1}{3} \times 3 \times 2{\cdot}75 \times 6 = 16{\cdot}50$

$\tfrac{1}{3} \times {\cdot}5 \times \overline{{\cdot}25^2} \times \frac{14}{275} = {\cdot}05$

$16{\cdot}55 = s$

$$\frac{4s}{33} = 2{\cdot}06 \text{ tons}$$

$$\frac{{\cdot}22}{2{\cdot}75} \times \frac{4}{3} = {\cdot}106 \text{ deflection.}$$

The following experiments have been made subsequently to the publication of my Report to the London and Birmingham Directors.

NORTH UNION OR GRAND JUNCTION RAIL.

182. *Experiments, with the Proving Machine in Woolwich Dockyard, to ascertain the Strength and Stiffness of the Parallel Rail, with double flanch, for the North Union Railway Company. Weight, per yard,* 60 lbs.*; Area of Section,* $6\frac{1}{8}$ *inches; Depth,* $4\frac{1}{2}$ *inches.*

Figure of Section as in the following page.

Note.—The dotted line shows the assumed equivalent right-lined section.

Results obtained from three single Experiments.									Results obtained from the Means of three Experiments.								
Weight.	Deflections by index.	Deflections for each ton.	Weight.	Deflections by index.	Deflections for each ton.	Weight.	Deflections by index.	Deflections for each ton.	Weight.	Deflections by index.	Deflections for each ton.	Weight.	Deflections by index.	Deflections for each ton.	Weight.	Deflections by index.	Deflections for each ton.
1	·028		1	·035		1	·009		1	·027		1	·021		1	·018	
2	·031	·003	2	·039	·004	2	·016	·007	2	·031	·004	2	·026	·005	2	·024	·006
3	·036	·005	3	·044	·005	3	·020	·004	3	·036	·005	3	·031	·005	3	·028	·004
4	·038	·002	4	·048	·004	4	·029	·009	4	·039	·003	4	·036	·005	4	·033	·005
5	·043	·005	5	·054	·006	5	·033	·004	5	·044	·005	5	·041	·005	5	·037	·004
6	·046	·003	6	·059	·005	6	·034	·001	6	·048	·004	6	·044	·003	6	·040	·003
7	·050	·004	7	·064	·005	7	·038	·004	7	·052	·004	7	·048	·004	7	·044	·004
8	·055	·005	8	·069	·005	8	·042	·004	8	·057	·005	8	·053	·005	8	·048	·004
9	·060	·005	9	·076	·007	9	·046	·004	9	·063	·006	9	·059	·006	9	·053	·005
10	·066	·006	10	·082	·006	10	·050	·004	10	·079	·007	10	·064	·005	10	·059	·006
11	·074	·008	11	·086	·004	11	·055	·005	11	·077	·007	11	·071	·007	11	·067	·008
12	·084	·010	12	·096	·010	12	·066	·011	12	·087	·010	12	·081	·010	12	·077	·010
Mean deflection, with 11 tons,		·051			·056			·051			·055			·055			·054

Computed Strength and Deflection by the rule, p. 332. *See also Example,* p. 335.

$$\text{Head} \quad \left\{\begin{array}{l}(2{\cdot}25 - {\cdot}65) \times 10 = 16 \\ (4\tfrac{1}{2} - \tfrac{1}{2}) \times 12 = 48\end{array}\right\} \quad \ldots \quad = 0{\cdot}33$$

$$\text{Middle rib} \quad \tfrac{1}{3} \times 4 \times 4\tfrac{1}{2} \times {\cdot}65 \times 10 \quad \ldots \quad = 39{\cdot}00$$

$$\text{Lower web} \quad \left\{\begin{array}{l}3\tfrac{1}{2} \times (2{\cdot}25 - {\cdot}65) \times 10 = 56 \\ 12 \times \overline{3\tfrac{1}{2}}|^2 + 21 \ldots = 168 \\ 168 - 6 \ldots\ldots = 162\end{array}\right\}$$

$$168 : 162 :: 5{\cdot}6 : 54 \ldots\ldots = 54{\cdot}00$$

$$\overline{93{\cdot}33}$$

$$\frac{4 \times 93{\cdot}33}{33} = 11\tfrac{1}{4} \text{ tons, computed weight or strength;}$$

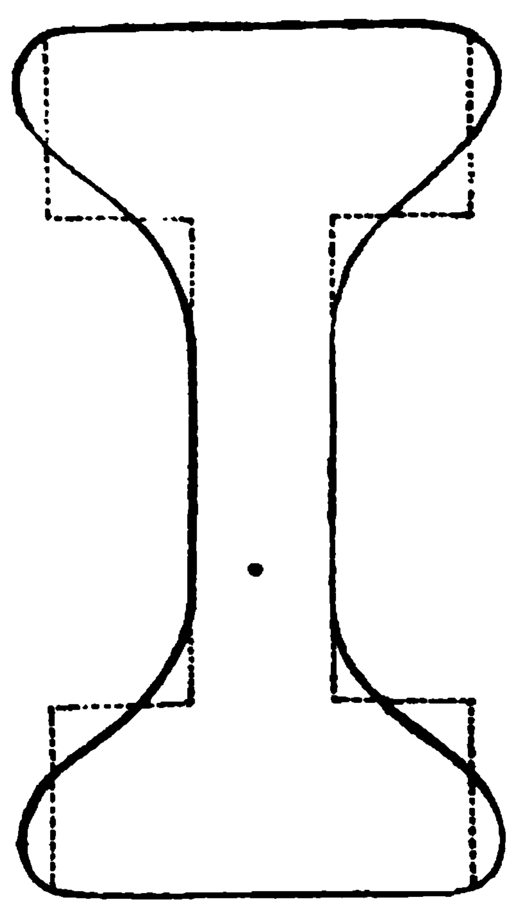
$$\frac{{\cdot}22}{4} = {\cdot}055 \text{ computed deflection.}$$

Section of Equivalent Straight-lined Rail.

Weight 60 lbs. per yard.

$nn = 2{\cdot}25$
$nn - pq = 1{\cdot}6$
$ns = 4$
$nx = {\cdot}5$
$hr = 1$
$hs = 4{\cdot}5$

(See fig. Art. 168.)



183. *Report of Experiments made with the Proving Engine in the Royal Dockyard, Woolwich, to ascertain the Strength and Stiffness of Three Specimens of Railway Bars designed for the Southampton Railway.*

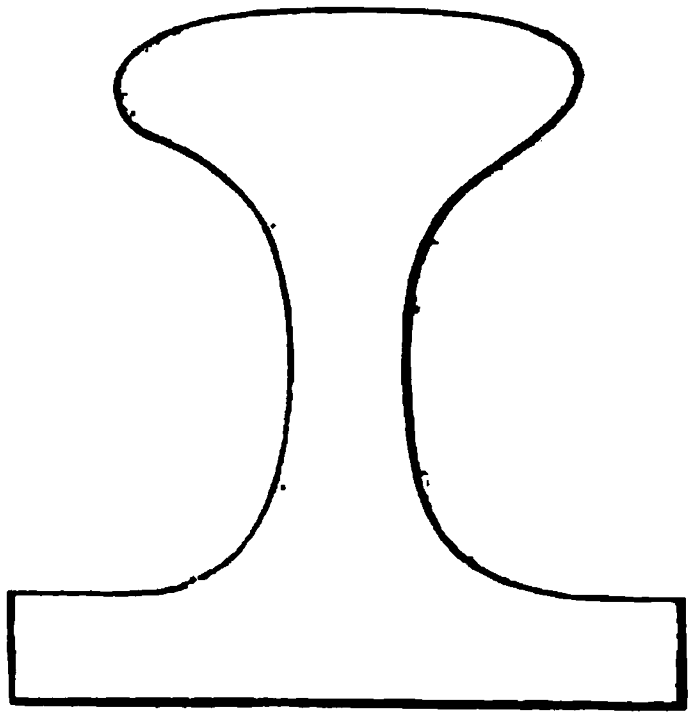
Fig. of Section as in following page.

Present,—Col. Henderson, Acting Director: Mr. Giles, Engineer; and Wm. Reed, Esq., Secretary.

The experiments were made precisely in the same way as is described in my Report addressed to the Directors of the London and Birmingham Railway Company, except that, in consequence of the greater breadth of the lower flanch, the frame I had hitherto used was too narrow to admit the Southampton rail. Another frame was therefore made by Messrs. Gordon and Company for the purpose; like the other frame, except in the above particular, and that the opening of the frame to form the points of bearing was by mistake made 34 inches instead of 33 inches. For the sake of comparison, I have therefore reduced the observed strength to 33 inches' bearing; and also, as the engineer proposes, to have the chairs 5 inches long, giving only a bearing of 31 inches clear. I have also reduced the strength to this bearing. The deflection requires no correction, being measured by the same instrument; and the observed deflections are those which take place be-

tween the feet of the instrument, independently of the points of bearing. The following are the detail of the experiments.

Section.



Depth of rail, 3½ inches.
Thickness, centre rib, ·8 inch.
Breadth, lower flanch, 3½ inches.
Depth of ditto, ·6 inch.
Weight, 57 lbs. per yard.

Bar No. 1.			Bar No. 2.			Bar No. 3.		
Strain.	Index readings.	Deflection per ton.	Strain.	Index readings.	Deflection per ton.	Strain.	Index readings.	Deflection per ton.
tons.			tons.			tons.		
2	·076		2	·043		2	·030	
3	·087	·011	3	·052	·009	3	·040	·010
4	·097	·010	4	·066	·014	4	·052	·012
5	·110	·013	5	·077	·011	5	·065	·013
6	·122	·012	6	·094	·017	6	·076	·011
7	·137	·015	7	·109	·015	7	·093	·017
8	Quite destroyed.		8	·137	·026	8	·116	·023
						9	·167	·051

The above bars were in 7½-feet lengths, and the experiments were all made on their middle point. In the following, the experiment was first made on

the middle of the length, and then on the middle of one half-length.

Middle, Bar No. 4.			Half-length, Bar No. 4.		
Strain.	Index readings.	Deflection per ton.	Strain.	Index readings.	Deflection per ton.
tons.			tons.		
2	·041		2	·014	
3	·053	·012	3	·024	·010
4	·063	·010	4	·030	·006
5	·071	·008	5	·041	·011
6	·077	·006	6	·054	·013
7	·083	·006	7	·070	·016
8	·108	·015	8	·094	·024
			9	·166	·076

From the above results, it appears, that the mean strength of the bars cannot be stated at more than 7 tons, four out of the five bars showing indications of weakness with that weight. But this is with 34 inches' bearing.

	tons.
This reduced to 33 inches, gives . . .	$7\frac{1}{5}$
and reduced to 31 inches . . .	$7\frac{2}{3}$
Mean deflection, estimated per ton . .	·011
Deflection with 3 tons	·033

Computed Strength.

Here the equivalent right-lined section may be taken as follows:

$$hs = 3{\cdot}5,\ ns=3,\ pq={\cdot}8,\ nn=2{\cdot}25,\ hx=1,\ nx=\tfrac{1}{2},\ nn-pq=1{\cdot}45,$$
$$nm=2{\cdot}7,\ mm=3{\cdot}5,\ rs={\cdot}6.$$

Hence by the rule, p. 329,

	tons.
Resistance of rib $\frac{1}{3} hs \,.\, ns \,.\, pq \,.\, t$ .... =	28·0
Head $\frac{1}{3} hx \,.\, \overline{nx}\rceil^2 \,.\, \frac{nn-pq}{ns} t$ .... =	0·6
Lower web $\begin{cases} nm + \frac{\overline{rs}\rceil^2}{12\,nm} + cn = \delta'' \\ nm \,.\, rs \,.\, (mm-pq) \frac{\delta''}{d'} t = \end{cases}$	43·7
	72·3

Whence $\frac{72{\cdot}3 \times 4}{34} = 8\frac{1}{2}$ tons, the computed strength for 34 inches. Whereas the experiment shows a strength of only 7 tons.

I had no hesitation, on this ground, in reporting the iron bad; and that I was justified in so doing, is shown by the following experiments, which were other specimens from a different maker. Bars 1 and 2, of good medium quality; and bar No. 3, a higher priced iron of superior quality. The character of the section the same, but the centre rib $\frac{1}{2}$ inch more between the flanches.

184. *Report of Experiments made with the Proving Machine in His Majesty's Dockyard, Woolwich, on Three Bars of Iron sent as Specimens for the Railway Bars of the London and Southampton Railway.* *Dec.* 26, 1835.

Equivalent rectilinear dimensions of the section.
- Head $2\frac{1}{4}$ inches by 1 inch deep.
- Whole depth 4 inches.
- Thickness, middle rib $\frac{3}{4}$ inch.
- Lower web: Depth $\frac{3}{16}$ do.; Breadth $3\frac{1}{4}$ inches.

Mean weight per yard 60 lbs.

The experiments were performed exactly in the same manner as described in my former Report, in the presence of Col. Henderson, R.E., P. Giles, Esq., Engineer, and W. Reed, Esq., Secretary.

The bearing distance in the frame made for the London and Southampton Railway experiments being 34 inches, and the frame on which my other experiments were made being only 33 inches, I have determined the strength for 33 inches by computation, that these strengths may be more readily compared with the bars, of which the experimental results are given in my printed Reports. I have also found the strength at 31 inches, the bearing proposed by Mr. Giles. The deflections require no correction.

BAR No. 1.

Weight of 9 feet . . . 479 lbs.

Position of bar direct. First trial.			Strain left on 2 hours; experiment repeated in the same place.		
Strain in tons.	Index readings.	Deflections with each ton.	Strain in tons.	Index readings.	Deflections with each ton.
2	·030		2	·037	
3	·0325	·0025	3	·042	·005
4	·034	·0015	4	·045	·003
5	·037	·003	5	·0515	·0065
6	·043	·006	6	·056	·0045
7	·0475	·0045	7	·063	·007
8	·057	·0095	8	·070	·007
9	·065	·008	9	·075	·005
10	·0765	·0115	10	·083	·008
			11	·092	·009
			12	·132	·040

It appears, from these experiments, that although the bar shows great stiffness with the first strains, it yields considerably to the last strains, and that it had taken a permanent set with 10 tons.

The mean deflection per ton of this bar, taken between 5 and 10 tons, 1st experiment	·0079
Ditto ditto 2nd experiment	·0063
Mean	·0071

Mean strength . . 10 tons at 34 inches.
$10\frac{1}{2}$ tons at 33 inches.
11 tons at 31 inches.

To try the effect of the lower web, the bar was reversed in position, and another part submitted to the strain.

POSITION REVERSED.

Strain in tons.	Index readings.	Deflections with each ton.
2	·012	
3	·016	·004
4	·024	·008
5	·027	·003
6	·031	·004
7	·036	·005
8	·041	·005
9	·051	·010
10	·067	·016
11	·082	·015
12	·125	·043
Mean between 5 and 10 tons . .		·008

EXPERIMENTS ON BAR No. 2.

DIRECT AND REVERSED.

Weight of 9 feet . . . . 181 lbs.

POSITION DIRECT.			POSITION REVERSED.		
Strain in tons.	Index readings.	Deflections with each ton.	Strain in tons.	Index readings.	Deflections with each ton.
2	·035		2	·041	
3	·040	·005	3	·047	·006
4	·045	·005	4	·052	·005
5	·049	·004	5	·061	·009
6	·055	·006	6	·0655	·0045
7	·062	·007	7	·072	·0065
8	·072	·010	8	·077	·005
9	·0795	·0075	9	·0825	·0055
10	·086	·0065	10	·0905	·008
11	·0905	·0045	11	·099	·0085
12	·105	·0145	12	·113	·014
13	·127	·022	13		
Mean between 5 and 10 tons		·0074	Mean between 5 and 10 tons		·0059

EXPERIMENTS ON BAR No. 3.

BOTH DIRECT.

Weight of 9 feet . . . . 179 lbs.

POSITION DIRECT.			POSITION DIRECT.		
Strain in tons.	Index readings.	Deflections with each ton.	Strain in tons.	Index readings.	Deflections with each ton.
2	·037		2	These readings were missed, the strain being brought on too quickly.	
3	·044	·007	3		
4	·052	·008	4		
5	·056	·004	5	·031	
6	·064	·008	6	·040	·009
7	·070	·006	7	·046	·006
8	·0765	·0065	8	·052	·006
9	·084	·0075	9	·060	·008
10	·089	·005	10	·0685	·0085
11	·096	·007	11	·077	·0085
12	·107	·011	12	·084	·007
13	·127	·021	13	·105	·021
Mean between 6 and 11 tons		·0064	Mean between 6 and 11 tons		·0074

GENERAL MEAN RESULTS.

	Mean strength at 34 in. bearing. Tons.	33 inches. Tons.	31 inches. Tons.	Mean deflection. Per ton.
Bar No. 1.	10	$10\frac{1}{8}$	11	·0071
No. 2.	11	$11\frac{1}{8}$	12	·0074
No. 3.	12	$12\frac{1}{8}$	$13\frac{1}{4}$	·0069

Computed Strength.

Here the equivalent right-lined figure gives

$$hs = 4,\ ns = 3{\cdot}5,\ pq = {\cdot}75,\ nn = 2{\cdot}25,$$

$$hx = 1,\ nn - pq = 1{\cdot}5,\ nn = 3{\cdot}2,\ mm = 3{\cdot}5,\ rs = {\cdot}6.$$

Hence by the rule, page 329,

$$\begin{array}{lr} & \text{tons.} \\ \text{Resistance of rib } \tfrac{1}{3}\, hs\,.\,ns\,.\,pq\,.\,t \quad .\ .\ .\ .\ . & = 35{\cdot}0 \\ \text{Do. head } \tfrac{1}{3}\,.\,hx\,.\,nx^2\,.\,\dfrac{nn-pq}{ns} \quad .\ . & = 0{\cdot}7 \\ \text{Lower web}\left\{\begin{array}{l} nm + \dfrac{rs^2}{12\,nm} + cn = \delta'' \\ nm\,.\,rs\,.\,(mm - pq)\,\dfrac{\delta''}{d'}\,t \quad .\ .\ . \end{array}\right. & \\ & = 52{\cdot}8 \\ \hline & 88{\cdot}5 \end{array}$$

And then $\frac{88{\cdot}5 \times 4}{34} = 10\frac{1}{2}$ computed strength, which agrees with the mean of the two medium bars as nearly as possible.

The third bar, as has been stated, was of a superior description of iron.

185. *Report of Experiments made on Three Bars, for the Southampton Railway Company, from the same Iron Works as the first set; March 12th*, 1835.

Present, W. Reed, Esq., Secretary.

	inches.		inches.
Depth	3½	Breadth of centre rib	$\frac{6}{10}$
Depth of lower flanch	·6	Breadth of lower flanch . . .	3½

Weight 57 lbs. per yard.

Strain in tons.	Index readings.	Deflection for each ton.	Strain in tons.	Index readings.	Deflection for each ton.	Strain in tons.	Index readings.	Deflection for each ton.
1	·0575		1	·0050		1	·0340	
2	·0680	·0105	2	·0150	·0100	2	·0420	·0080
3	·0790	·0110	3	·0250	·0100	3	·0460	·0040
4	·0900	·0110	4	·0360	·0110	4	·0510	·0050
5	·0970	·0070	5	·0450	·0090	5	·0600	·0090
6	·106	·0090	6	·0540	·0090	6	·0700	·0100
7	·120	·0140	7	·0660	·0120	7	·0860	·0160
8	·128	·0080	8	·0880	·0220	8	·110	·0240
9	·149	Destroyed	9	·109	Destroyed	9	·190	

By comparing the above results with those obtained on the bars first tested, the strength and stiffness will appear to be very nearly the same, except bar No. 1, which retained its elasticity with 8 tons. Bars No. 2 and No. 3 cannot be said to have borne more than 7 tons at 34 inches' bearing; but reduced to a bearing of 31 inches, the strengths will be as follow:

			tons.
Bar No. 1.	Strength at 31 inches' bearing	.	8¾
No. 2.	do. do.	. .	7⅔
No. 3.	do. do.	. .	7⅔
Bar No. 1.	Deflection 3 tons	. . . . . .	·024
No. 2.	do. do.	. . . . . .	·023
No. 3.	do. do.	. . . . . .	·020

It appears, therefore, that No. 1 is the strongest bar, and No. 3 the stiffest. Upon the whole, the bars are nearly the same as those first sent; as will be observed in referring to my Report on them.

I believe that some improvement was attempted to be made in the manufacture of these bars, but it is clear that the metal itself is defective. And nothing, perhaps, could have better proved the accuracy of the rules I have given, nor the propriety of testing the bars when delivered from the maker, as recommended in my first Report to the Directors of the London and Birmingham Railway, than the preceding experiments.

186. The following are experiments made on two specimens of iron in bars of 75 to 77 lbs. per yard, intended for 5-feet bearings.

Report of Experiments made on the Testing Machine in His Majesty's Dockyard, Woolwich, on two Specimens of Railway Bars, viz.

Two bars, maker not known.
Two bars, from Messrs. Solly, best patented.

First specimen.
Section, double flanch with centre rib, similar to fig. Art. 182.
Greatest breadth of flanch 2·6 inches.
Mean depth $1\frac{1}{4}$ inch. Whole depth of rail 5 inches.
Mean breadth of flanch 2·125 inches.
Thickness of centre rib ·85 inch.
Weight not stated, but about 75 lbs. per yard.

Best patented.	The same dimensions, rather more full. Thickness of centre rib ·9 inch. Weight of one of these bars, 3 cwt. 1 qr. 20 lbs., or 77 lbs. per yard; of the other, 3 cwt. 1 qr. 12 lbs., or 75½ lbs. per yard.

The experiments were performed as before, except, that in consequence of these bars being intended for 5-feet bearings, the iron frame was obliged to be altered; and that it might answer both for those bars designed for 4-feet bearings as well as 5-feet, it was lengthened to 4 feet 6 inches, and proportionally strengthened, which, as I understand the experiments to be only comparative, seemed to answer both cases without having a new frame made.

The difference in the strength of the two specimens, it will be seen, is very considerable, although the stiffness at first is nearly the same: the first specimen is, however, rather the stiffest, but the other much the strongest; the elasticity or restoring power being preserved up to a strain of 10 tons in the latter, and only to 8½ tons in the former, at a bearing of 4 feet 6 inches. Or reducing both to 5 feet bearing, we have for the greatest load that can be safely borne,

	Tons.
First specimen	7·65
Best patented	9·00

But the deflection per ton with

	Inch.
First specimen	·0165
Best patented	·0175

In the computation I made in my last Report, it was intended the bars should bear 8 tons, at 5-feet bearings. It appears, therefore, that the strength of the former is rather less than ought to be expected of good medium iron, and that the other is in excess of strength 1 ton.

The following are the experiments from which these deductions have been made:

FIRST SPECIMEN.

BAR No. 1.

Strain in tons.	Index readings.	Deflection per ton.	
1			
2	·050		
3	·067	·017	
4	·075	·008	
5	·092	·017	
6	·107	·015	
7	·122	·015	
8	·142	·020	
9	·165	·023	Injured very little.
10	Elasticity quite destroyed.	·016	Mean deflection per ton.

BAR No. 2.

Strain in tons.	Index readings.	Deflection per ton.	
1			
2	·032		
3	·045	·013	
4	·062	·017	
5	·085	·023	
6	·102	·017	
7	·121	·019	
8	·136	·015	Mean per ton ·017.
9	·171	·035	
10	·255	·084	

BEST PATENTED.

Bar No. 1.

Weight, 3 cwt. 1 qr. 20 lbs.

Strain in tons.	Index readings.	Deflection per ton.	
1			
2	·036		
3	·045	·009	
4	·066	·021	
5	·086	·020	
6	·096	·010	
7	·110	·014	
8	·128	·018	
9	·149	·021	
10	·168	·019	
11	·188	·020	Mean per ton ·017.
12	·210	·023	

Bar No. 2.

Weight, 3 cwt. 1 qr. 12 lbs.

Strain in tons.	Index readings.	Deflection per ton.	
1			
2	·054		
3	·064	·010	
4	·084	·020	
5	·105	·021	
6	·120	·015	
7	·140	·020	
8	·161	·021	
9	·180	·019	Mean per ton ·018
10	·207	·027	
11	·244	·037	
12	·315	·071	

*To the Directors
of the London and Birmingham
Railway Company.*

Woolwich, Oct. 31st, 1836.

187. *Report of Experiments on Two Railway Bars received October 27th; manufacturer's name not stated, nor the weight, but by the section about 65 lbs. per yard. Double flanch, whole depth 4½ inches, intended for 4-feet bearings.*

Tested at 4½-feet bearings, the same as those tested on the 26th and 27th instant.

Bar No. 1.			Bar No. 2.		
Strain in tons.	Index readings.	Deflection per ton.	Strain in tons.	Index readings.	Deflection per ton.
1			1		
2	·048		2	·064	
3	·072	·024	3	·082	·018
4	·091	·019	4	·105	·023
5	·110	·019	5	·125	·020
6	·131	·021	6	·145	·020
7	·153	·022	7	·165	·020
8	·177	·024	8	·186	·021
9	·199	·022	9	·211	·025
10	·222	·023			
11	Elasticity destroyed.		10	·275	·065 Elasticity gone.
Mean deflection per ton. Bar No. 1, ·022 inch.			Mean deflection per ton. Bar No. 2, ·021 inch.		

Mean strength of the two bars 9½ tons, at 4 feet 6 inches bearing, or 10·2 tons at 4 feet.

The Directors cannot but observe the striking fact elicited by these and the preceding experiments on the bars Nos. 1 and 2; viz.

That 65 lbs. per yard is 1 ton stronger at the same bearing distance with these bars than with the other at 75 lbs. per yard; that is, with 13½ per cent. less weight there is 12 per cent., very nearly, more strength. Now whether this proceeds from a difference of the ore, a difference in the mode of

manufacture, or from the difficulty of manufacturing such large bars, I cannot tell; but it is a question which appears to me to be very deserving attention.

Taking into account the difference in the depth of the two specimens, the proportional stiffness is very nearly the same.

These experiments, again, as compared with the preceding, show the strong necessity of some mode of testing; as a Company may otherwise be liable to purchase bars at a great expense actually weaker than others of less cost, not only in the gross, but per ton; for I have since learned that these latter bars were bought at less per ton than the former.

MISCELLANEOUS EXPERIMENTS CONNECTED WITH RAILWAYS.

(EXTRACTED FROM A SECOND REPORT ADDRESSED TO THE DIRECTORS OF THE LONDON AND BIRMINGHAM RAILWAY COMPANY.)

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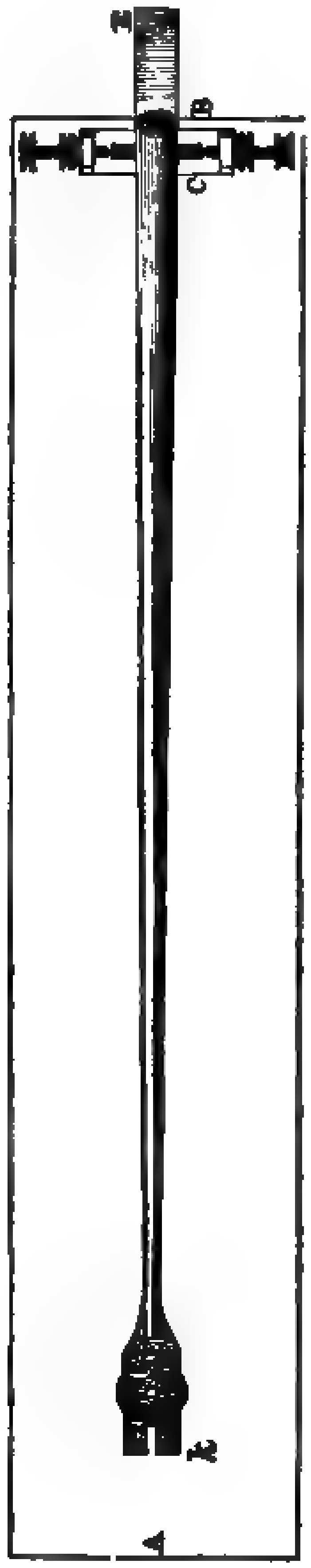
188. The first and most important point which required to be decided was, the strength of iron necessary to insure the most ample safety, at any practicable speed, with any given load and given length of bearing. The strain which any quiescent load impresses on a bar, is, I think, now well known; but what is the effect of velocity? This was one of those questions on which I found opinions greatly divided; and it was a question, perhaps, considered merely hypothetically, in which there was great room for doubt. My first object, therefore, was to reduce it to a matter of experimental fact: this rendered it necessary to construct an instrument for the purpose, and I feel myself much indebted to Mr. King, of the Liverpool Gas Works, for the ready attention he paid to my suggestions, and for the ingenuity he exercised in giving it its first form, the whole of which was left to his own invention, after being simply informed of its object, and the general mode of its intended operation.

This instrument, which it is proposed to call a *deflectometer*, is represented in plan and elevation in the following diagram. A B is a plain board about 27 inches long and 6 inches broad, with two pillars or standards, one of which is seen in the elevation; and between them is suspended the lever D E by screw points, divided in C, in the proportion of 10 to 1; G H is a slightly inclined stout wire, on which slide the two indexes *i*, *i*, but with sufficient friction to remain in their places.

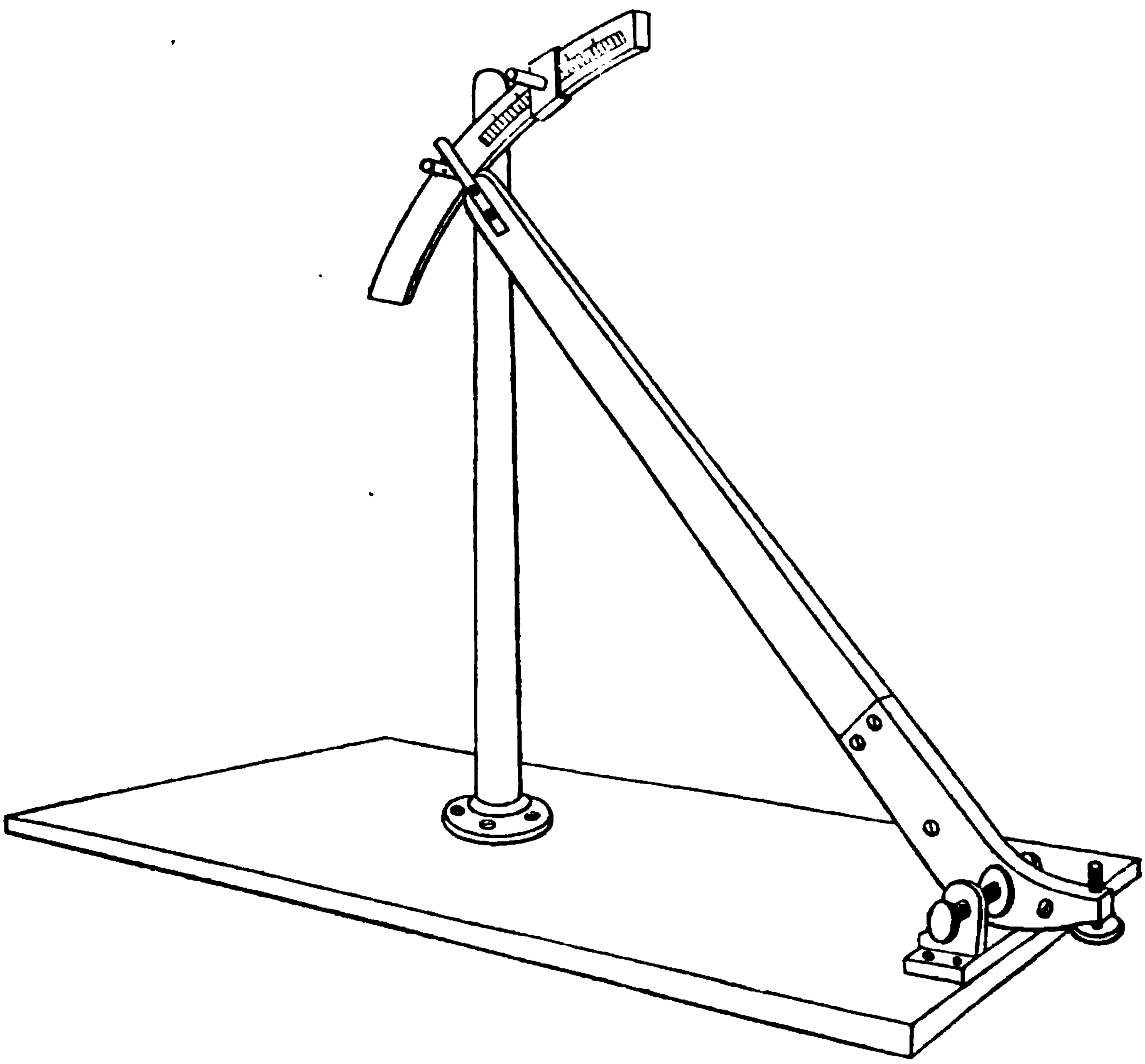
DEFLECTOMETER.

PLAN.



The manner of using the instrument is by levelling the ground under the centre of the rail, and placing the point E under its lower edge; the preponderance then being on the side of the long arm, the point E is kept in contact with the lower edge of the bar, and the lower index *i* is moved up to the metal plate *k*; the upper one is then, in like manner, brought down and placed in contact also. It is obvious, now, that whatever deflection the rail may sustain during the passage of an engine, or a train of waggons, the index *i* will be lifted ten times the quantity the bar is deflected, and remaining in its place, the greatest deflection the bar has sustained will be truly and distinctly indicated.

An improved form of this instrument is represented in the following page, but the principle of its action is the same. We found in the first instrument an inconvenience from the index being so near the ground, and in order to avoid this, the late Mr. W. Gilbert gave it the form shown in the figure. The register here is by a sliding vernier on an arc; the latter also being raised, the result may be read with great ease and convenience. The upright stand carrying the arc is a brass tube which fits tightly over a brass pin on the base-board. It may, therefore, be easily removed, and the whole packed very close for convenience of carriage.



189. *Experiments made with a view to ascertain the Strain which a Load in rapid Motion produces upon the Rail over which it passes, in order to compare the same with the known Strain produced by an equal quiescent Load.*[1]

Mr. King's little instrument was admirably suited to this inquiry, for by this the greatest deflection

[1] The experiments were made in 1835.

the bar sustained, from whatever cause it proceeded, was accurately registered, and by comparing this deflection with the experiments made on the same bar with quiescent loads, the effects due to velocity, and those proceeding from irregularities in the joints, &c., became known, at least in the aggregate, and this aggregate is of course the strain against which it is necessary to provide.

Experiments on the central Deflection of Railway Bars during the passage of a heavy Load at different Degrees of Speed, and on different Lengths of Bearings.

190. Our observations were commenced in and near the cutting at Wavertree Hill, in rock cutting, the ground being as sound, and the bearings as firm, as in any part of the line.

The first trials were made on the Grand Junction Rail laid down in May, on 3 feet 9 inches bearings. The weight of rail 62 lbs. per yard. A deflectometer was accurately placed under each of four bearing lengths—one having been selected next the bearing end, the other three were middle lengths. The following were our recorded observations:

FIRST EXPERIMENT.

With the passage of the *Speedwell* engine and

train, at a medium velocity, or about 20 miles per hour: this showed—

Deflection of joint length		·0625 inch.	
Ditto	middle length	·0425	mean ·0408
Ditto	ditto	·0400	
Ditto	ditto	·0400	

SECOND EXPERIMENT.

With the *Swiftsure* engine, furnished for the experiments: weight on driving wheels, 5 tons 16 cwt.; velocity about 20 miles per hour.

Deflection of joint length		·0800 inch.	
Ditto	middle length	·0320	mean ·0380
Ditto	ditto	·0400	
Ditto	ditto	·0420	

THIRD EXPERIMENT.

The same engine, very slow:

Deflection of joint length		·040 inch.	
Ditto	middle length	·024	mean ·027
Ditto	ditto	·025	
Ditto	ditto	·032	

FOURTH EXPERIMENT.

One trial, quite at rest . . . . . . ·040

The mean of the above three means is . . . . . ·0353

To compare this with the mean deflection of such a bar, with a quiescent load, I may refer to the experiments on the same bars at Woolwich, forwarded for the purpose by the Directors of the Grand Junction line, (Art 182,) by which it appears that the mean deflection per ton, at 33 inches clear bearing,

was ·0050; consequently, for three tons, ·0150; and reducing this to the clear bearing of $45 - 3 = 42$ inches, we have as $33^3 : 42^3 :: \cdot 0150 : \cdot 0314$, the deflection with three tons at rest; and the mean of the preceding deflections in motion is ·0353, a close agreement, which shows, that when every thing is well fixed and secure, the deflection, and consequently the strain, is nearly the same, whether the load be in motion or at rest, and that each rail is only pressed with half the weight of one pair of wheels.

Experiments on the same Bars at Five-Feet Bearing.

FIFTH EXPERIMENT.

SWIFTSURE ENGINE.—VELOCITY ABOUT TWENTY-TWO MILES.

	v. = 22.	v. = 22.	v. = 22.
Deflection, middle length	·093	·077	·080
Ditto joint length	·083	·080	·123
Ditto ditto	·108	·143	·130
Ditto middle length . . .	·082	·070	·077

WITH GREATER VELOCITIES.

	Speedwell.		Fury train.
	v. = 30.	v. = 32.	v. = 23.
Deflection, middle length	·112	·122	·083
Ditto joint length	·080	·105	·085
Ditto ditto	·250	·120	·095
Ditto middle length . . .	·091	·115	·085

In obtaining a mean from these results, the deflections on the joint lengths are, as in the preceding case, rejected, being obviously in excess. The mean of the rest, that is, of the central length, is ·089.

In my experiments at Woolwich, the deflection per ton at 33 inches bearing being ·0050, or, for 3 tons, ·0150, we have, deducting 3 inches from 60, to obtain the clear bearing—

$$33^3 : 57^3 :: \cdot 0150 : \cdot 079,$$

while the mean determined by the deflectometer, as we have seen, is ·089.

Nothing can be expected much more satisfactory; as it is thus proved, *independently of any opinion*, that while the blocks and fixings are secure, the strain from a passing load is but little in excess of that from a quiescent load: whereas the effect on the joint ends amounts, from a mean of the preceding, to ·121, being in excess nearly 40 per cent. This, however, is not all strain, part being due to the looseness of the chair or block.

191. *Continuation of the Experiments on the Deflections of different Rails and Blocks on the Liverpool and Manchester Railway.*

DUBLIN AND KINGSTOWN PARALLEL RAIL.

Weight, 45 lbs. per yard, with a lower web; bearing distance, 3 feet, fixed by vertical keys; depth, $3\frac{1}{8}$.

SWIFTSURE ENGINE.

	Deflections in parts of inches.						Means.
Joint length .	·120	·120	·105	·167*	·177*	·105	·114
Ditto . .	·120	·084	·098	·090	·080	·098	
Middle length .	·125	·110	·130	·130	·156*	·130*	·120
Ditto .	·110	·103	·108	·112	·120	·108	

The deflections marked with an asterisk are re-

markable instances of the effect of the lurching of the engine and carriages, spoken of in the Report as amounting to nearly double the smaller and more natural deflections.

In the above experiments the blocks were sounded, and found firm; the fixings also appeared to be secure at the time of making the experiment; but generally the vertical keys used with this rail require, according to the report of the workmen, incessant attention.

MR. STEPHENSON'S FISH-BELLIED RAIL.

Weight, 43½ lbs. per yard; bearings, 3 feet, fixed by iron keys on the side; greatest depth, 4½; less ditto, 3½.

SWIFTSURE ENGINE.

	Deflections.				
1 Joint length . .	·032	·040	·038	·027	·045
2 Ditto . .	·070	·170	·068	·130	·077
3 Middle length . .	·125	·130	·130	·170	·093
4 Joint length . .	·030	·025	·030	·028	·056

The blocks of Nos. 2 and 3 were loose.

The mean of the other deflections is ·034, but we have no experiments to compare with.

The same Experiments repeated on four other Rails: velocities not recorded.

Middle length	·105	·135	·100	·150	
Ditto .	·035	·050	·047	·053	Mean ·062
Ditto .	·075	·075	·070	·085	
Ditto .	·065	·060	·070	·060	

The great discrepance between the means in these

two sets of experiments is very remarkable; I am quite unable to explain the cause from any fact I am acquainted with.

THE RAILS ON THE ST. HELEN'S LINE.

Parallel, with lower bead; weight, 43 lbs. per yard; bearings, 3 feet.

SWIFTSURE ENGINE.

Joint length . .	·110	·092	·115	·095
Middle ditto .	·060	·075	·100	·068
Joint ditto . .	·070	·080	·148	·135
Middle ditto .	·082	·045	·063	·045

Mean deflection of joint lengths, ·105; of middle lengths, ·067.

MR. BOOTH'S NEW RAIL.

Parallel, with equal upper and lower flanch; weight, 60 lbs. per yard; depth, 4 inches; bearing distance, 3 feet.

SWIFTSURE ENGINE.

Middle length . .	·066	·062	·066
Joint ditto . .	·038	·084	·050
Ditto ditto . .	·100	·042	·144 lurch.
Middle ditto . .	·040	·052	·044

The deflectometers were removed from the above two joint lengths; the other two remained the same.

Middle length . .	·052	·064	·064
New joint ditto . .	·048	·064	·042
Ditto ditto . .	·074	·082	·050
Middle ditto . .	·056	·060	·054

Mean of the four middle lengths, ·056.

Parallel Plain T Rail.—Huyton Plane.

Weight, 50 lbs. per yard; bearing, 3 feet; laid down ten months; depth, 3½ inches.

	Vesta train.	Samson train.	
1st Middle length	·088	·070	
2nd ditto	·072	·066	Mean
3rd ditto	·052	·044	·067
4th ditto	·068	·080	

SWIFTSURE ENGINE.

	Slow.	Velocity 12.	V. 15.	
1st	·064	·084	·082	
2nd	·065	·080	·082	Mean
3rd	·048	·060	·060	·072
4th	·072	·080	·086	

General mean, ·0695.

On Chat Moss.

MR. R. STEPHENSON'S FISH-BELLIED RAIL CHAIR.

Weight, 44 lbs. per yard; 3-feet bearing on wooden sleepers. The four deflectometers were here applied to two blocks and two rails, but not adjacent, and the disturbance on the blocks and rails observed together as below:

SWIFTSURE ENGINE.

	Deflections.				Means.
1 Block	·058	·060	·060	·060	·059
2 Middle rail .	·176	·178	·200	·198	·188
3 Block	·030	·028	·040	·032	·032
4 Joint block . .	·152	·160	·160	·170	·160

2 B

Experiments repeated.

The rails and blocks being now selected so as to have one rail between the two blocks, and the other adjacent, the results were—

	Deflections.					Means.
1 Block . .	·018	·018	·018	·022	·023	·019
2 Rail between	·178	·195	·190	·194	·196	·191
3 Block . .	·050	·056	·060	·056	·060	·056
4 Rail adjacent	·136	·124	·154	·130	·124	·134

These last results, as in the other fish-bellied rails, are very anomalous. In the present instance, we may suppose a great deal is to be attributed to their peculiar situation, as the whole road trembled under our feet as the engine passed; but still the great excess of deflection of the rail, beyond that of the disturbance shown by the block, is very unaccountable, although some of it may be due to the working of the segmental piece in this particular chair. Still, however, after every allowance, I must think there are obvious indications of the rails being much more strained in such a situation as this, than on a good bottom; and should this be verified by further observations, it would certainly be advisable in future, in such cases to strengthen the rails, either by enlarging them beyond the dimensions given in the other part of the line, or, which would amount to the same, preserving the dimensions, and reducing the bearing distance.

The speeds, in the last two sets of experiments, varied from 15 to about 21 miles per hour.

192. *Experiments on the lateral Deflection of Railway Bars.*

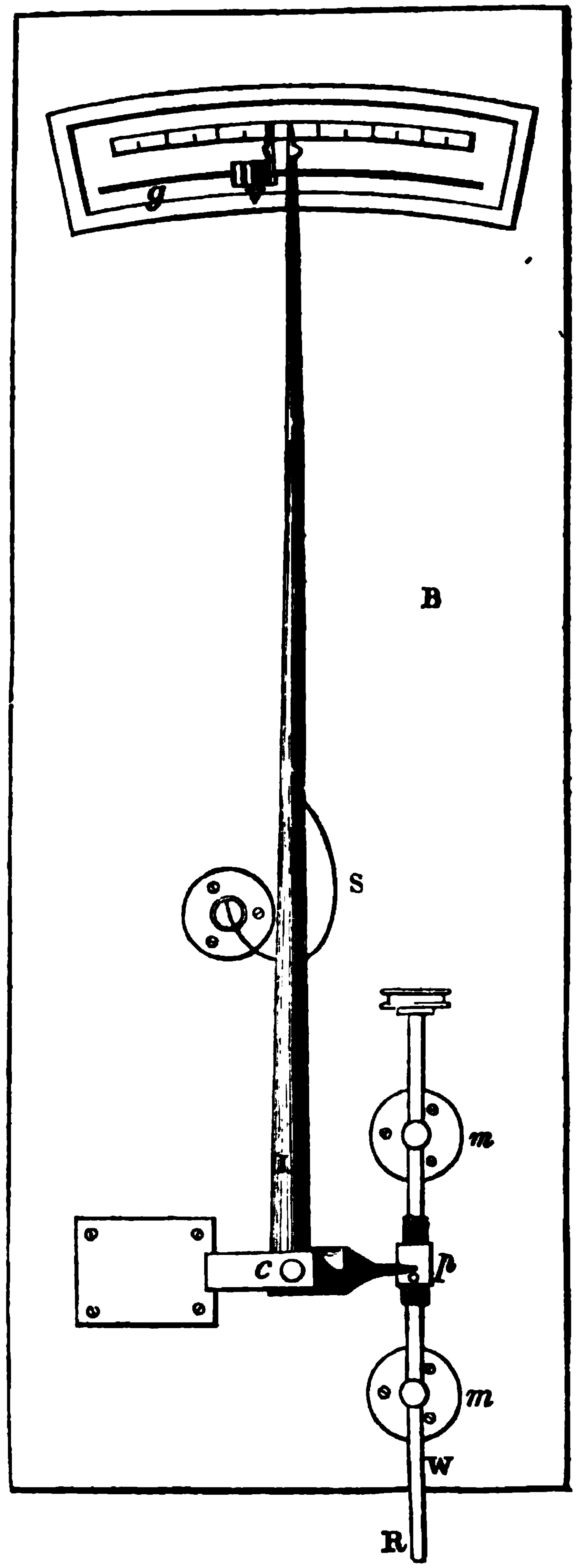
Having ascertained the deflection of the bars in a vertical direction, it occurred to me that it would be very desirable to determine also to what extent the rails were deflected laterally on the outer sweeps of curves, in order that I might, if it should be found necessary, increase the thickness in the longer bearing rails, beyond what mere strength required, in order to counteract this necessarily greater strain.

The whole of these experiments have a tendency to show, that the stress which the bars have to sustain in this direction is not such as to require to be more amply provided for than the increased thickness the bar must have, to meet the greater vertical strain due to the longer bearing. In other words, the additional strength given to the bar, for the purpose of meeting the vertical strain, will be amply sufficient to meet and resist the lateral strain. It will therefore not be necessary, in proportioning the weights and sections of bars for different lengths of bearing, to attend to more than the vertical strength.

The following description of the instrument, and one set of experiments, will be sufficient for illustration.

Description of the Instrument.—In the following

figure, L is a bent lever, turning on a centre *c*; V, a vernier, sliding in the groove *g*; S, a steel spring, to keep the short end of the lever in contact with the stud *p*, to a wire sliding in the standards *m*, *m*, having an adjusting screw at *p*, to set the index to zero. The end R being now brought into contact with the rail, the stud *p*, on the passage of the engine, will press upon the short arm of the lever to the extent of its deflection, the amount of which, ten times multiplied, will be read on the scale or vernier at V.


g
B
S
m
c
p
m
W
R

"The experiments were made on the Wigan Railway, with the engine *Experiment:* the rail parallel weighing 42 lbs. per yard; the bearing distances, 3 feet.

"The instrument being adjusted, the following results were obtained:

	Deflection.	Velocity.	Direction of the engine.
Exp. 1. . .	·047 . .	8 miles per hour.	Back.
2. . .	·045 . .	10 „	Forward.
3. . .	·038 . .	11 „	B.
4. . .	·036 . .	12 „	F.
5. . .	·040 . .	10 „	B.
6. . .	·035 . .	12 „	F.

"The same experiment repeated, after the middle chair between two others was removed; the clear bearing now being 5 feet 10½ inches:

	Deflection.	Velocity.	Direction of the engine.
Exp. 1. . .	·070 . .	4 miles per hour.	Back.
2. . .	·078 . .	6 „	Forward.
3. . .	·093 . .	7 „	B.
4. . .	·097 . .	8 „	F.

Continuation of the Experiments on lateral Deflection, made on the Wigan Railroad, 10*th September,* 1835. *By Mr. Edward Woods.*

"The rails are of the parallel form; weight, 42 lbs. per yard; bearings, 3 feet.

" *1st Series.*—On the curve near the junction to the Liverpool and Manchester Railway.

Curve = 2 feet 4 inches per chain.
= to a radius of 622 yards.

" The outer rail of the curve $1\frac{3}{8}$ inch higher than the inner rail, to counteract the centrifugal force of the trains.

" Deflection (lateral) of an outside rail, 1 ft. 6 in. from the bearing. Engine, *Experiment*.

	Defl. in inches.	
No. 1.	·040	10 miles per hour.
2.	·024	8 ditto.
3.	·026	8 ditto.
4.	·022	14 ditto.
5.	·007	10 ditto.

" *2nd Series.*—Another rail on the outside of the curve, same engine, &c. as before.

	Defl. in inches.	Miles per hour.	
No. 1.	·000	13	F.
2.	·018	10	B.
3.	·000	9	F.
4.	·023	9	B.
5.	·017	11	F.
6.	·060	8	B.
7.	·031	10	F.
8.	·055	9	B.
9.	·042	12	F.
10.	·086	11	B.

" N. B.—The letters F. and B. denote whether the engine was working forwards or backwards.

" *3rd Series.*—With a rail exactly opposite that of the second series, viz., on the inner rail of the curve.

"In this and in all the other experiments, the deflection was measured outwards from the centre of the road.

"In this instance the deflection seemed to arise solely from the wedge-like action of the conical tire on the wheels, as some paint which had been smeared for a few yards on the inner side of the rail had not been wiped off; showing that the flanch had not come into contact with the rail. Engine, the *Experiment.*

	Deflect. inches.	Miles per hour.	
No. 1.	·030	8	B.
2.	·030	9	F.
3.	·040	9	B.
4.	·040	10	F.
5.	·030	4	B.
6.	·000	2	F.
7.	·037	3	B.
8.	·002	2	F.
9.	·033	3	B.
10.	·001	2	F.
11.	·006	6	'Jupiter,' with a coach train.

"*4th* and *5th Series* are given in the Report.

"*6th Series.*—With a rail on the straight road. Engine, the *Experiment.*

	Deflect. inches.	Miles per hour.	
No. 1.	·010	8	B.
2.	·010	14	F.
3.	·010	15	B.
4.	·007	10	F.

"*7th Series.*—Another rail near the same place. Engine, the *Experiment;* weight of working wheels, 5 tons 15 cwt. 1 qr.

	Deflect. inches.	Miles per hour.	
No. 1.	·032	16	B.
2.	·032	12	F.
3.	·020	13	B.
4.	·010	5	F.
5.	·008	4	B.
6.	·010	4	F.
7.	·046	25	B.
8.	·020	18	F.

(Signed) "EDWARD WOODS."

As the velocities are not the same in these experiments, except the first of the first series and the last of the second, we can only make this one comparison, and by this the deflection appears to be about double, which is certainly less than calculation would lead us to expect; but the amount is so far within the elastic power of the iron, and the strength of the rail experimented on so inferior to what will probably be adopted, that I am quite satisfied no additional strength will be required to meet this strain.

The above experiments were made by Mr. Edward Woods and Mr. King, in the presence of T. W. Rathbone, Esq., Dr. S. Trail, of Edinburgh, and J. Reynolds, Esq., of Swansea.

DEDUCTIONS.

193. It would be useless to go through a comparison of all the experiments noted in this and the preceding section; I shall therefore only observe, referring to the vertical deflections, that the obvious deduction from them is, that with firm blocks, chairs well fixed, and with joints well made, the road itself being firm, the rail is only deflected at the greatest velocity a little more than is due to a quiescent load equal to half the weight on the two wheels; but that in consequence of the imperfection of these parts, a strain is occasionally thrown on the rail which produces a deflection about double that which belongs to the load in question. This effect was frequently and obviously exhibited in the experiments with the trains. In many cases the deflectometer showed only the common amount of deflection when the engine (by far the heaviest load) passed over; whereas, perhaps in the middle, or at the end of the train, a waggon would lurch over from some irregularities, and throw up the index to double its former amount. This effect was very particularly noticed by the Deputation, Directors, Proprietors, and other parties present. It follows, therefore, that till greater perfection can be obtained in railways, a strength of bar more than double that due to the mean strain must be provided. In my original Report I have allowed 50

per cent. beyond the double as a surplus; but from these experiments, it appears that this allowance is in excess, and that from 10 to 20 per cent. beyond the double will be sufficient.

194. *On the proportional increased Strength with increased Distance of Bearings.*

In apportioning the quantity of metal for each length, regard must of course be had to the limits prescribed by practice, that is, we must only employ such sections as may be subject to no substantial practical objections; but with this condition, the form of section is unlimited.

The first limitation which practice enforces is, that whatever be the bearing length and weight of rails, the head ought to have the same certain weight.

It is not necessary to go far along the Liverpool and Manchester line to see that the heads of the original 35-lbs. elliptical rails are far too small for the present weight of the engines, the outside flanch of the upper table being, in numerous instances, nearly separated from the central rib. The Dublin 45-lbs. parallel rail, which has a broader and somewhat larger head, does not show the same defects; still, however, it is generally, I find, considered too small. The 50-lbs. parallel plain T rail, and the Grand Junction rail, are perhaps the best propor-

tioned heads in the line; their area of section, to an inch deep, occupying about 2¼ square inches. In the following calculations, therefore, I shall lay it down as a practical limit, that the head ought not to occupy less than 2·25 inches area, or, which is nearly the same, not weigh less than 22·5 lbs. per yard.

Another practical limit, in which I believe most engineers agree, is, that the depth of the rail ought in no case to be more than 5 inches.

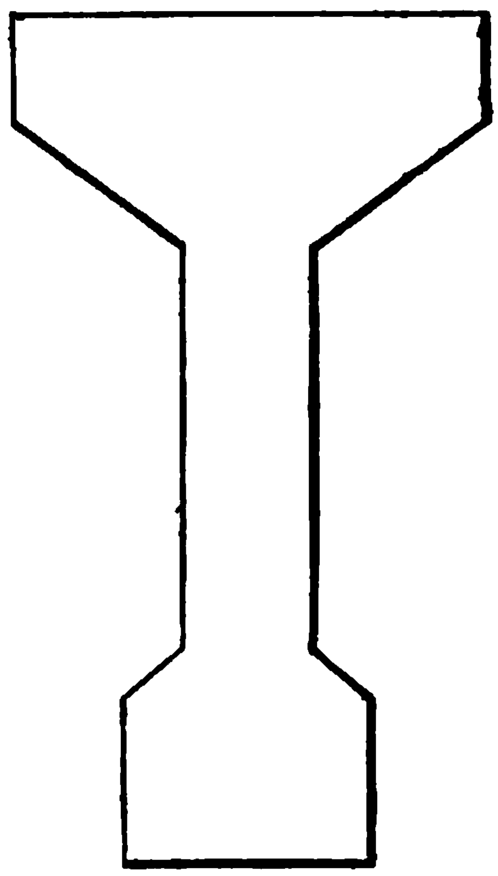
Abiding, therefore, by these conditions, I propose to compute the weight of iron per mile, on four lines of rails, preserving in all cases a constant strength of 7 tons, at the several bearing lengths of 3 feet, 3 feet 9 inches, 4 feet, 5 feet, and 6 feet; distributing the iron in each bar most economically for strength.

The lightest rail in the line, which appears to approach towards the required degree of strength, is the Dublin parallel rail, of 45 lbs. per yard; but as the head is lighter than the present practice seems to point out as the best, I would increase this by 2½ or 3 lbs., and with a little addition to the rail itself, make the whole about 52 lbs., which is, perhaps, the least weight that ought to be given to a rail on 3-feet bearings; and the best disposition of this weight, according to the solution of the problem on the principle of *maxima* and *minima*, regard being had to the practical limits above stated, is given in Art. 174; and on similar principles, although not

strictly following the minutia of the solution, have been arranged the proportions for the other bearings, the section at half-size and the several particulars being as follow:

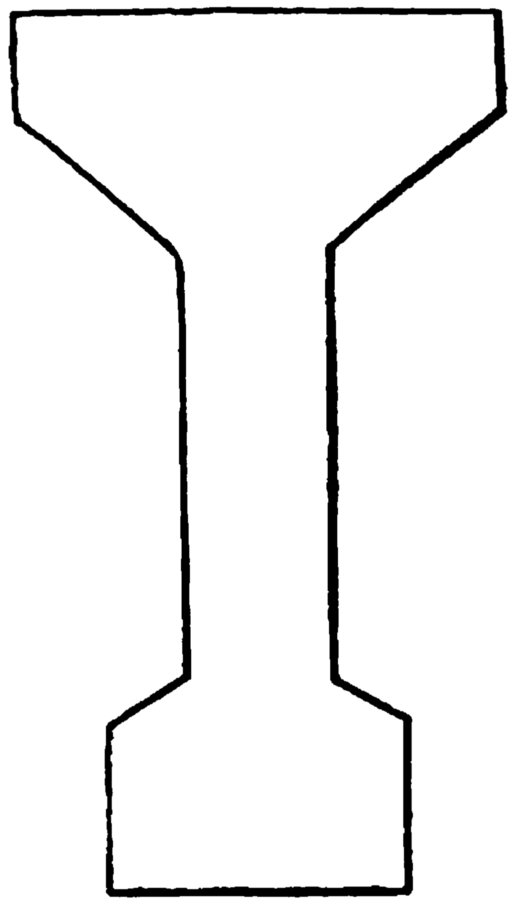
Section for a Three-Feet Bearing.

ON A SCALE OF HALF THE LATERAL DIMENSIONS.



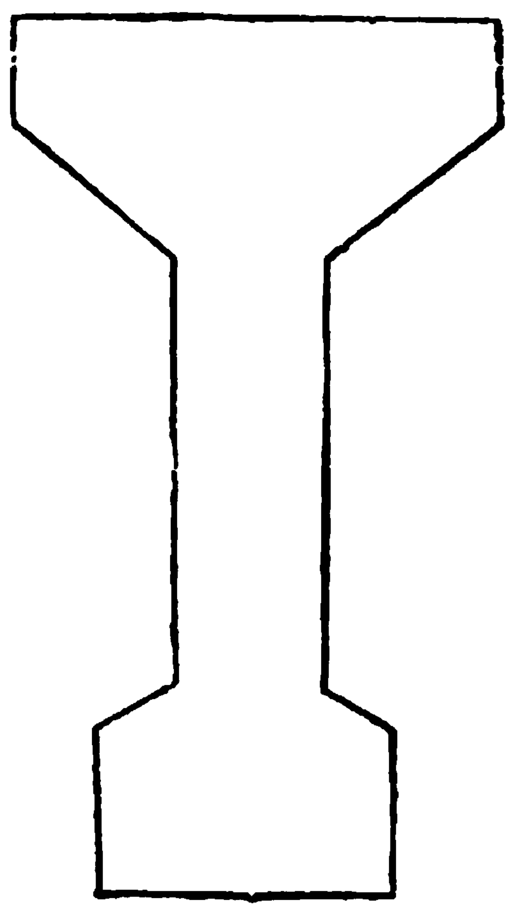
Head to 1 inch depth, 22·5 lbs. per yard; whole depth, 4½ inches.
Ditto bottom web, 1 inch.
Breadth ditto, 1·25 inch.
Thickness of middle rib, ·6 inch.
Whole weight, 51·4 lbs. per yard.
Strength, 7 tons.
Deflection with 3 tons, ·024 inch.

Section for a Three-Feet Nine-Inch Bearing.



Head to 1 inch depth, 22·5 lbs. per yard.
Whole depth, 4⅝ inches.
Ditto of bottom web, 1 inch.
Breadth ditto, 1½ inch.
Thickness, middle rib, ·75 inch.
Whole weight, 58·8 lbs. per yard.
Strength, 7 tons.
Deflection with 3 tons, ·037 inch.

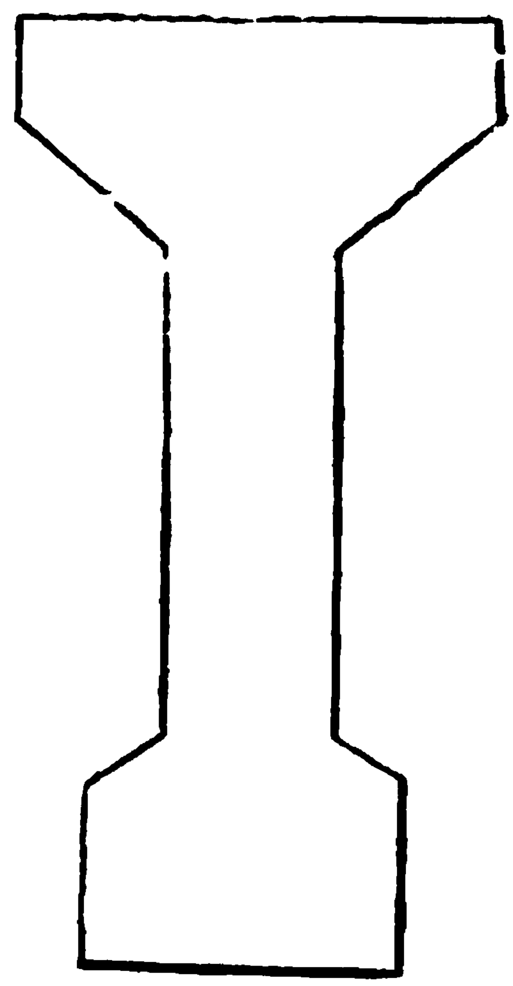
Section for a Four-Feet Bearing.



Head to 1 inch depth, 22·5 lbs. per yard.
Whole depth, 4¾ inches.
Ditto of bottom web, 1 inch.
Breadth of ditto, 1½ inch.
Thickness of middle rib, ·8 inch.
Whole weight, 61·2 lbs. per yard.
Strength, 7 tons.
Deflection with 3 tons, ·041 inch.

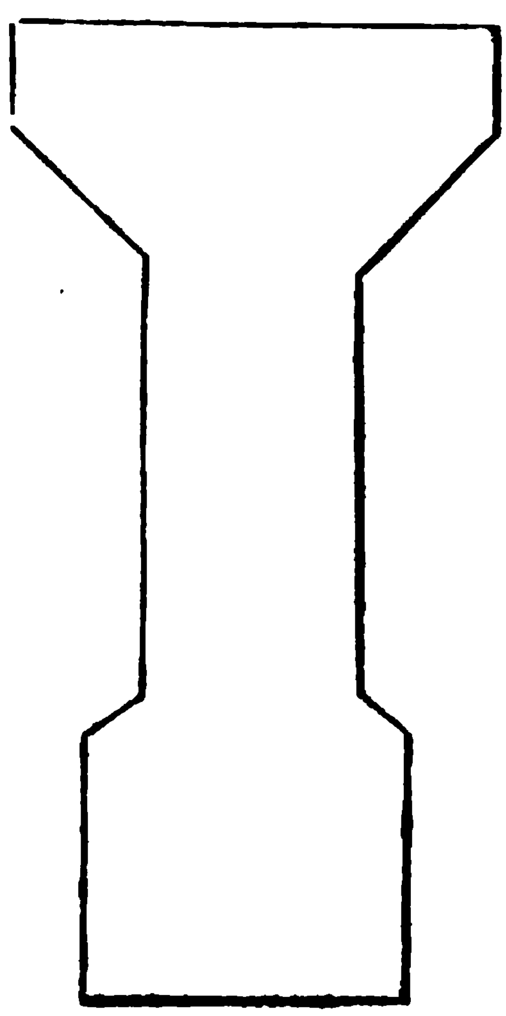
Section for a Five-Feet Bearing.

Head to 1 inch depth, 22·5 lbs. per yard.
Whole depth, 5 inches.
Ditto of bottom web, $1\frac{1}{2}$ inch.
Breadth of ditto, 1·66 inch.
Thickness of middle rib, ·85 inch.
Whole weight, 67·4 lbs. per yard.
Strength, 7 tons.
Deflection with 3 tons, ·064 inch.



Section for a Six-Feet Bearing.

Head to 1 inch depth, 22·5 lbs. per yard.
Whole depth, $5\frac{1}{10}$ inches.
Ditto of bottom web, $1\frac{1}{2}$ inch.
Breadth of ditto, 1·66 inch.
Thickness of the middle rib, $1\frac{1}{8}$ inch.
Whole weight, 79 lbs. per ton.
Strength, 7 tons.
Deflection with 3 tons, ·082 inch.



It will be seen, by the above statement, that although I have preserved the same strength or resistance in each of the rails, the longer bearings are less stiff than the shorter; indeed, unless this increased deflection be allowed, all thoughts of greatly increasing the distance of the bearings must be given up; for, in order to preserve a proportional deflection, either the breadth of the rail must be so increased as to require a weight of iron altogether inadmissible, or the depth must be increased in the same proportion as the length of bearing, which is impracticable. The deflections, however, of the longer bearings, although greater than the shorter, do not amount to a large quantity; the deflection of several of the rails at present on the line being much greater, as may be seen by referring to the several experiments on this subject.

On the Best Form of Rail.

195. In the sections given in a preceding page for rails at different lengths of bearings, it will be seen that I have confined the breadth of the lower web to $1\frac{1}{2}$, or, at most, to $1\frac{2}{3}$ inch; and this has been done, although I am well aware that, to extend the breadth of the lower web, and to reduce its depth, would theoretically give the strongest rail; in fact, that the double T is, on paper, a stronger rail than the deep and less broad flanched rail, but I am quite

convinced it is not so in practice. The lower web comes no other way into use than as it is brought into a state of tension by the action of the centre rib; and, although the fibres of the lower web lying immediately below the centre rib are brought into action by it, and these fibres excite a similar action laterally in those immediately contiguous to them, and these again to the next, and so on, yet in a ductile metal like malleable iron this lateral effect is soon lost; so that the extreme fibres of the extended lower flanch become inefficient.

The fact is, this particular form of rail was proposed with a view to a certain advantage it was supposed to possess, viz., that it might be turned when the upper table had been worn down, but this has been shown in my former Report to be impracticable; and not fulfilling this condition, while in other respects it is disadvantageous, it should be at once rejected: I know it is said it may still be turned and used in side rails; but I reply, wherever it is used, it will be strongest if not turned. Again, it is stated, that both sides being alike, the rail-layers may select the side that fits best; but it would surely be better to have the rails made so uniform that no such choice was requisite. Again, it gives a broad bearing, in which, however, I see no advantage when carried to excess. And, lastly, it admits of the rail being fixed by a wooden key or wedge; but is it not better, if possible, to avoid the wedge altogether? In fact, I can see no advantage this

form of rail possesses, to compensate for its actual and obvious defects.

The proportions I have shown in the preceding diagrams, which resemble nearly the form of rail to which the prize was awarded, make, I am persuaded, the strongest and best rail; it being of course understood that these diagrams give only angular outlines, the salient and re-entering angles of which may be softened down or fortified according to the taste or other considerations of the engineer.

To convince Mr. Locke, and some other gentlemen, of the defect of the double ⌶ form, I had one of the rails taken up, and ½ an inch cut away on each side from the lower flanch, reducing its breadth at the point of greatest strain, that is, in the middle of the bar, to 1½ instead of 2½ inches. It was then put into the press, and the strains brought on as usual, under the superintendence of Mr. Edward Woods and Mr. John Gray; Mr. Locke himself being obliged to leave just at the time the experiment was in progress.

Mr. Rathbone, Mr. Edward Cropper, and myself were also present, and the result was, that the bar thus mutilated showed greater strength than the mean strength which Mr. Locke found to belong to it when whole. Now, although I am ready to grant that the bar was actually weakened, and that this apparent anomaly is attributable to the imperfection of the press already pointed out, yet, on the other hand, it must be admitted that it could, with

such a result, have lost but little of its strength, and that the iron thus abstracted, viz., nearly $\frac{1}{8}$th of the whole section, if judiciously introduced elsewhere, would undoubtedly give a much stronger rail.[2]

[2] It is since this was written that the experiments have been made on the Southampton rails, which are still more objectionable from their extended lower web; but it must be admitted that these, where the iron was good, did not indicate the weakness anticipated from their extensions.

APPENDIX:

CONTAINING

THEORETICAL INVESTIGATIONS ON THE EFFECT OF THE DEFLECTION OF RAILS, INCLINED PLANES, GRADIENTS, &c.

To determine the Influence of the Deflection of an Elastic Bar to the Motion of a Body passing over it, the Bar being supported at its two extremities.

1. LET A C B represent an elastic bar, supported at its middle point, and loaded at its extremities with two equal weights, w, w. Then the deflection of the two ends will be exactly the same as that of the same bar supported at its ends and loaded with a weight $2w$ at its middle point.

Fig. 1.

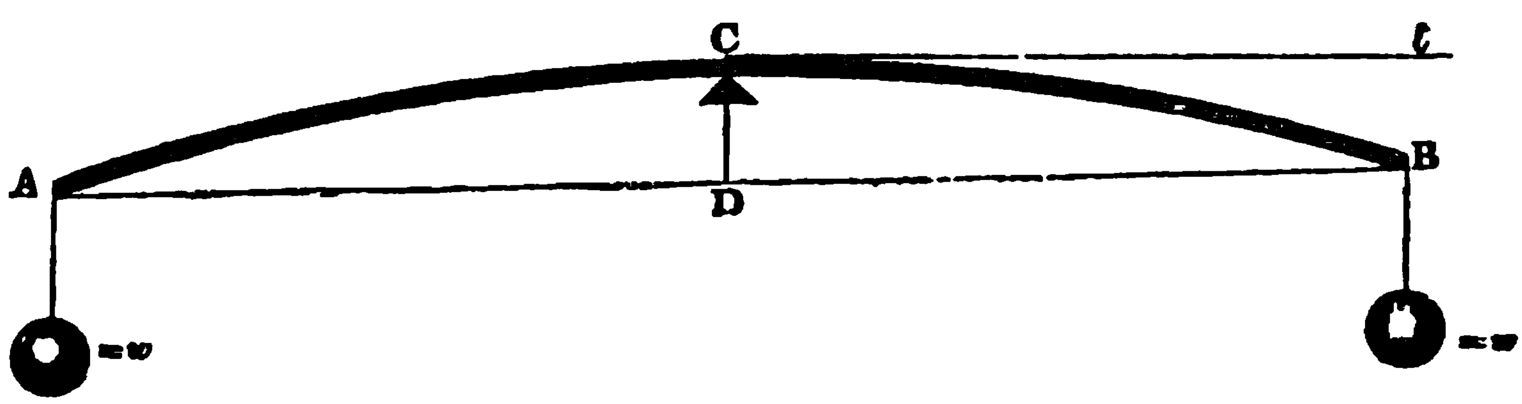
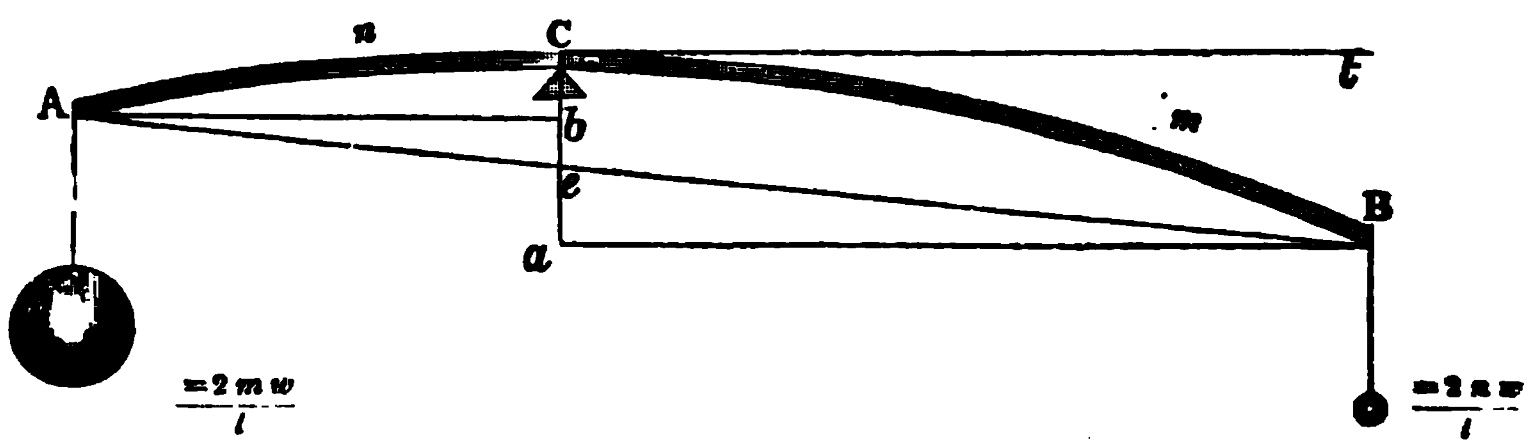


Fig. 2.



2. Let ACB, fig. 2, be the same bar supported at any point C, dividing the beam into two lengths m, n, and loaded at B by a weight $\frac{2nw}{l}$, and at A by a weight $\frac{2mw}{l}$ (l being the whole length), so that the beam may be still in equilibrio on the support C, and the sum of the two weights equal to $2w$, as before. Then Cb will be the deflection of the point A, and Ca, that of the point B, Ce being a mean deflection, as referred to the oblique line AB; and this deflection Ce will be the same as if the beam was supported at A and B in a horizontal line, and loaded at C with a weight $2w$, the deflections being considered as very small in comparison with the length.

In fig. 1, let the element of deflection at C be Δ, then the whole deflection, being as the element of deflection into the square of the length, we may represent $CD = \delta$ by $\frac{1}{4}l^2\,\Delta$. But the element of deflection in the same beam is as the strain; and the strain at C in fig. 2, is to that in fig. 1, as $mn : \frac{1}{4}l^2$. Therefore, in fig. 2,

the element of deflection $\Delta' = \frac{4mn}{l^2}\Delta$,

and the deflection $Ca = \frac{4m^2n}{l^2}\Delta = \delta'$,

the deflection $Cb = \frac{4mn^2}{l^2}\Delta = \delta''$,

and $ba = \frac{4mn(m^2 - n^2)}{l^2}\Delta = \delta' - \delta''$.

Consequently, the sine of the inclination, or of the angle ABa

$$= \frac{4mn(m^2 - n^2)}{l^3}\Delta.$$

And this is precisely the inclination the tangent Ct would have, if the beam were turned about C till AB became horizontal, and therefore the same as the tangent Ct would have, if the beam were supported at its ends, and loaded at C with a weight $2w$; and it is this inclination which forms the impediment to the motion of the body along the plain face of the bar.

3. To find the point where this inclination is the greatest, we have

$$\begin{cases} m + n = l \\ m\,n\,(m^2 - n^2) = \text{a max.} \end{cases}$$

$$\text{or,} \quad m\,(l - m)\,(2\,l\,m - l^2) = \text{a max.}$$

$$\text{or,} \quad -\,2\,l\,m^3 + 3\,l^2\,m^2 - l^3\,m = \text{a max.}$$

$$\text{whence,} -\,6\,l\,m^2 + 6\,l^2\,m - l^3 = 0,$$

$$m^2 - l\,m = -\tfrac{1}{6}\,l^2$$

$$m = \tfrac{1}{2}\,l\,(1 \pm \sqrt{\tfrac{1}{3}})$$

$$n = \tfrac{1}{2}\,l\,(1 \mp \sqrt{\tfrac{1}{3}}).$$

When m and n have these values, the inclination of the tangent is the greatest, and consequently at that point the resistance to the motion is the greatest. It is shown that the sine of the angle of inclination is expressed generally by

$$\frac{4\,m\,n\,(m^2 - n^2)}{l^3}\,\Delta.$$

Calling $l = 1$, this is $\frac{2}{3} \times \sqrt{\frac{1}{3}} = \cdot 384\ \Delta$.

Now the sine of the inclination of a plane of half the length of the bar, viz. $\frac{1}{2}\,l$, whose altitude is equal to the central deflection, viz. $\frac{1}{4}\,l^2\Delta$, (with which this case is frequently but erroneously confounded,) would, when $l = 1$, be proportional to $\dfrac{\frac{1}{4}\,l^2}{\frac{1}{2}\,l}\ \Delta = \cdot 5\ \Delta$. That is, the greatest resistance a heavy load experiences in consequence of the deflection of the bar over which it passes, is to the constant resistance it would experience in ascending an inclined plane whose height is equal to the central deflection, as ·384 to ·50, or nearly as 3 to 4. The former, moreover, acts only for an instant, and begins and terminates in zero, while the other remains constant throughout.

To compare the sum of all the resistances in the two cases, let us consider still $l = 1$, then the general expression for the resistance at any point, viz.

$$\frac{4\,m\,n\,(m^2 - n^2)}{l^3}\,\Delta$$

$$\text{becomes } 4\,\Delta\,(-\,2\,m^3 + 3\,m^2 - m),$$

and this multiplied by the differential of m,

$$\text{gives } 4\,\Delta\,(-\,2\,m^3 + 3\,m^2 - m)\,d\,m,$$

the integral of which between the values

$$m = \tfrac{1}{2} \text{ and } m = 1, \text{ is } \tfrac{1}{8}\,\Delta\,;$$

while the sum of all the constant resistances ·5 Δ for the half-length

$$= \tfrac{1}{2} \times \tfrac{1}{2}\,\Delta = \tfrac{1}{4}\,\Delta.$$

That is, the sum of all the variable resistances to a load by the deflection of the bar over which it passes, is exactly half the resistance the load would experience in ascending a plane of the same half-length, and whose height is equal to the central deflection of the same bar.

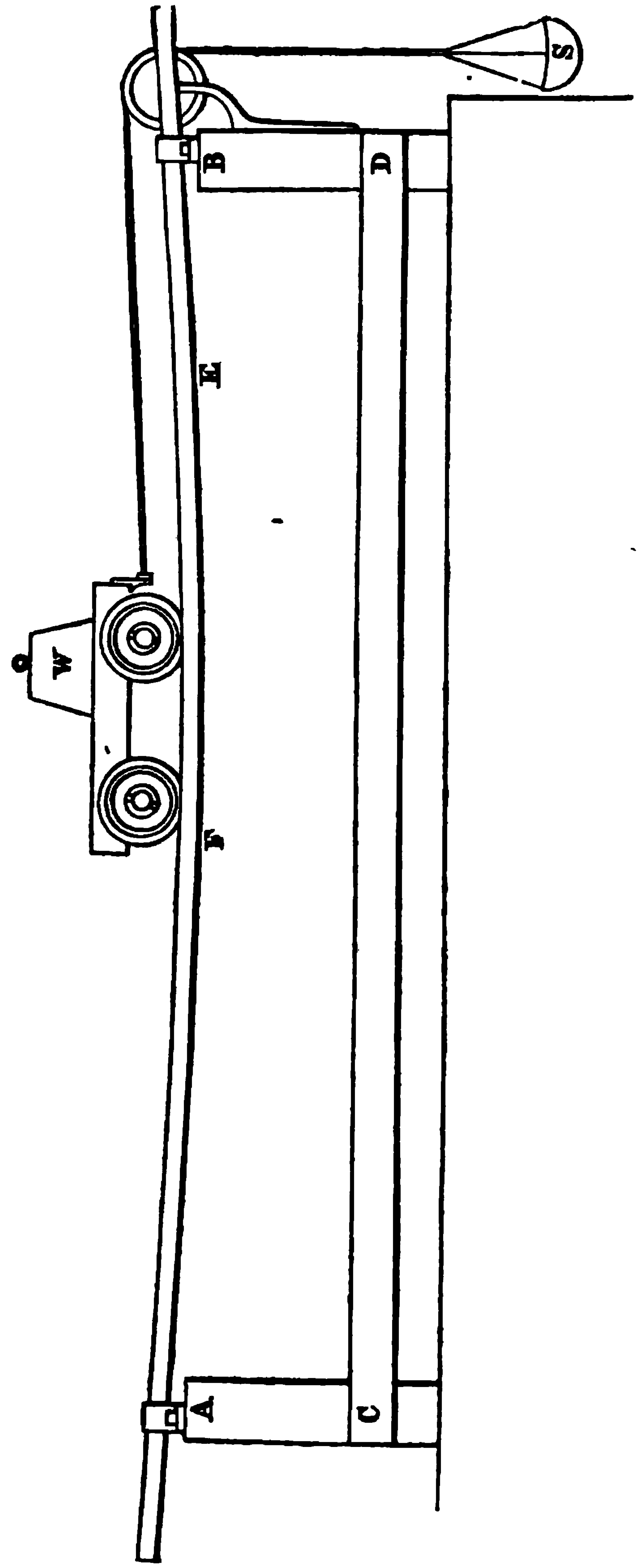
Now the resistance on such a plane, the central deflection being δ, which is to be considered the height of the plane, its length being $\frac{1}{2} l$, is $\frac{2\delta}{l}$, consequently the resistance of a bar only deflected to the same extent will be $\frac{\delta}{l}$.

4. It will be understood that this is the resistance to the ascent of the body from the middle of the bar up to the prop; and if, as has been assumed by some persons, as much power was gained in the descent as was lost in the ascent, the odds would be made all even, and the deflection of the bar would be no impediment; but this assumption is altogether erroneous, both in theory and practice. In fact, the gain from descent is so exceedingly small in such short planes as we are here considering, that it may be wholly rejected; so that in a plane supposed perfectly horizontal, the retardation, or additional resistance to the carriages, caused by the deflection of the bar, will be equivalent to the carriage being carried up a plane of half the whole length on a slope equal to $\frac{\delta}{l}$, the other half being horizontal, or, which is the same, on one entire ascending plane, whose slope is $\frac{\delta}{2l}$, where l is the distance between the props, and δ the central deflection. Having, thus, the resistance due to deflection estimated on a continually rising plane, the resistance per ton becomes known, and consequently the exact numerical increase of engine power which is necessary to overcome that resistance. Computing in this way, it appears that the effect of deflection on the several bars whose sections are given in p. 397 et seq., produced resistances equivalent to planes of the following slopes; viz.

Bearing distances.		Deflections.	Equivalent planes.	Increase power per ton.
ft.	in.			lbs.
3	0	·024	1 in 3000	·75
3	9	·037	1 in 2432	·92
4	0	·041	1 in 2341	·95
5	0	·064	1 in 1875	1·2
6	0	·082	1 in 1756	1·3

5. These being important considerations in the economy of railways, and feeling that what is perfectly satisfactory to a mathematician cannot be equally so to persons not in the habit of following such trains of reasoning, I had a little model made, representing one length of rail, the distance of the supports being 30 inches: the bars are drawn steel, $\frac{1}{4}$ inch by $\frac{1}{8}$; the load with the carriage weighs 134 ounces, and the deflection with that weight is nearly $\frac{1}{2}$ an inch. The model is represented in the following page, with the scale in which weights are placed for illustrating the points in question. From A to B was laid a well-planed piece of wood, on which, in the first instance, the railway bars were secured at their proper parallel distance. The end of the model A being now raised, this plane was made to be truly horizontal; weights were then gradually put into the scale till that weight was found which just balanced the friction, and which was found to be exactly 5 ounces, including the scale.

The model was then placed in its natural position, the base CD accurately levelled, and the carriage placed on the unsupported bars, the weight being thrown as nearly as possible over the front wheels only; 5 ounces due to friction were introduced, and weights gradually added: as each ounce was introduced the carriage advanced, and with 16 ounces it rose over the point E, where the resistance was the greatest, and was then accelerated to the end. E, according to the preceding investigation, was a little beyond the half of the half-length, and the same was distinctly indicated by the experiment. At the lowest point of the curve the resistance was the same as on the horizontal plane, as it was also at the end B, which are both likewise consistent with the investigation.


S
B
D
E
W
F
A
C

The bars were now removed, and the plane already mentioned placed from A to B, inclining so that the bars passed exactly through the point F, when it was found that the weight necessary to balance the carriage and friction was $19\frac{1}{2}$ ounces. The greatest resistance, therefore, on the deflected bars was to the resistance on this plane as

$$(16-5) \text{ to } (19\tfrac{1}{2}-5), \text{ or as } 11 \text{ to } 14\tfrac{1}{2},$$

which is also very closely approximative to what is given by the theory. The only doubt, therefore, which can remain, is how far I ought to reject as inconsiderable any increase of power on the descending side. This point cannot be met experimentally, and I am therefore obliged here to depend only on demonstration. The case certainly involves no great difficulty of conception as a mere question of theoretical mechanics: having, however, been treated on different principles by persons of considerable scientific eminence, I should have been glad to have exhibited the effect experimentally; but as the whole turns upon velocity, this is impossible. The demonstration alluded to is involved in the principles explained in the following section.

On the Laws which govern the Action of Locomotive Engines on Railways.

6. At this time, when a novel application of a powerful mechanical agent is being made over so many miles of this country, and different public companies are competing with each other to effect the same object by different lines, it is desirable that some certain rule should be established of estimating the effects of the same engine on different loads, and of the several ascending and descending planes which necessarily occur in all, in order thereby to form a just comparison of their respective mechanical merits. These questions have been examined by different writers, but unfortunately without coming to any fixed conclusion; in fact, both the theory and practice in this branch of mechanics involve points of consideration which are liable to lead to some discrepancies, according to the views which may be taken of them.

One of the prevailing defects in many of these solutions is, that of assuming that the engine power required for different loads on a horizontal plane is proportional to the power of trac-

tion requisite to produce the motion; whereas the expense of engine power has no definite ratio to the force of traction, in consequence of the different forces which must be overcome before any motion can be impressed on the load.

Thus, for example, before any motion can be produced on the load, whether it be great or small, the following resistances must be overcome: viz.

1st. The friction of the engine gear.

2nd. The friction of the wheels and axles of the engine and tender.

3rd. The pressure of the atmosphere upon the surface of the pistons.

The power or quantity of steam thus expended every stroke of the engine, before any effect can be transmitted to the load, is very considerable, in many cases quite as much as is employed for actual traction.[1]

7. Amongst the writers who have contributed most to elucidate the laws of action in locomotive engines, we ought to distinguish M. Pambour, a French engineer, who, after many judiciously conducted experiments on the Liverpool and Manchester and on the Darlington lines of railway, has arrived at numerical results which appear in every respect to be entitled to entire confidence: according to these—

1st. The friction of the engine gear alone, that is, without a load, amounts on an average of several engines, to 6 ℔s. per ton of the weight of the engine, as applied to the circumference of the wheel.

2nd. That the friction of the wheels, axles, &c. of the engine and tender is 9 ℔s. per ton.

3rd. That the friction of the waggons, without the engine and tender, is 8 ℔s. per ton, including the weight of the waggons and load.

[1] Our engineers are in the habit of speaking of the power of high-pressure engines by the pressure of the steam as exhibited or limited by the safety-valve, that is, by the pressure above the atmosphere, and this is quite correct while comparing the effective power of different engines; but in estimating the expenditure of steam to produce this disposable power, the whole elasticity of the steam must be considered.

4th. That the friction on the engine gear is, at a medium, 1 lb. additional per ton for every ton weight of the load and waggons.

5th. M. Pambour, who, as far as I know, is the first writer who has distinctly introduced the pressure of the atmosphere on the pistons, estimates that pressure at 14·7 lbs. per square inch.

6th. Lastly, it is assumed, that equal quantities of steam are producible in equal times; and consequently, that the pressure on the piston, at any time, is inversely as the velocity.

8. Let now

W denote the tons weight of the engine.

w the tons weight of tender.

L the tons weight of the waggons and load.

L′ the gross load, including the engine, tender, &c.

Then the force necessary to be applied at the circumference of the wheel to balance these resistances alone, will be

$$6\,W + 9\,(W + w) + 9\,L = 6\,W + 9\,L'.$$

To this is to be added the pressure of the atmosphere, or its resistance to the motion of the pistons, viz.

$$\tfrac{1}{2} d^2 \pi \times 14{\cdot}7,$$

$d^2 \pi$ being the area of one piston in inches, and 14·7 the number of lbs. pressure per inch.

But this last resistance being only overcome with the velocity of the piston, must be transferred to the circumference of the wheel, where the other resistances are estimated. Taking therefore D to denote the diameter of the wheel, and l for the length of stroke, we have

$$D\pi : 2l :: \tfrac{1}{2} d^2 \pi \times 14{\cdot}7 : \frac{14{\cdot}7\, d^2 l}{D},$$

which is the force that must be applied at the circumference of the wheel to balance the pressure on the piston.

Let this be denoted by A, then the whole force requisite to balance the resistance on a horizontal plane is

$$A + 6\,W + 9\,L'.$$

And as the sum of the first two terms is constant, call

$$A + 6\,W = C,$$

then the whole resistance will be expressed simply by

$$C + 9\,L'.$$

And suppose that the observed horizontal velocity with this load is v, and it be required to determine the velocity the same engine would impress on a gross load L″, we should have

$$(C + 9 L') v = v' (C + 9 L'').$$

$$\text{Whence} \quad v' = \frac{C + 9 L'}{C + 9 L''} v.$$

9. In an observed experiment, let the weight of the engine W = 12 tons, of the tender w = 6 tons, and L = 82 tons; and consequently L′ = 100 tons, and the velocity v = 25 miles per hour. And in another case, let the load be one-half, or 41 tons, and therefore the gross load L″ = 59 tons; and let the dimensions of the engine be as follows, viz., diameter of piston 12 inches = d, the length of stroke $l = 1\frac{1}{2}$ foot, and diameter of drawing wheels D = 5 feet.

$$\text{Then } A = \frac{14 \cdot 7\, d^2\, l}{D} = 635 \text{ lbs.}$$

$$6 W \qquad = \underline{\ 72\ }$$

$$\text{Then } C \qquad = 707$$

$$\text{And in the first case } 9 L' = 900 \text{ lbs.}$$

$$\text{in the second } 9 L'' = 531 \text{ lbs.}$$

And substituting these numbers in the above expression, we find

$$v' = \frac{C + 9 L'}{C + 9 L''} v = 32\tfrac{1}{2} \text{ miles.}$$

So that diminishing the load by one-half only increases the velocity about $7\frac{1}{2}$ miles per hour.

If, on the other hand, the velocity v = 25 was that observed on the half-load, we should have

$$v = \frac{707 + 531}{707 + 900} \times 25 = 19\tfrac{1}{4} \text{ miles.}$$

That is, the double load is carried by the same engine, and with the same expenditure of power, at nearly $\frac{4}{5}$ths the speed of the single load,—results which are by no means inconsistent with practical experience.

On the Effect of Gradients.

10. As some difference of opinion exists on this subject, probably arising more from imperfect definition than from any other

cause, it may be well to examine the subject rather more in detail than would be otherwise requisite.

Let us therefore take a very simple theoretical case, by supposing a body free from friction and resistance to be moving along a horizontal plane with a certain velocity, which we may assume to be 32 feet per second, and that it arrives at the foot of a plane, rising 16 feet; then, by the known laws of mechanics, the body in this particular case will arrive at the top of the plane, and at that point will have lost all its velocity; but if there it meets an equal descending plane, it will, in its progress down, acquire at the bottom the same velocity it had at first. In this respect, therefore, it may be said to have lost no force, because its first and last velocities are equal; but, as the time of the body ascending one plane and descending the other, will be double that with which it would have passed over the same horizontal distance with its first velocity, it will have *lost time;* and a loss of mechanical effect is thus sustained.

11. If now, instead of a body free from friction and resistance, we take the case of a locomotive engine, moving with the same velocity, and suppose it to possess, within itself, a power so exerted as just to balance the friction at all velocities, that is, as acting upon the piston throughout the journey with a uniform pressure, then this body will not mechanically differ from the former; that is, it will ascend and descend the plane according to the same laws, and there would be still no loss of power, but a loss of time only; for, according to this view of the question, the quantity of steam power expended would be the same as if the body had passed along the base of the two planes (rejecting the difference in the length of the base and plane itself as altogether inconsiderable).

It will be observed, however, that the nature of the steam power thus assumed, is not that which occurs in the actual machine; for, as the steam itself can only be generated at a certain rate, it follows, that its pressure will vary according to the rate of motion, and therefore, instead of being applied, as supposed above, only to overcome the friction, it will act on the ascending plane to aid in the ascent; and, on the other hand, on the descending plane the natural gravitating power will assist in overcoming the friction. The two forces thus act conjointly,

and being subject to different laws, the question of gain or loss of power becomes rather complicated. If we examine our first two supposititious cases, it will be found, that the restoration of the original velocity depends upon the time of ascent and descent being equal, so that all the velocity lost by the ascent is regained in the descent; but in the actual case, the time of ascent exceeds that of the descent, and there is not therefore time for gravity to restore on the descending side all the velocity lost on the ascending side; and a loss both of time and power (which are equivalent in a locomotive engine) is sustained accordingly.

12. It is clear, that when a locomotive engine and train, proceeding with a given horizontal velocity, arrive at the foot of an ascending plane, the motion from that point will be retarded till the increased pressure of the steam is sufficient to balance the increased force of traction and friction, after which the motion will continue uniform. And when the engine and train, proceeding at the same velocity, arrive at the top of a descending plane, the motion down will be accelerated till the reduced pressure of the steam due to the increased velocity is just such as to balance the difference between the two opposite forces of friction and gravity, when the descending velocity will become uniform also.

13. Let us now endeavour to get an expression for the accelerating forces above referred to.

We have seen, that with a gross load L', the force of traction on a horizontal plane is expressed in lbs. by $C + 9\,L'$; and let $\frac{C + 9\,L'}{2240\,L'} = \frac{1}{f}$, be taken to denote the force as a fraction of the load, the corresponding velocity being v, and let $\frac{1}{s}$ denote the slope of the plane, or the height divided by the length, and let v' be the velocity of ascent at any time, then the steam pressure being inversely as the velocity, and being equal to $\frac{1}{f}$, with a velocity v will, at the velocity v', be expressed by $\frac{v}{v'f}$.

The increased force of traction in lbs. will be $\frac{2240\,L'}{s}$, and this

will bring on an increased friction on the engine gear of $\frac{2240\,L'}{8\,s}$.

For we have seen, that the friction on the engine gear amounts to $\frac{1}{8}$th of the whole force of traction: if, therefore, we again divide these terms by 2240 L′, as before, we find that the actual forces in operation are,

Urging force . . $\frac{v}{v'f}$, or steam pressure.

Retarding force $\frac{1}{f}$, the original retarding force.

Do. do. . . $\frac{1}{s}$, the weight of body on the plane.

Do. do. . . $\frac{1}{8\,s}$, increased friction of engine gear.

And therefore the whole variable force is

$$\frac{v}{v'f} - \frac{1}{f} - \frac{1}{s} - \frac{1}{8\,s} = \frac{v - v'}{v'f} - \frac{9}{8\,s}.$$

14. Precisely the same forces are in action on the descending plane, but $\frac{1}{s}$ is now an urging force, and $\frac{1}{8\,s}$ acts as a reduction of the force $\frac{1}{f}$. The expression, therefore, for the descending force is

$$\frac{v - v'}{v'f} + \frac{9}{8\,s}.$$

And therefore,

$$\frac{v - v'}{v'f} \pm \frac{9}{8\,s} = \phi$$

will be a general expression for the variable force with which the engine is urged along any plane ascending or descending.

15. From this expression we may in all cases determine at once the velocity of ascent or descent after the acceleration ceases, that is, after the motion becomes uniform; for in this case the preceding value of the force ϕ becomes zero, so that

$$\frac{v - v'}{f v'} \pm \frac{9}{8\,s} = 0, \text{ or that}$$

$$\frac{v - v'}{f v'} = \mp \frac{9}{8\,s}.$$

And from this we may ascertain the uniform velocity due to any slope, or the slope which will give any proposed velocity.

Suppose, for example, it were required to find the inclination which would produce a final uniform velocity $= 2v$. Substituting $2v$ for v', we find

$$\frac{1}{2f} = \frac{9}{8s}. \quad \text{Or } \frac{1}{s} = \frac{4}{9f}.$$

Again, to find the slope that will give an ultimate uniform velocity $\frac{1}{5}$th greater than the uniform velocity v, we have only to substitute $v' = \frac{6}{5}v$, and we obtain

$$\frac{1}{6f} = \frac{9}{8s}. \quad \text{Or } \frac{1}{s} = \frac{4}{27f}.$$

And this is perhaps the greatest increased speed that can, with a due regard to safety, be admitted on a descending plane; and it is therefore the greatest slope that can be safely descended with the steam admission valve fully open.

16. In order to form a correct estimate of the practical effect of gradients, we must confine ourselves wholly to the question as limited by considerations of prudence, that is, by claiming no more advantage for the descending planes than is consistent with safety.

These limitations must be somewhat arbitrary, but the following are perhaps agreeably to the usual practice.

1. That no plane on which the train would be accelerated with the steam wholly shut off, ought to be descended with more than the uniform horizontal velocities. Such are all planes having a slope $\frac{1}{s}$ greater than $\frac{8}{9f}$, and on which of course the brake must be applied to prevent acceleration.

2. That all those planes on which the ultimate velocity would exceed $\frac{1}{5}$th of the original horizontal velocity, and in descending which, therefore, the admission of steam must be partly shut off, ought not to be descended with more than $\frac{6}{5}$ths of the original velocity. Such are all planes between

$$\frac{1}{s} = \frac{8}{9f} \text{ and } \frac{1}{s} = \frac{4}{27f}.$$

All planes of less slope than this last will, soon after the descent of the body commences, take up their uniform velocity without shutting off any steam, and the speed down them may be computed from the formula

$$\frac{v - v'}{v' f} = \frac{-9}{8s}$$

without any sensible error.

And in all cases the ascending velocity, which soon becomes uniform, may be computed by the formula

$$\frac{v - v'}{v' f} = \frac{9}{8s},$$

the former of which gives

$$v' = \frac{8vs}{8s - 9f},$$

and the latter

$$v' = \frac{8vs}{8s + 9f}.$$

17. Hence, in estimating the mechanical advantage of a descending plane, we must claim nothing for those whose slopes are equal to or exceed $\frac{1}{s} = \frac{8}{9f}$.

For all planes whose slopes fall between

$$\frac{8}{9f} \text{ and } \frac{4}{27f}$$

we may claim an increased velocity of $\frac{1}{8}$th.

For planes of less slope than $\frac{4}{27f}$ the advantage may be computed by the first of the above formulæ;

And in all cases the reduced velocity on the ascending plane by the latter formula.

18. The best way of exhibiting these effects will be by computing the lengths of equivalent horizontal planes, that is, the lengths of horizontal planes which would be passed over in the same time, and with the same power as the ascending or descending planes in question, and taking these lengths as the measure of their mechanical effects.

Thus, planes sloping more than $\frac{8}{9f}$ (descending), will have for their equivalent horizontal plane one of equal length to the planes themselves; descending planes having slopes between

$$\frac{8}{9f} \text{ and } \frac{4}{27f},$$

will have their equivalent horizontal planes $\frac{1}{8}$ths of their own lengths. And planes of less slope than $\frac{4}{27f}$ will have their equivalent planes $\frac{8s-9f}{8s}$ times their own length; and

Lastly, all ascending planes will have their equivalent planes $\frac{8s+9f}{8s}$ times their own length.

19. By way of illustration, the following Table has been computed, taking the dimensions already given of the locomotive, page 413, with a gross load of 100 tons.

According to those data,

$$\frac{C+9L'}{2240L'} = \frac{707+900}{224000} = \frac{1}{139} = \frac{1}{f};$$

and taking the several planes, each 1 mile, the lengths of the equivalent planes for the ascending side are given in column 2, and the equivalent descending planes in column 3; and column 4 shows the mean of two, ascending and descending.

Thus the time and power required to ascend a plane of 1 in 90, one mile in length, would carry the train 2·74 miles on a horizontal plane. The time to descend it would be the same as to go over the same mile horizontally, and the mean of the two 1·87, that is, a mile of such plane would require the same time to pass and repass it as would admit the train to pass and repass 1·87 mile on a level.

20. *Table showing the equivalent horizontal lines to the several ascending and descending planes as given below; the power and dimensions of the engine being as stated in* p. 413. *The gross load, including engine, &c.,* 100 *tons.*

Gradients or inclined planes.	Equivalent horizontal lines.		Mean effect.
	Ascending.	Descending.	
1 in 90	2·74	1·00	1·87
1 100	2·57	1·00	1·78
1 120	2·31	1·00	1·65
1 140	2·12	1·00	1·56
1 160	2·00	·83	1·41
1 180	1·87	·83	1·35
1 200	1·78	·83	1·30
1 250	1·63	·83	1·23
1 300	1·52	·83	1·17
1 350	1·46	·83	1·14
1 400	1·39	·83	1·11
1 500	1·31	·83	1·07
1 750	1·21	·83	1·03
1 1000	1·16	·85	1·01
1 1500	1·10	·90	1·00

It will have been observed that as the expression $C + 9\,L'$ involves a constant quantity C, the value of the fraction $\frac{1}{f}$ will vary with the load. Thus, supposing the gross load to be 50 tons instead of 100 tons, we should have

$$\frac{C + 9\,L'}{2240\,L'} = \frac{1}{97} = \frac{1}{f}.$$

The length of the equivalent planes, therefore, change with the load, and the following Table is computed for the same engine, with a load of 50 tons.

21. *Table showing the equivalent horizontal lines to the several ascending and descending planes, as given below; the power and dimensions being as stated in* p. 413. *The gross load, including the engine, &c.,* 50 *tons.*

Gradients or inclined planes.	Equivalent horizontal lines. Ascending.	Equivalent horizontal lines. Descending.	Mean effect.
1 in 90	2·21	1·00	1·61
1 100	2·09	1·00	1·54
1 120	1·91	1·00	1·45
1 140	1·78	·83	1·39
1 160	1·68	·83	1·25
1 180	1·60	·83	1·21
1 200	1·54	·83	1·18
1 250	1·44	·83	1·13
1 300	1·36	·83	1·09
1 350	1·31	·83	1·07
1 400	1·27	·83	1·05
1 500	1·22	·83	1·03
1 750	1·15	·85	1·00
1 1000	1·11	·89	1·00
1 1500	1·07	·93	1·00

22. The two cases above computed, of gross weights of 100 tons and 50 tons, are about the mean of the luggage and passenger trains on the Liverpool and Manchester line. And in estimating the loss occasioned by gradients on any proposed line, we may take the one or the other accordingly as the traffic may be expected to consist mostly of luggage or passengers.

The following Table shows the computed equivalent length of a line of railway from Croydon to Dover; the data being assumed as stated in the Table.

23. *Table showing the lengths of the equivalent horizontal planes for the several gradients on the South Eastern line, between Croydon and Dover. Engine as before; assumed gross weight,* 100 *tons.*

Distance.		Gradients.		Rise or Fall.	Equivalent horizontal lines from Croydon.		Equivalent horizontal lines from Dover.		Data employed.
M.	Ch.				M.	Ch.	M.	Ch.	
0	22			Level.	0	22	0	22	Weight of engine.. 12 tons.
1	12	1 in	150	Rise.	2	28	1	12	Do. tender.. 6 „
1	58	1	100	Ditto.	4	34	1	56	Waggons and loads 82 „
1	14	1	150	Ditto.	2	22	1	14	Gross weight ....100 „
2	56	1	330	Ditto.	3	79	2	20	Friction of load 8 lbs. per ton.
1	68	1	360	Fall.	1	43	2	53	Engine and tender 9 lbs. do.
1	50	1	100	Ditto.	1	50	4	14	
7	0	1	330	Ditto.	5	67	10	28	Engine gear without load } 72
5	0	1	528	Ditto.	4	14	6	40	
1	0	......		Level.	1	0	1	0	Additional at 1 lb. per ton.
1	40	1	880	Rise.	1	61	1	24	
2	40	1	330	Fall.	2	7	3	56	Diameter of wheel....5 ft.
3	0	1	880	Fall.	2	49	3	43	Length of stroke ....1 „
4	0	1	1320	Rise.	4	38	3	53	Diameter of piston 12 inches.
1	0	1	2640	Fall.	0	77	1	4	Pressure of atmosphere,
3	0	......		Level.	3	0	3	0	14·7 lbs. per inch.
9	0	1	609	Rise.	11	27	7	40	
2	68	1	2950	Fall.	2	60	2	79	
3	49	1	330	Rise.	5	27	3	1	
5	40	1	380	Ditto.	7	60	4	47	
1	42	1	100	Fall.	1	42	3	73	
1	71	1	330	Ditto.	1	46	2	63	
5	53	......		Level.	5	53	5	53	
0	76	1	338	Fall.	0	63	1	32	
69	37				79	9	79	37	Mean 79 23

Whence it appears that the effect of the several gradients will cost an expenditure of time and power which would have carried the train 10 miles further on a horizontal plane; being a loss of power of about 10 per cent.

It will be observed, that in the preceding Tables the whole time of ascent is considered as if it were made with the uniform velocity, whereas the commencement of the ascent is more rapid in consequence of the original velocity; it is, however, assumed that the little time thus gained is lost after the train reaches the top of the

plane, by its having to regain its original horizontal velocity. A similar remark applies to the time of descent.

To obtain a practical case, in order to compare the preceding rules with practice, I wrote to Mr. R. Stephenson, and was furnished by him with the following:

WHARNCLIFFE ENGINE.

	ft.	in.
Diameter of driving wheels	4	6
Length of stroke	1	6
Diameter of piston	0	12

Mean speed, horizontal plane, with a load of 100 tons . . . . 20 miles.

Mean speed up the Rainhill plane of $\frac{1}{96}$, with a load of 50 tons . 12 „

Weight of engine, 12 tons; tender, 6 tons.

Let us now assume the horizontal velocity of 20 miles, as given, and compute what the ascending velocity ought to be:

First, $100 + 18 = 118$ gross load,

$$\left.\begin{aligned} 6\,w &= 72 \\ A = \frac{14{\cdot}7\,d^2\,l}{D} &= 705 \\ 118 \times 9 &= \underline{1062} \end{aligned}\right\} \text{with 118 tons.}$$

$$C + 9\,L' = 1839, \text{ and } \frac{1839}{2240\,L'} = \frac{1}{140} = \frac{1}{f}.$$

Again,

$$\left.\begin{aligned} 6\,w &= 72 \\ A &= 705 \\ 68 \times 9 &= \underline{612} \end{aligned}\right\} \text{with 68 tons.}$$

$$C + 9\,L' = 1389, \text{ and } \frac{1389}{2240\,L'} = \frac{1}{110} = \frac{1}{f}.$$

And

		Miles.	Miles.
1389 :	1839 : :	20 :	$26\frac{1}{2}$

the rate a load of 50 tons would be carried on a horizontal plane by the same engine: we have, therefore, by the formula

$$\frac{v - v'}{v' f} = \frac{9}{8 s}$$

$$v' = \frac{8 s}{8 s + 9 f} = 11\tfrac{1}{2} \text{ miles,}$$

the velocity of ascent, which, according to Mr. Stephenson's practical experience, is 12 miles per hour; as close an approximation as can be expected in such a case.

The following Table contains a number of other practical examples, which will enable the reader to form a comparison of the results with the formula. They are taken from the experiments by M. Pambour, on levels and planes of $\frac{1}{89}$ and $\frac{1}{96}$.

FROM PAMBOUR.

		Load and tender.	Descent or level.	Speed, miles.	Press. of steam.	Ascent.	Speed, miles.	Press. of steam.
July 17, 1834.	Atlas	27·45	[illegible]	26·47	54	[illegible]	14	56
July 23.	Do.	39·40	not given			[illegible]	6	55
July 31.	Do.	40·15	level	16	27½	[illegible]	7·5	51
Aug. 4.	Do.	44·26	not given			[illegible]	3·75	61
July 24, 1834.	Fury	56·16	level	17·14	55	[illegible]	6·31	66·5
July 24, 1834.	Do.	48·8	level	17·50	55	[illegible]	15	67
Aug. 4.	Do.	37·97	level	25·00	52·5	[illegible]	13·33	55
Aug. 1, 1834.	Vesta	33·15	level	29	50	[illegible]	14·11	55
Aug. 16, 1834.	Do.	37·45	not given			[illegible]	3·25	58
Do.	Do.	39·05	not given			[illegible]	3·0	56·5
Aug. 15, 1834.	Leeds	38·15	level	22·5	46·5	[illegible]	10	48·5
July 22, 1834.	Vulcan	39·07	not given			[illegible]	11·42	57·5
July 22.	Do.	41·32	not given			[illegible]	18·75	57·5

		Inches.	Stroke.	Diam. W.	Weight.
Atlas . .	Diam. piston	12	16 in.	5 feet.	11·40 tons.
Fury		11	16	5	8·20
Vesta		11½	16	5	8·71
Leeds		11	16	5	7·07
Vulcan		11	16	5	8·34

Atlas . .	26·47	:	14	or	1	:	·53	Mean 1 : ·52
Do. . . .	16	:	7·5		1	:	·47	
Fury . .	17·14	:	6·31		1	:	·37	
Do. . . .	17·50	:	15		1	:	·85	
Do. . . .	25·00	:	13·33		1	:	·53	
Vesta . .	29·00	:	14·11		1	:	·48	
Leeds . .	22·5	:	10		1	:	·44	
							7)367	
							·52	

TABLE

Showing the Specific Gravity and the Weight of a Cubic Foot of various Building Materials.

The specific gravity of rain water being 1000.

MATERIALS.	Specific gravities.			Weight of a cubic foot in lbs.		
WOODS.	From		To	From lbs.		To lbs.
Acacia (false)	748	..	820	46·75	..	51·25
(three-thorned)	Mean	..	676	Mean	..	42·25
Ash (dry)	690	..	845	43·12	..	52·81
Beech (mean sort)	696	..	854	43·50	..	53·37
(dry)	690	..		43·12	..	
Birch	720	..		45·00	..	
Box (Dutch)	1030	..	1328	64·37	..	83·00
(Turkey)	950	..	1024	59·37	..	64·00
Cedar (Indian)	1315	..		82·18	..	
(various countries)	453	..	753	28·31	..	47·06
(of Libanus)	486	..	603	30·37	..	37·68
Cherry Tree	672	..	741	42·00	..	46·31
Chestnut (Sweet)	535	..	685	33·45	..	42·81
(Horse)	483	..	657	30·18	..	41·06
Cowrie	579	..		36·20	..	
Cypress	644	..	655	40·25	..	40·93
Elm (green)	693	..	940	44·41	..	58·75
(seasoned)	553	..	588	34·56	..	36·75
Fir (Norway Spruce)	512	..		32·00	..	
(American)	465	..		29·06	..	
Larch (seasoned) red	496	..	640	31·00	..	40·00
(white)	364	..		22·75	..	
Mahogany (Spanish)	816	..	852	51·00	..	53·30
(Honduras)	560	..		35·00	..	
Oak (green)	1063	..	1216	66·43	..	76·03
(Irish Bog)	1046	..		65·37	..	
(Adriatic)	993	..		62·06	..	
(American)	752	..		47·00	..	
(English) dry	625	..		39·06	..	
(Dantzic)	755	..		47·24	..	
Pear Tree (dry)	646	..	708	40·37	..	44·25
Pine (American Pitch) dry	741	..	936	46·31	..	58·50
(Scotch) dry	529	..	696	26·81	..	43·50
(Memel and Riga)	466	..	553	29·12	..	34·56
(American)	368	..		23·00	..	
Plane	538	..	648	33·62	..	40·50
Poona	635	..		39·95	..	

TABLE—(CONTINUED).

Materials.	Specific gravities.		Weight of a cubic foot in lbs.	
	From	To	From lbs.	To lbs.
Woods.				
Poplar	374	529	24·37	33·06
Sycamore	590	645	36·87	40·31
Teak (dry)	657	832	41·06	52·00
Walnut Tree (green)	920		57·50	
(dry)	616	735	38·50	45·93
Willow (green)	619		38·68	
(dry)	404	568	25·25	35·50
Stones and Cements.				
Basalt	2478	3000	154·87	187·50
Brick (common)	1557	2000	97·31	125·00
(stock)	1841	2168	115·06	135·50
(Dutch clinker)	1482		92·62	
(Welsh fire)	2408		150·50	
Brickwork	Mean	1520	Mean	95·00
Chalk	2315	2657	144·68	166·06
(Clunch)	1869	2657	116·81	166·06
Flint	2580	2630	161·25	164·37
Granite	2624	3000	164·00	187·48
Marble	2580	2840	161·25	177·50
Mortar (hair) dry	1384		86·50	
(various) dry	1414	1393	88·37	118·31
Plaster, cast	1286		80·37	
Puzzolano	2570	2850	160·62	178·12
Serpentine	2561	2683	160·06	167·68
Slate	2512	2888	157·00	180·50
Stone (Bath)	1975	2494	123·43	155·87
(Blue lias limestone)	2467		454·18	
(Bramley Fall)	2506		156·62	
Stone (mean of various kinds)	2000	2686	125·00	167·87
Stonework	Do.	Do.	Do.	Do.
(Yorkshire paving)	2356	2507	147·25	163·37
Tile (common)	1815	1858	113·43	116·15
Earths, etc.				
Clay (common)	1919		119·93	
(with gravel)	2560		160·00	
Coke	744		46·50	
Coal	1269	1526	79·31	95·37
Earth (common)	1520	2016	95·00	126·00
Gravel	1749		109·80	
Lime (quick)	843		52·68	
Marl	1600	2870	100·	179·37
Sand (quartz)	2750		171·87	
(common)	1454	1886	90·87	117·87

TABLE—(CONTINUED).

Materials.	Specific gravities.		Weight of a cubic foot in lbs.	
	From	To	From lbs.	To lbs.
Earths, etc.				
Shingle	1424 ..		89·00 ..	
Water (Rain)	1000 ..		62·50 ..	
(Sea)	1027 ..		64·18 ..	
Metals.				
Brass (cast)	8100 ..		506·25 ..	
(wire, plate)	8441 ..	8544	527·56 ..	534·00
Copper (cast)	8607 ..		537·93 ..	
(sheet)	8785 ..		549·06 ..	
Iron (bar)	7600 ..	7800	475·00 ..	487·50
(cast)	7200 ..	7600	450·00 ..	475·00
Lead	11352 ..	11407	709·50 ..	712·93
Pewter	7248 ..		453·00 ..	
Platina	21531 ..		1345·63 ..	
Steel	7780 ..	7840	486·25 ..	490·00
Tin	7291 ..	7299	455·68 ..	456·18

PLATE I

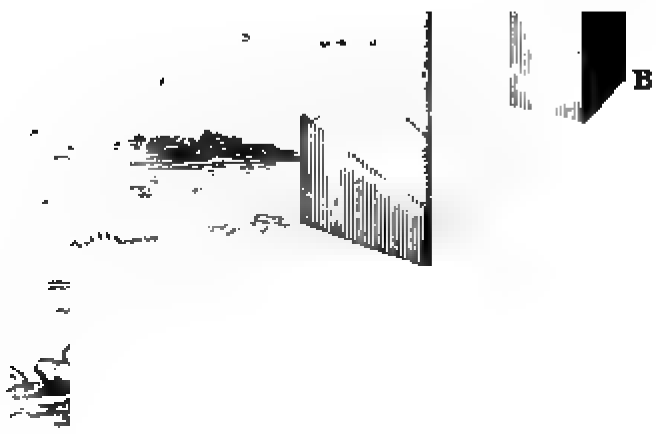
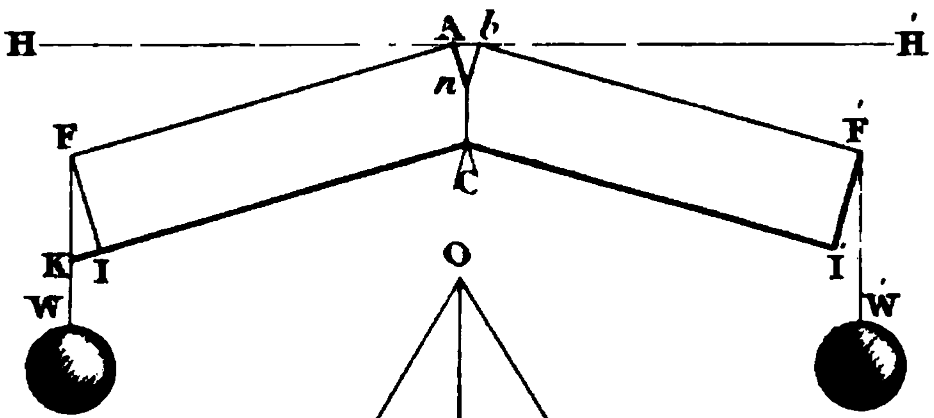


Fig. 3.



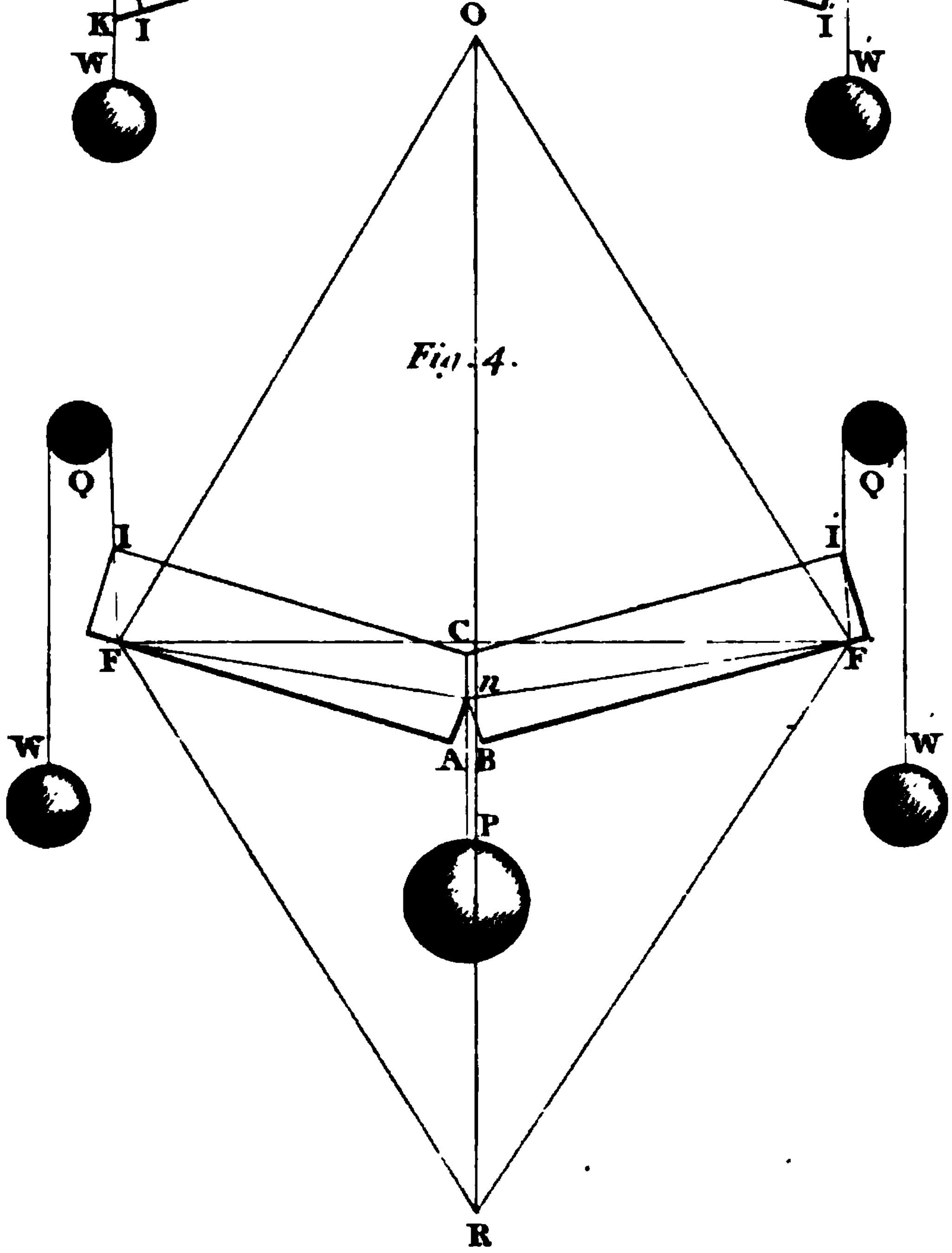


Fig. 8.

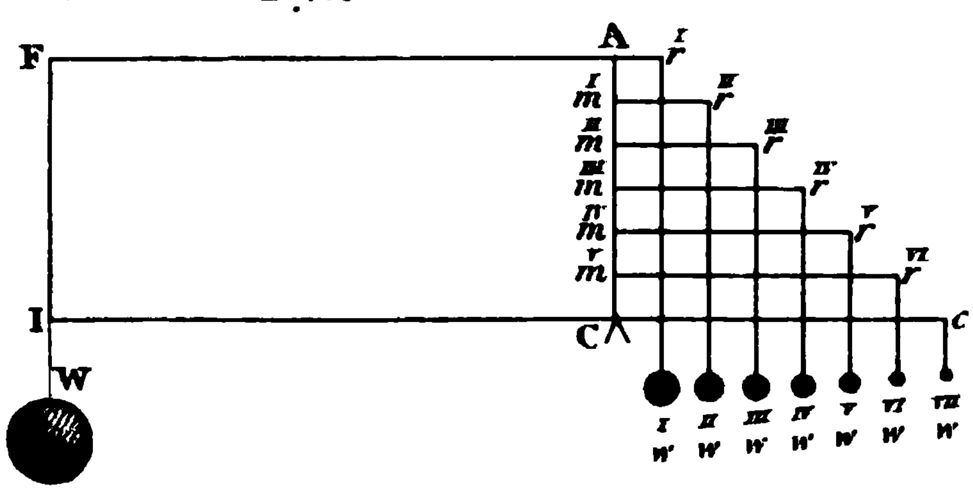


PLATE III.

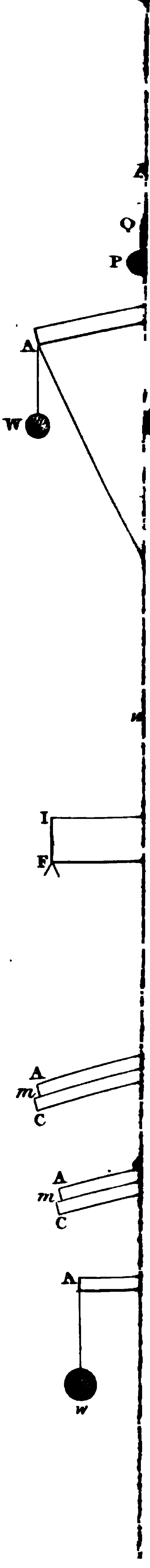


Fig. 6.

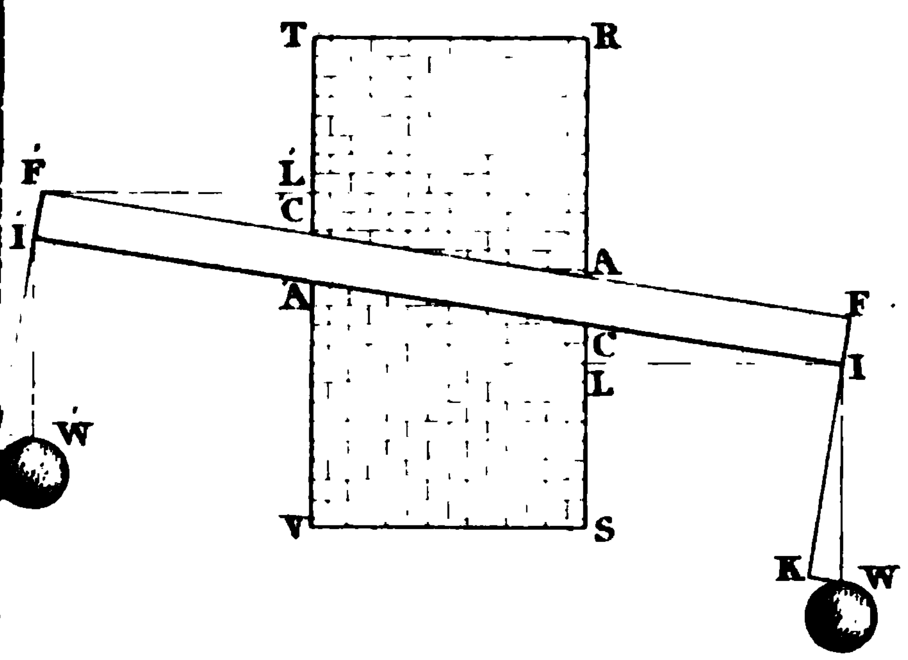


Fig. 7.

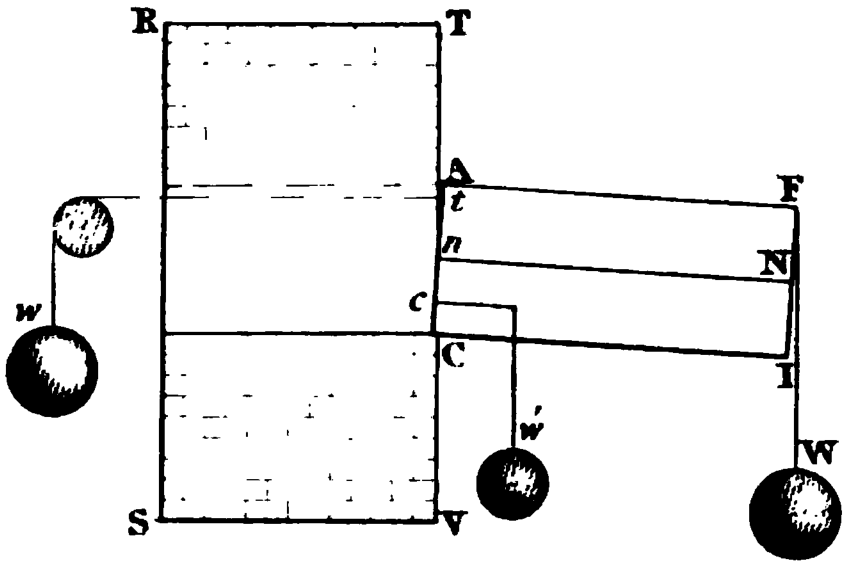
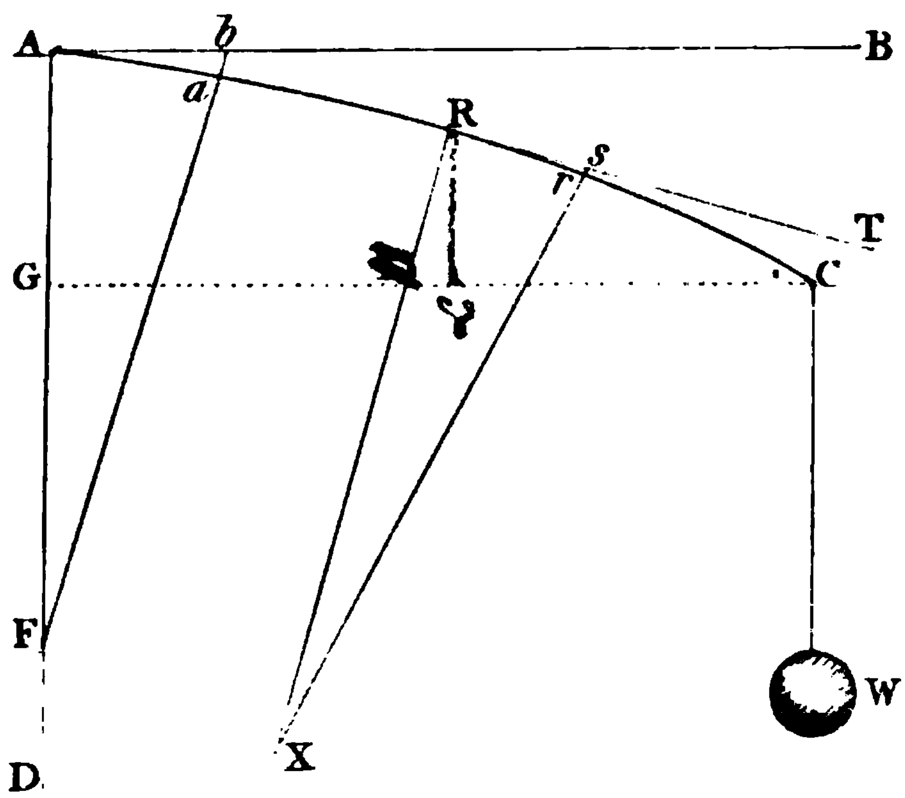


Fig. 9.



IV.

PLATE V

WOOLWICH.

Pl. VI.

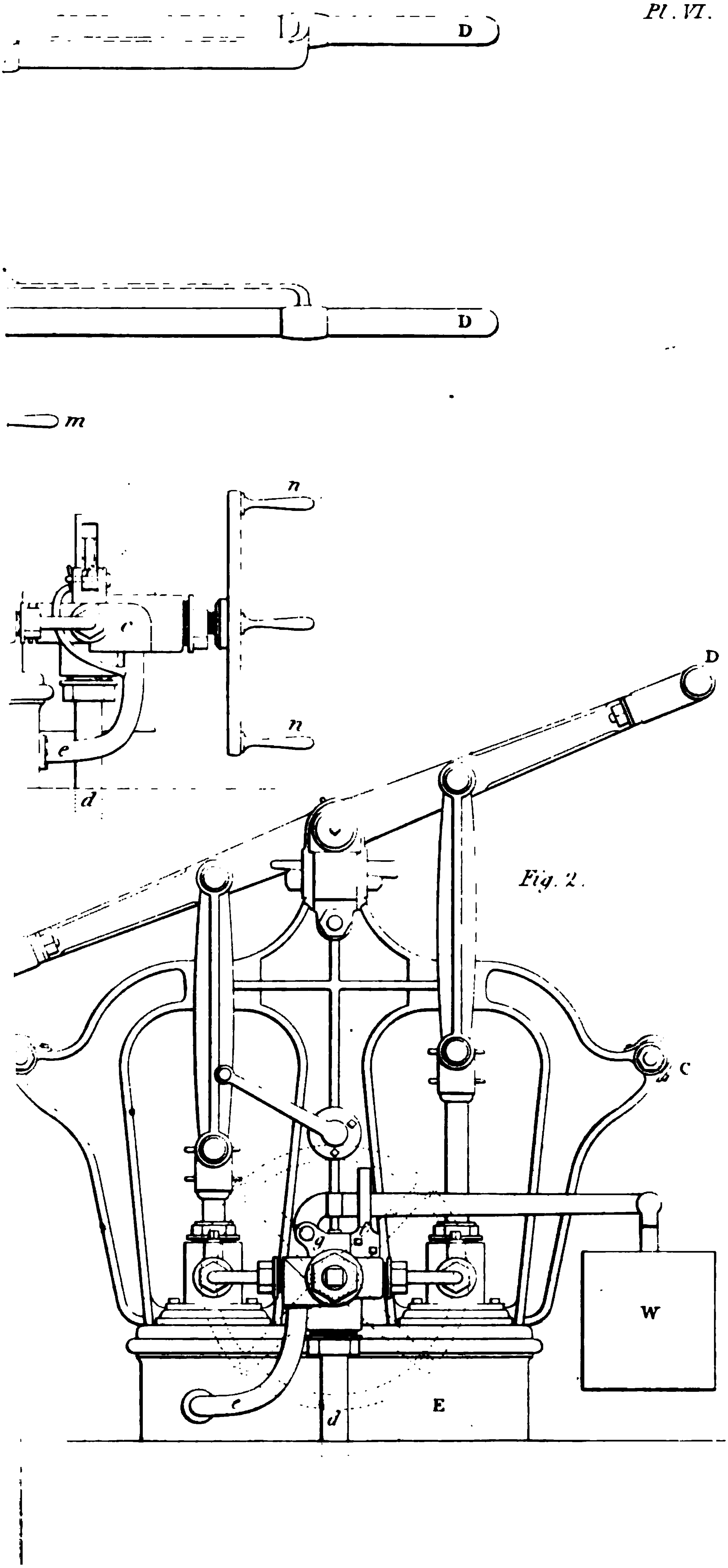
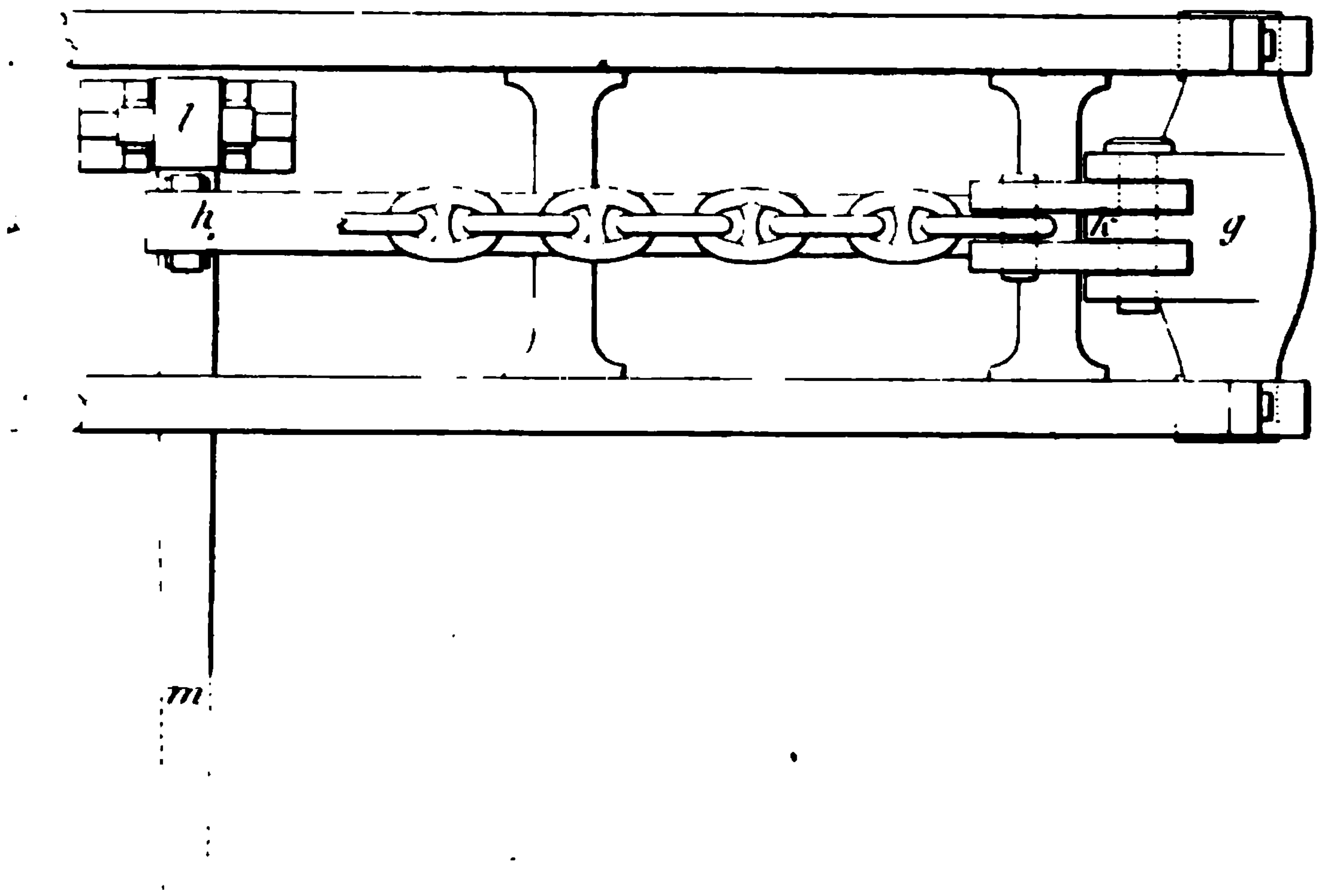


Fig. 2.



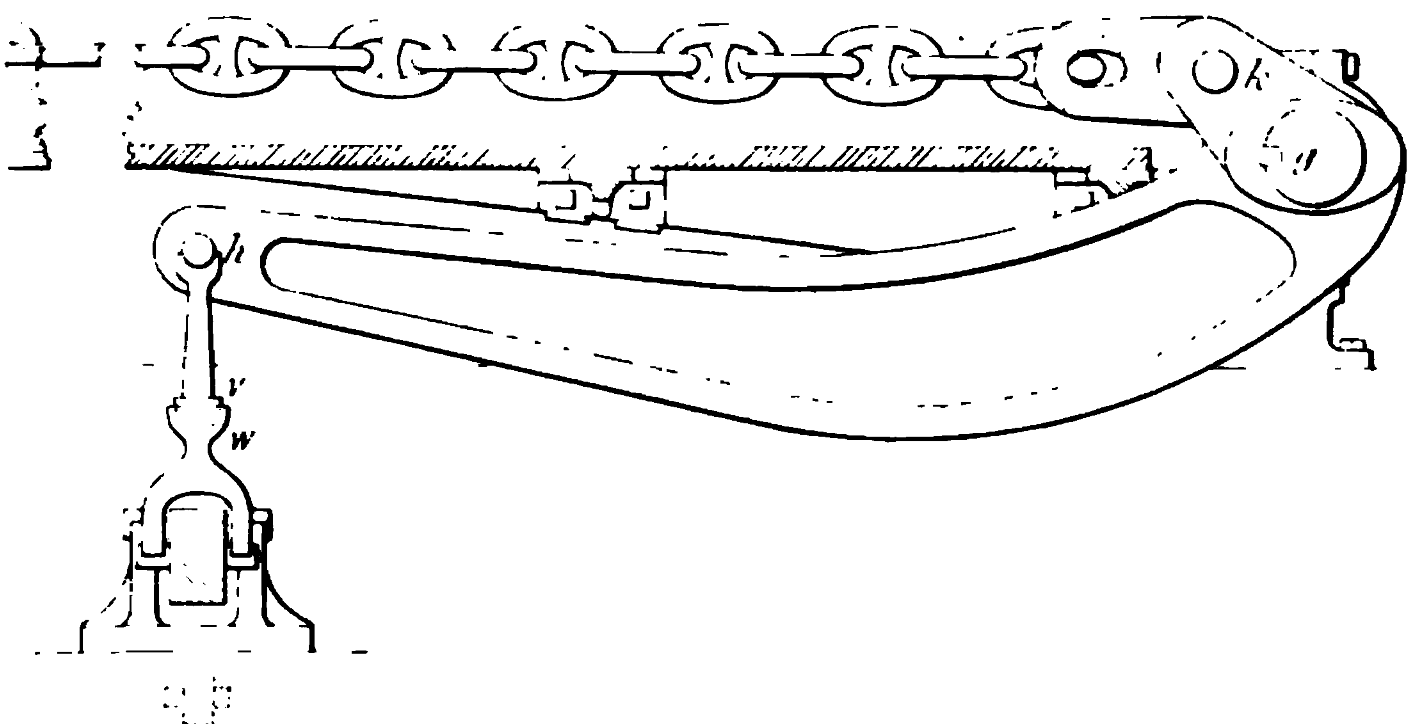
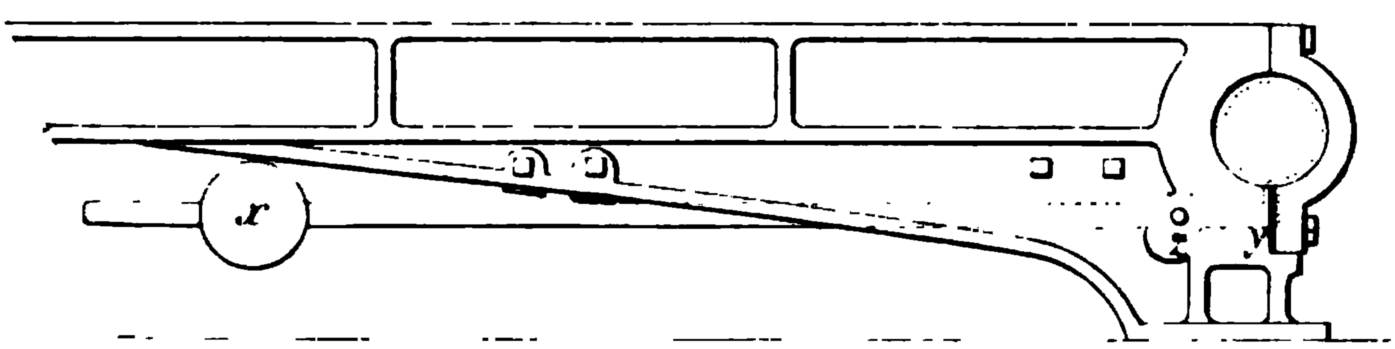


Fig. 5.



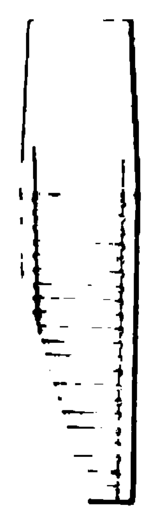
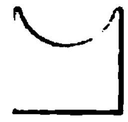


Fig. 8.

Fig. 9.

Fig. 10.



AN ESSAY

ON

THE EFFECTS PRODUCED BY CAUSING WEIGHTS TO TRAVEL OVER ELASTIC BARS.

BY

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(*Extracted from the Appendix to the Report of the Commissioners appointed to inquire into the Application of Iron to Railway Structures.* —*July* 26, 1849.)

ESSAY ON THE EFFECTS PRODUCED BY CAUSING WEIGHTS TO TRAVEL OVER ELASTIC BARS.

CHAPTER I.

General Remarks and Description of the Apparatus erected in Portsmouth Dockyard, and of the Experiments performed with it by Captain James and Lieutenant Galton.

ONE of the objects to which the attention of the Commission was directed by the terms of its appointment, was "to illustrate by theory and experiment the action which takes place under varying circumstances in iron railway bridges." Now a bridge has necessarily to sustain the action of loads which pass over it, and, in the case of railway bridges, the velocity of transit is exceedingly great.

The effects of loading elastic bars with weights appended to them at rest have been very fully investigated, both by theory and experiment, as is perfectly well known; but the effects produced upon such bars by causing the weights with which they are loaded to travel with more or less velocity along them had never been, as far as the Commissioners were aware, made the subject of research either practically or theoretically. It was therefore resolved that experiments should be arranged for the purpose of determining the influence of velocity communicated to a load upon the deflection and fracture of the structure over which it is transmitted, and which has, therefore, to sustain its pressure during its transit.

It was thought desirable, at the beginning of the investigation, that the experiments should be made on a large scale, so as to give a practical value to the results, whatever they might be,

that should be obtained. The object in view was to subject bars of cast iron to the action of passing loads for the purpose of examining how the velocity of any given load would operate to increase or to diminish its pressure upon the bars, and consequently of determining its power in deflecting or fracturing them as compared with the effects of the same load, placed at rest upon the bars in the usual manner of experiments upon the strength of materials.

An apparatus was therefore required which admitted of having bars which were to be the subjects of the experiments readily fixed to receive the passing load, the latter being capable of adjustment to various weights at pleasure; and it was also requisite to have the means of giving any desired velocity to this load. Lastly, contrivances were required for the purpose of registering the effects.

A liberal permission had been granted to us by the Lords Commissioners of the Admiralty to make use of Portsmouth Dockyard for our experiments, and as the apparatus in question required considerable space, it was determined to erect it in that place. Captain H. James, one of Her Majesty's Commissioners for carrying out the present inquiry, also resided at Portsmouth, holding the office of Director of Works in the Dockyard. He, therefore, was requested to undertake the construction of the apparatus required for the purposes already mentioned, and the mechanism about to be described was wholly contrived and set up under his direction. Of this mechanism it is sufficient to say that from the beginning it answered its purpose most admirably, requiring only a few alterations, the necessity for which became evident after the preliminary experiments had shown more clearly the points of the investigation that required to be developed. The experiments themselves were wholly carried out under the personal superintendence of Captain James and Lieutenant Galton, the Secretary to the Commission.

The apparatus was principally designed to experiment on bars of nine feet in length, and the load consisted of a small ordinary railway car, adapted to run on rails three feet asunder, and to receive pigs of cast iron, by which the weight of the whole could be adjusted from half a ton to two tons at pleasure. It was determined to employ an inclined plane as the simplest mode of giving

a manageable velocity to the load, and the space at command in the Dockyard enabled this plane to be erected upon a scale that raised its upper extremity 40 feet above the lower part.

The entire machine, together with details of every portion of it, is shown in Plates I. and II. The form and proportions of the car and its rails are sufficiently shown by its side elevation, plan and section, in figs. 4, 5, and 6, respectively. The general form and arrangements of the scaffold are given in figs. 1, 2, and 3.

Figs. 1, 2. This inclined plane or scaffold supported the railroad, of which thirty feet of the upper part were straight and inclined to the horizon at an angle of 46°. The course of the bars was then bent into an arc of a circle of 50 feet radius, by which the upper and inclined part (*A*) of the railroad was gently and imperceptibly connected with (*D D'*) the horizontal portion beneath, which from the point of its junction with the curves was extended 18 feet to the place (*C*) where the ends of the trial bars were fixed. These were laid horizontally so as to form a continuation of the railway, with this difference, that whereas the railway bars were supported by chairs of the ordinary kind, fixed at intervals of 4′ 6″ to the framework of the scaffold, the trial bars were sustained by chairs of a peculiar construction (*F F'*) at each end only.

One of these chairs is represented in plan and section on a larger scale in figs. 10, 11, and 12; from which it appears that the end of each trial bar (*C*) was cast with a projection beneath, and kept in its place laterally by a pair of wedges, which were not driven sufficiently tight to impede its vertical deflections. The lower surface of the above-mentioned projection, which formed the bearing surface, could be readily adjusted by the file so as to insure continuity between the upper edges of the fixed rail and of the trial bar respectively at their junction, and thus to avoid the jumping or jerking of the wheels of the car: for it is of the utmost importance to the accuracy of experiments of this kind that the car should enter upon the trial bar without jolting. A wooden wedge was also dropped between the extremities of the rail and trial bar for a similar purpose.

Beyond the farthest end of the trial bars a portion of a similar railway was laid (as will be presently described), for the purpose of receiving the car after it had passed over the bars. Thus the

bars formed a part of the railway for the time being, and to determine the effect of any required load and velocity upon the bars, it was only necessary to load the car accordingly, and draw it up to such an altitude of the plane as would correspond to the desired velocity, and, lastly, to release it suddenly. It then ran down the plane and passed over the bars with the velocity acquired, deflecting or fracturing them, as the case might be. From the nature of this apparatus it is necessary to fix a *pair* of trial bars into the frame, for as the car in its passage deflects the bars, it necessarily sinks downwards. If only one trial bar were employed, and the corresponding opposite one stiffened by resting on a sleeper or otherwise, the car would be thrown laterally over. Some inconvenience arises from this necessity for employing two flexible bars at once; but a greater one was occasioned by the fracture of the bars whenever that took place, which of course it frequently did, since one object of the research was to discover the load that would fracture the bars with given velocities. But whenever either bar broke, the car, having lost its support, rolled head over heels into the yard, and usually some hours were consumed in repairing the consequent mischief; also, the fear of such accidents made it necessary for the observers to escape to a safe distance before the car was released, instead of closely watching the phenomena of its passage.

In estimating the load upon the trial bars, it must be remembered that the weight of the car was equally divided between the two, and therefore, although the car was capable of being loaded to two tons, each trial bar could only be exposed to the action of half those weights.

The vertical height of the top of the railway has been said to be 40 feet above the horizontal portion; but the centre of gravity of the car could not, of course, be raised to the very top; and deducting also the retarding effect of friction, it was found that the greatest actual velocity with which the car could be made to pass the trial bars was not greater than 43 feet per second (or about 30 miles per hour), a velocity due to a fall from only 30 feet when resistances are neglected.

The actual velocity of the car was measured by Lieutenant Galton in the following manner:—A distance of 12 feet 6 inches was marked out on each side of the centre of the trial bar (see

Plate II., figs. 4, 5, and 6), on entering which a roller *P*, attached to the car, struck a lever *M*, which, by means of the link rods *M' M'*, pushed the plate *K* from under the pencil *L*, and allowed the latter to come in contact with and trace a line upon the cylinder *O*, which was maintained in equable rotation by an equatorial clock. The arrangement of the pencil, cylinder, and guard-plate *K*, is shown at large in fig. 9. The clock was kindly lent by Dr. Lee, F. R. S., of Hartwell House. When the car had passed to the end of the assigned distance, the roller *P*, striking the lever *N*, raised the pencil by means of the connecting link rod *N'*, the end of which was jointed to an arm hanging from the axis to which the pencil carriage was fixed.

We must now consider the mode of checking the velocity of the car and bringing it to rest, after it had passed over the trial bars. For this purpose the railway was continued beyond the trial bars, exactly in the same manner as in front of them, namely, by a curve and an inclined plane, which is represented in fig. 1, from *D'* to *B*. In the earlier experiments, the car, after passing the trial bars, ran up the second inclined plane, nearly as high as the point whence it had been released from the first. Then it ran down again, again passed over the trial bars and up the first plane, and so backwards and forwards until its velocity became so far subdued that it could be stopped by hand.

But these repeated journeys, besides wasting time, were found to interfere so seriously with the registering apparatus and the adjustment of the trial bars, that a better scheme was carried out at the suggestion of Lieutenant Galton, which is represented in figs. 4 and 5.

A second railway was laid parallel to the first on the horizontal portion, having its bars respectively about nine inches distant from those of the first, and upon the same level. This railway, about 50 feet in length, was curved horizontally to meet the first at its two extremities, and connected to them by switches; the levers and connecting rods of which are shown at *D D*, *D' D'*, figs. 4 and 5. In the position of the apparatus represented in the plan, the switches are set in a position which does not disturb the continuity of the direct line of the rails. If the switches at each end are shifted to the position shown by the dotted lines, the horizontal portion of the direct line which contains the trial bars

will be completely cut off, and the railway, descending the inclined plane and curves from each side, will be conducted by the switches to the intermediate railway. (It is plain that the two sets of switches must be shifted.) The mode of performing an experiment with this improvement was as follows:—The switches were, in the first instance, set in the position of the figure, so as to continue the original direct line of rails, and the car, when released, ran down the left-hand inclined plane, and having passed over the trial bars, ran up the second plane to the right. Immediately the two switch levers were shifted so as to cut off the trial bars, and the car, returning, was thus diverted upon the intermediate line upon which it travelled backwards and forwards, running up and down the two curves and inclines as before, but without repassing the trial bars or deranging the registering apparatus.

It remains to describe the apparatus represented in the figures 4, 5, 7, 8, by which the effects and results of the experiments were registered. In the earlier trials it was only thought necessary to ascertain the central deflection of the trial bar, in order to compare its amount as produced *statically* by placing the loaded car at rest upon it, with its amount when obtained *dynamically* by running the same loaded car over it. This deflection was simply obtained by a horizontal lever set at right angles to the middle of the trial bar, and having one end in contact with its lower surface. The other end of the lever carried a pencil, which, when the bar was depressed either statically or dynamically, traced a line upon a piece of paper, the length of which line was proportional to the deflection.

But upon investigating the theory of these experiments I soon perceived that the information thus conveyed was wholly inadequate, and that much more information was required of the movements imparted to the bar by the passing weight. At my suggestion, therefore, the registering apparatus represented in the figures was substituted for the simple deflectograph above described. The reasoning which led me to the contrivance of this apparatus will be fully explained below (see p. 457), and although, as it will appear, its construction was not sufficiently delicate to carry out my purposes as originally intended, it was employed for the whole of the subsequent series of experiments. Five pencils were attached to as many points of the trial bar, equidistant from each

other and from the ends of the bar. In the section, fig. 7, *C* is the trial bar, and a spring pencil appears beneath it, the tube of which is fixed to a clamp that can be readily screwed to the lower part of the bar so as not to be displaced by the flanges of the car-wheels in their passage along the upper surface of the bar.

A long board, *E E*, is placed in front of and parallel to the bar at a distance of about an inch and a quarter. This board, six inches in width (or rather height), is arranged as shown by the section, so as to run easily upon rollers in the direction of its length. Its inner vertical surface (or that which lies next to the bar) is covered with paper and receives the traces of the five pencils; for, as shown in fig. 5, the board *E* is sufficiently long to be in contact with all the pencils *a, b, c, d, e,* at the same time. If the board remained at rest during the passage of the car, it is plain that each pencil would trace a line upon the paper, which would be equal to the deflection of the corresponding point of the bar to which that pencil was fixed; and thus, instead of recording merely the central deflection of the bar, the apparatus would inform us of the deflection of each of the five points of the bar. Let us now suppose that a slow equable motion is given to the board, which, as already explained, is mounted on rollers. In this case each pencil will, in lieu of a simple vertical line, trace a curve in the form of a loop or irregular U, the inflection of which will, when properly analyzed, inform us of every particular respecting the motion of the bar, as I shall explain at length below.

However, to do this completely, the board must be maintained in motion with a constant velocity such as an equatorial clock or similar contrivance alone can effect, and that only when the board and its rollers are so mounted as to move with small and equable friction, a condition which the general roughness of the apparatus in question rendered inadmissible. The board, therefore, was simply fitted to receive its motion from the descent of a weight at *G*, (figs. 4, 5, 8,) fastened to a string, which, passing over three pulleys, was thereby conducted into the proper horizontal direction, and also to the level of the board, to the end of which it was tied. The weight was temporarily prevented from descending by a small board placed under it, and which was connected to a lever *H*, as shown in the figures, in such a manner that when the car

in its course arrived at this lever, near to the trial bar, it struck it aside, and thus drawing the board from beneath the weight, the latter began its descent, dragging with it the board. The board thus received a travelling motion, of course considerably accelerated, but which enabled it to receive from the pencils curves of the nature of those above described.

These curves, although from the irregular motion of the board they were inadequate to convey the entire information for which I sought, did yet suffice to record the simultaneous deflections of each of the five points, and were used for this purpose alone. The remaining information I contrived to obtain by means of my own, which will be presently described.

Upwards of four hundred* experiments were made with this apparatus by Captain James and Lieutenant Galton, and the results which they obtained were equally new and important, developing, for the first time, the fact that a given weight passing rapidly along a bar produces a greater deflection in that bar during its passage, than it would have done had it been suspended at rest from the centre of the bar.

The three first series of experiments were made upon bars of Blaenavon cast iron, nine feet long, of which those of the first series were an inch broad and two inches deep. In the second they were one inch broad by three inches deep, and in the third four inches broad by an inch and a half deep. As these three series were each managed in the same manner, it will only be necessary to describe one at length, for which purpose I shall select the second series.† In describing the load of the car, it must be remembered that its actual weight is distributed upon four wheels, two of which rest on each trial bar. Thus, when the weight of the car and its load amount to 2240 lbs., each bar is loaded with 1120 lbs. In describing the experiments, therefore, the weight mentioned must be understood to be the total weight of the loaded car.

In the first place, a sufficient number of bars having been cast

* In this enumeration each journey of the carriage is reckoned as one experiment. But in the Tables the experiments are arranged in groups of seven or eight of such journeys, each group being numbered as one experiment, so that the total number of experiments appears much less.

† See the Tabular Summary below, p. 445, Second Series, Experiment No. 7.

of the above-stated dimensions, a pair of them were placed in the chairs, and the car having been set at rest upon their centre, was loaded with gradually increasing weights from 1120 lbs. upwards, until one of the bars broke, the deflections and sets having been carefully noted for each accession of weight. This preliminary experiment, which was repeated upon three pairs of bars, was made for the purpose of testing in the usual manner the actual strength of the bars which were to be the subject of the dynamical experiments. It was thought better to test in this manner the quality of specimens taken at random from the actual parcel of bars provided for the dynamical experiments than to trust to calculated results.

A pair of bars were, in the next place, selected for a dynamical trial, and placed in the chairs. The car was loaded with 1120 lbs. and placed at rest in the centre of the bars. The statical deflection was 0·32 inch. The car was then drawn up to the point of the inclined plane which corresponded to a velocity of 29 feet per second, or 20 miles per hour, and suddenly released. The transit over the bars produced a deflection of 0·36 inch. The velocity given to the load had thus added one-tenth to the statical deflection. The car was then loaded to 1778 lbs., 2348 lbs., 2670 lbs., and so on, adding 56 lbs. each time, and always releasing it from the same point of the plane, the deflection meanwhile steadily increasing at each increase of weight, until, with a load of 2999 lbs., it became 2·67 inches. This load would, calculating from the statical deflection of the same bar by 1120 lbs., have produced a statical deflection of 1·30 inch. The velocity, therefore, in this case more than doubled the statical deflection due to the load. The car, as already observed, was always drawn up to the same point, so that the velocity remained constant in each set of experiments with a given pair of bars, and the load was increased at each successive trial until one or both bars broke. In the set of experiments we are now considering, the next load of 3167 lbs. fractured both bars at once. The mean statical breaking weight of bars of these dimensions is about 4200 lbs. Thus it is shown that the motion of the load over the bars increases the deflection, and, as would naturally follow, enables a smaller weight to fracture them. When higher velocities are given to the car, the above effects are increased.

A pair of bars received from a load of 1120 lbs. a statical deflection of 0·27 inch.* When a velocity of 43 feet per second (30 miles per hour) was given to the car, the deflection became 0·52 inch; and with loads of 1778 lbs. and 2066 lbs., it reached 1·07 inch and 1·87 inch respectively. The bars were fractured with 2122 lbs.; their mean statical breaking weight being about 4200 lbs. Calculating the statical deflection due to the above loads, it appears that this high velocity enabled 1778 lbs. to effect more than double that deflection, and 2066 lbs. to increase it threefold.

To estimate the increase of the statical deflection, produced by the velocity of the load in the above examples, it is necessary, as I have shown above, to know the statical deflection due to each load. Now the object of the experimenters was simply to ascertain the breaking weight of each pair of bars under a given velocity. They, therefore, only tried the statical deflection of each pair with the first load of 1120 lbs.; for in dealing with cast iron, the imperfection of its elasticity and the consequent amount and irregularity of the set makes it necessary to avoid as much as possible the repeated deflections of the bars. On this account they did not ascertain the statical deflection for each successive load, but contented themselves, after the first trial, with releasing the car from its constant altitude, increasing the load at each trip until the bars broke. The statical deflection, therefore, after the first, can only be calculated by comparing that first deflection due to 1120 lbs. with the deflections in the preliminary statical experiments, already described, which were made for this purpose.

The irregularities introduced by the set of the bars, which our imperfect knowledge of that phenomenon makes it impossible for us to remove from the calculations, must prevent this method from being very accurate, but it will be found sufficiently exact to enable us to compare roughly the statical with the dynamical deflection, considering the other sources of irregularity and error which are inseparable from experiments of this nature, as I shall point out below.

If indeed the whole of the bars could be cast of the same strength, the deflection of one bar would correspond so nearly to those of the other that no sensible error need be apprehended,

* See Second Series, p. 446, Experiment 14.

but this can never be the case. Compare, for example, the three experiments in the second series upon a velocity of 15 feet with the three following upon a velocity of 29 feet, and it will appear that the statical deflections due to 1120 lbs. in these six experiments vary from ·29 to ·42, although all the bars were cast in the same mould.

But to compare the effects of velocity upon the deflections with more accuracy, some experiments were subsequently undertaken upon a different principle, namely, that in each set the velocity should be varied, and the load remain constant; thus the statical deflection due to this constant load, being ascertained at the beginning, was applicable without error to the whole: these are contained in the sixth and seventh series of experiments.*

Within the limits employed in the previous experiments, the increase of velocity had been constantly accompanied by an increase of deflection, but it was conceivable that with a very high velocity the load might pass over the bar without having time even to fall through the space required for the statical deflection, and that thus there must be a limit to the increase of the deflection, so that beyond the velocity corresponding to this limit, the deflection would diminish. It was clear that this limit would be approached more nearly by employing shorter bars, and those as flexible as possible, for the purpose of at once diminishing the time of passage, and increasing the space through which the load must fall vertically. Bars of wrought iron were tried, 4 ft. 6 ins. in length; and in order to get rid of the complication of effect produced by having two wheels pressing on the bar at once, the car was elongated, so as to render the distance between its axles 6 ft. 6 ins.

The load therefore still pressed upon each rail with two wheels, but as the trial bar was shorter than the distance between these wheels, the travelling load could only press upon it in one point at a time, for the front wheel had completely passed off the bar before the hind wheel entered upon it. The load was laid so as to press much more upon the front than upon the hind wheel, and thus the effect of the passage of the latter was insignificant. The desired maximum deflection was not, however, reached by these bars, as will be seen by referring to the Tables in the Report (pp. 239, 240). But a pair of steel bars 2 ft. 3 ins. long, 2 ins.

* See Table X. below, p. 488.

broad, and ¼ in. deep, gave the following results, and exhibited the effects which were sought for:

Velocity, in feet, per second . ..		15	24	29	34	44
Central Deflection	·70	1·02	1·32	1·45	1·30	1·03

A bar of wrought iron 9 ft. long, 1 in. broad, and 3 ins. deep, with a load of 1778 lbs., gave the following relations between the velocities and deflections, in which the latter pass the maximum limit:

Velocity, in feet, per second		15	29	36	43
Central Deflection	·29	·38	·50	·62	·46

The following Tables contain a Summary of the central deflection in the three first Series of the Portsmouth Experiments, showing the velocities and weights employed, the statical deflections due to those weights, the dynamical deflections obtained, and the ratio between the statical and dynamical deflections in each case.

The bars were all 9 feet long between the supports: the first column in the following Tables contains the number corresponding to each experiment (or rather set of experiments) in the detailed Tables given in the Appendix to the Report, p. 215. The second column gives the weight upon each pair of bars. The third column contains the statical central deflection due to the weight. The first deflection in each experiment which corresponds to the weight of 1120 lbs. was obtained by trial, the remainder for the higher weights, calculated as explained above. The fourth column contains the dynamical deflections given by the experiment. Finally, the fifth column is the ratio of the dynamical to the statical deflection.

Each experiment was terminated necessarily by one or both bars breaking. This fact is recorded by the word "*broke*," inserted in that part of the Table which belongs to the fractured bar.

TABLE I.—FIRST SERIES.

Bars 1 inch broad, 2 inches deep.

No. of Experiment.	Weight in lbs.	Left Bar.			Right Bar.		
		Statical deflection.	Dynamical deflection.	Ratio.	Statical deflection.	Dynamical deflection.	Ratio.
Velocity 15 feet per second.							
4	1120	·88	1·24	1·41			
	1240	1·10	1·70	1·54			
	1440	1·48	1·98	1·34			
	1560	1·71	2·51	1·47			
	1760	2·09	3·00	1·43			
	1876	Broke.					
5	1120	·86	1·11	1·28			
	1240	1·06	1·41	1·33			
	1440	1·45	1·94	1·24			
	1560	1·66	2·50	1·55			
	1760	2·03	3·06	1·51			
	1788	2·10	3·53	1·68			
	1816	2·12	3·61	1·70			
	1844	2·18	4·17	1·91	Broke.		
6	1120	·62	·74	1·19			
	1240	·77	·88	1·14			
	1356	·93	1·10	1·18			
	1460	1·07	1·34	1·25			
	1560	1·20	1·76	1·47			
	1680	1·36	2·37	1·74			
	1792	1·51	2·90	1·92			
	1816	Broke.					
Velocity 24 feet per second.							
7	1120	·64	1·02	1·59			
	1240	·80	1·55	1·94			
	1356	·96	2·70	2·81			
	1412	1·04	3·16	3·04			
	1440	Broke.	..	..	Broke.		
8	1120	·65	·87	1·43	·88	1·10	1·25
	1240	·81	1·10	1·35	1·10	1·30	1·18
	1356	·98	2·32	2·37	1·32	1·60	1·21
	1412	1·06	2·85	2·69	1·43	2·43	1·70
	1440	1·09	3·81	3·50	1·48	2·76	1·86
	1468	1·13	..	..	1·54	2·88	1·87
	1496	1·17	3·94	3·37	1·59	2·94	1·85
	1524	Broke.	..	..	Broke.		

FIRST SERIES—(*continued*).

Bars 1 inch broad, 2 inches deep.

No. of Experiment.	Weight in lbs.	Left Bar.			Right Bar.		
		Statical deflection.	Dynamical deflection.	Ratio.	Statical deflection.	Dynamical deflection.	Ratio.
Velocity 24 feet per second.							
9	1120	·74	1·14	1·54	·72	1·10	1·52
	1240	·92	1·47	1·59	·90	1·30	1·44
	1356	1·10	1·74	1·58	1·08	1·68	1·55
	1412	1·20	2·02	1·70	1·17	1·70	1·45
	1440	1·24	2·23	1·80	1·21	2·00	1·65
	1468	1·29	2·41	1·87	1·25	2·36	1·89
	1496	1·33	2·54	1·90	1·29	2·74	2·12
	1520	1·37	2·68	1·96	1·33	3·00	2·26
	1552	1·42	2·77	1·95	1·38	3·24	2·35
	1580	1·46	3·08	2·11	1·42	3·60	2·54
	1604	Broke.	..	..	Broke.		
Velocity 29 feet per second.							
10	1120	·95	1·80	1·89	1·80	2·10	2·10
	1240	Broke.	..	..	Broke.		
11	1120	1·17	2·54	2·17	·75	2·04	2·71
	1176	1·31	3·36	2·56	·84	2·65	3·15
	1204	Broke.	..	..	·89	3·10	2·76
12	1120	·96	2·30	2·39	1·18	2·04	1·72
	1176	1·08	3·03	2·80	1·32	2·68	2·03
	1204	Broke.	..	..	Broke.		
Velocity 33 feet per second.							
13	1120	·84	2·02	2·40	·73	1·86	2·55
	1176	·94	2·67	2·83	·82	2·26	2·76
	1204	Broke.	..	..	·87	2·60	2·99
14	1120	·81	1·31	1·61	·70	1·15	1·64
	1176	·91	1·86	2·05	·78	1·50	1·92
	1204	·96	2·44	2·54	·83	1·91	2·28
	1232	1·00	3·02	3·02	·87	2·46	2·83
	1260	1·04	3·65	3·51	·90	2·80	3·11
	1288	Broke.	..	..	·95	2·90	3·05
15	1120	1·30	3·04	2·34	1·12	2·48	2·21
	1148	Broke.					
Velocity 36 feet per second.							
16	1120	·86	1·86	2·16			
	1148	·94	2·25	2·38	Broke.		

FIRST SERIES—(*continued*).

Bars 1 inch broad, 2 inches deep.

No. of Experiment.	Weight in lbs.	Left Bar. Statical deflection.	Left Bar. Dynamical deflection.	Left Bar. Ratio.	Right Bar. Statical deflection.	Right Bar. Dynamical deflection.	Right Bar. Ratio.
Velocity 36 feet per second.							
17	1120	·72	1·64	2·28	·70	1·50	2·14
	1148	·76	2·26	2·97	·74	2·08	2·80
	1176	Broke.	..	..	Broke.		
18	1120	·70	1·50	2·14	·70	1·40	2·00
	1148	·74	2·10	2·83	·74	1·73	2·33
	1176	·78	2·31	2·76	·78	2·14	2·74
	1204	Broke.	..	..	Broke.		

TABLE II.—SECOND SERIES.

Bars 1 inch broad, 3 inches deep.

No. of Experiment.	Weight in lbs.	Left Bar. Statical deflection.	Left Bar. Dynamical deflection.	Left Bar. Ratio.	Right Bar. Statical deflection.	Right Bar. Dynamical deflection.	Right Bar. Ratio.
Velocity 15 feet per second.							
4	1120	·37	·41	1·1	·39	·41	1·05
	1778	·69	·58	·87	·73	·70	·96
	2348	1·02	·97	·95	1·07	1·00	·93
	2955	1·47	1·65	1·11	1·55	1·46	·94
	3296	1·74	2·35	1·34	1·84	1·95	1·06
	3352	1·80	2·70	1·5	1·90	2·38	1·25
	3408	Broke.					
5	1120	·38	·42	1·10	·44	·47	1·06
	1778	·71	·69	·97	·82	·72	·88
	2348	1·05	1·02	·97	1·21	1·02	·84
	2955	1·51	1·66	1·10	1·74	1·58	·91
	3296	Broke.	..	..	..	1·72	
6	1120	·29	·31	1·07	·27	·36	1·33
	1778	·54	·60	1·11	·51	·60	1·18
	2348	·80	·83	1·04	·75	·80	1·07
	2955	1·19	1·50	1·26	1·07	1·15	1·07
	3296	1·37	1·85	1·35	1·28	1·32	1·03
	3408	1·46	2·22	1·52	1·36	1·45	1·06
	3464	1·50	2·65	1·76	1·40	1·56	1·11
	3496	Broke.	..	..	..	1·82	
Velocity 29 feet per second.							
7	1120	·32	·36	1·11	·32	·42	1·31
	1778	·60	·76	1·26	·60	·86	1·43
	2348	·88	1·36	1·52	·88	1·34	1·52
	2670	1·07	1·82	1·70	1·07	1·78	1·66
	2775	1·14	2·06	1·80	1·14	1·88	1·65

SECOND SERIES—(*continued*).

Bars 1 inch broad, 3 inches deep.

No. of Experiment.	Weight in lbs.	Left Bar.			Right Bar.		
		Statical deflection.	Dynamical deflection.	Ratio.	Statical deflection.	Dynamical deflection.	Ratio.
Velocity 29 feet per second.							
7	2831	1·18	2·16	1·83	1·18	1·91	1·62
	2887	1·22	2·27	1·86			
	2943	1·26	2·52	2·00			
	2999	1·30	2·67	2·05			
	3167	Broke.	..	..	Broke.		
8	1120	·42	·54	1·28	·45	·64	1·42
	1778	·79	1·19	1·50	·14	1·09	1·30
	2348	1·15	2·02	1·75	1·24	1·57	1·26
	2955	Broke.	..	..	Broke.		
9	1120	·33	·52	1·57	·32	·42	1·31
	1778	·62	·88	1·42	·60	·76	1·26
	2348	·91	1·59	1·75	·88	1·58	1·80
	2955	1·31	2·77	2·07	1·27	2·01	1·58
	3011	Broke.	..	..	Broke.		
Velocity 36 feet per second.							
10	1120	·39	·67	1·71			
	1778	·73	1·12	1·53			
	2348	1·07	2·08	1·94			
	2468	Broke.	..	..	Broke.		
11	1120	·34	·50	1·47	·37	·58	1·56
	1778	·62	1·09	1·75	·69	1·03	1·49
	2348	·92	1·90	2·05	1·02	1·78	1·73
	2404	Broke.					
12	1120	·49	·72	1·47	·40	·72	1·8
	1778	·93	1·31	1·42	·75	1·54	2·05
	2348	Broke.	..	..	Broke.		
Velocity 43 feet per second.							
13	1120	·34	·44	1·29	·30	·46	1·53
	1778	·62	·93	1·50	·56	1·18	2·10
	2066	·76	1·56	2·04	·68	1·84	2·67
	2182	Broke.	..	..	Broke.		
14	1120	·27	·52	1·92	·30	·68	2·27
	1778	·51	1·07	2·10	·56	1·30	2·31
	2066	·61	1·87	3·07	·68	2·00	2·94
	2122	Broke.	..	..	Broke.		
15	1120	·24	·38	1·58	·26	·50	1·92
	1776	·45	·86	1·90	·50	1·02	2·04
	2066	·55	1·30	2·35	·59	1·40	2·36
	2182	·60	1·86	3·09	·65	2·02	3·11
	2242	Broke.	..	..	Broke.		

TABLE III.—THIRD SERIES.

Bars 4 inches broad, 1½ inch deep.

No. of Experiment.	Weight in lbs.	Statical deflection.	Dynamical deflection.	Ratio.
Velocity 15 feet per second.				
3	1120	·43	·63	1·46
	1778	·83	1·35	1·64
	2348	1·27	2·00	1·57
	2955	1·88	3·78	2·01
	3191	2·17	4·65	2·14
	3247	2·23	4·85	2·17
	3303	Broke.		
4	1120	·57	·78	1·36
	1778	1·10	1·45	1·32
	2348	1·71	2·21	1·29
	2955	2·54	4·12	1·62
	3296	3·04	4·85	1·59
		Right bar broke.		
Velocity 29 feet per second.				
5	1120	·74	1·08	1·45
	1778	1·44	2·04	1·42
	2066	1·82	2·92	1·43
	2348	2·21	4·14	1·87
	2670	Both bars broke.		
6	1120	·60	1·01	1·68
	1778	1·16	2·17	1·87
	2348	1·80	3·72	2·06
	2670	Broke.		
Velocity 36 feet per second.				
7	1120	·52	·95	1·82
	1778	1·00	2·19	2·19
	2060	1·26	3·88	3·08
	2176	Broke.		
8	1120	·58	1·23	2·11
	1778	1·12	3·11	2·78
	2060	Both bars broke.		
Velocity 43 feet per second.				
9	1120	·63	1·54	2·45
	1778	Both bars broke.		
10	1120	·50	1·28	2·56
	1402	·69	2·31	3·35
	1522	·77	3·18	4·13
	1638	·85	4·39	5·14
		Right bar broke.		

The mode in which the bars were fractured in the above experiments is delineated in Plate III. It will be seen that the fractures took place, with few exceptions, at points beyond the centre of the bar, and that the bars were usually broken into three, and often into four or five pieces, thus indicating a great and violent strain towards the end of the transit of the load, which will be found in perfect accordance with the theoretical and experimental results given in the succeeding chapters. It must be observed that in all the examples of the third series, in which broad thin bars are used, there is but a single fracture, and that always beyond the centre.

The results which we have passed in review were obtained from horizontal straight bars. But it was suggested that if the bars were curved, or made convex upwards, the increase of deflection produced by the velocity of the load would be certainly diminished, and might be entirely removed; for as the effects in question are analogous to the centrifugal action of bodies moving on curves, if the bar were curved into such a form that the weight of the load should depress it exactly to the horizontal line, passing through its bearing points, then this centrifugal action would be completely destroyed. And if this were not exactly effected, the convex curvature would diminish the pressure of the moving load. It was for the purpose of following out these views that the 'Eighth Series of Experiments,' namely, upon curved bars, which will be found in pages 241 to 244 of the Parliamentary Report, were undertaken. They show a considerable reduction in the increment of deflection produced by the velocity of the load, but they were not carried far enough to lead to complete results.

It is very doubtful whether in practice difficulties would not be introduced by the attempt to curve the rails, that would counterbalance the diminution of deflection. A bad joint or sudden change of direction in the rails has a much greater effect in enabling the carriages to shake and strain the bridge than the velocity of the load can possibly produce. Now although the bridge may be, and indeed generally is, curved or cambered upwards in a slight degree, the rails are laid in straight lengths. Thus they form a portion of a polygon with very obtuse angles, and a carriage travelling with the high velocity employed on railways is necessarily at each angle of this polygon, that is, at each

joint of the rail, projected onwards in the direction of the rail it has left, so as to fall, in a small parabola, upon the next rail, with a blow that, repeated as it is by the continually passing carriages, gradually serves to deteriorate and disarrange the joints of the railway. This effect is very observable, and in the experiments of the Commission upon Ewell Bridge I was able to detect it by the jumping of the engine, &c. during the passage of the train, while I was stationed beneath the bridge to watch the deflectograph. The rails of this bridge are carefully laid with good joints, but the rails, as above described, are straight, and the bridge cambered.

The experiments upon Ewell and Godstone Bridges, the results of which are given below,* were made for the purpose of comparing the startling and unexpected results obtained at Portsmouth with some cases of real practice in order to discover

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* *Experiments made by the Commissioners on the Ewell and Godstone Bridges.*

The apparatus employed in making these Experiments is detailed in Plate IV.

Ewell Bridge. (*Epsom and Croydon Railway.*)

Span, 48 feet.
Two girders to support each line of rails.
Depth of girders at centre, 3 feet 6 inches.
Width of bottom flange, 20 inches.

	Tons.
Thickness of do., 3 inches.	
Weight of two girders	20
Weight of platform between these girders . . .	10
Total weight of half the bridge . . .	30
Weight of engine	25·2
Weight of tender	13·8
Total	39

Velocity in feet per second.	Deflection in decimals of an inch.
0	·215
25	·215
30·9	·23
32·3	·225
53·7	·245
75	·235

The deflections do not increase steadily, but this could hardly be expected from the many causes of disturbance.

whether an increase of deflection was to be found in actual bridges of the same nature and amount as those which exhibited themselves upon the 9-feet bars. It will be seen that in the Ewell Bridge, the span of which is 48 feet, the statical deflection produced by the engine and tender was only 0·215 in. This was increased to 0·245 in. by a velocity of 54 feet per second, or about 35 miles per hour. A velocity of 75 feet gave a somewhat less deflection, namely 0·235 inch:—

Hence $$\frac{\text{greatest dynamical deflection}}{\text{statical deflection}} = 1\cdot 14,$$

exhibiting an increase of about one-seventh.

In the case of the Godstone Bridge, the span was 30 feet, the statical deflection produced by the engine and tender was 0·19 in., and the dynamical deflection due to a velocity of 73 feet per second was 0·25 inch.

Hence $$\frac{\text{dynamical deflection}}{\text{statical deflection}} = 1\cdot 315,$$

showing an increase of little short of one-third.

In experiments of this kind the deflections must be ascertained

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Godstone Bridge. (South Eastern Railway.)

Span, 30 feet.
Three girders support the roadway.
Depth of girders at centre, 3 feet.
Width of bottom flange, 15 inches.
Thickness of do., 2¼ inches.

	Tons.
Weight of two girders	15
Weight of platform between these girders	10
Total weight of half the bridge . .	25
Weight of engine	21
Weight of tender	12
Total	33

Velocity in feet per second.	Deflection in decimals of an inch.
0	·19
22	·23
40	·22
73	·25

very carefully, for they are so small that the increase may escape notice altogether if roughly measured. Yet it must be remembered that the increase of pressure on the bridge produced by the dynamical action is measured by the increase of the deflections, however small the deflections themselves may be. We therefore selected bridges which were built to carry railways over roads, so that we could erect a temporary scaffold upon the road that should be perfectly independent of the flexure of the bridge above, and of easy access, (Plate IV.) Upon this scaffold was fixed a vertical drawing-board to receive the trace of a pencil, clamped to the lower edges of one of the girders of the bridge. Thus the pencil during the passage of the engine and tender traced a vertical line equal to the deflection. The board was constructed so as to admit of being shifted horizontally after each deflection had been traced, and thus to be ready to receive the trace of the next. The pencil was carefully watched during the passage of the load to guard against accidental jerks or shifts of the apparatus, which, however, were not found to happen.

TABLE OF VELOCITY.

Velocity in feet per second.	Velocity in miles per hour.	Height in feet due to Velocity.
10	6·82	1·55
15	10·2	3·49
20	13·6	6·21
30	20·5	13·97
40	27·3	24·8
44	30·	30·05
50	34·1	38·82
60	40·9	59·00
70	47·7	76·08
80	54·5	99·37
88	60·	120·24
90	61·4	125·77
100	68·2	155·27

The foregoing Table may be useful for reference during the reading of this Essay, to compare velocities, measured in feet and miles respectively.

CHAPTER II.

On the general Nature of the Problem, and on the Apparatus employed by me at Cambridge to obtain the Trajectory mechanically.

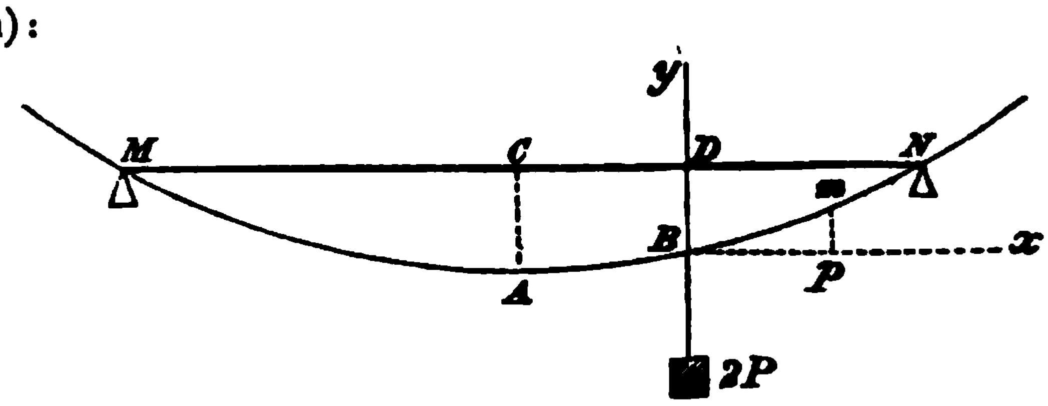
HAVING now explained the apparatus employed at Portsmouth, and the remarkable results which it has produced, it remains to examine the laws which connect the phenomena, in order to extend them to larger structures, and ascertain the effects of moving loads upon actual bridges. A few simple mechanical considerations will explain the method in which I shall proceed to investigate this part of the subject.

Let A, B, fig. 1, Plate V., be two fixed props at the same horizontal level, upon which an elastic bar, $A B$, rests. This bar is of equal section throughout, and its weight is supposed to be so small that it may be neglected. If a weight, W, be suspended to any given point, P, of the bar, it will depress it, and cause the bar to assume the form of a certain curve, $A P D E B$, of which the equation is known.* The principal properties of this curve with which we are at present concerned are as follows:—

1. It is convex downwards throughout.

2. The greatest curvature is at the point of suspension of the

* The equation to this curve is given by Navier, 'Application de la Mécanique à l'Etablissement des Constructions et des Machines,' Paris, 1833, tom. i. p. 231, in the following form (with a slight modification of the notation):



$M N$ the two props, $M A B N$ the bar loaded with a weight, $2 P$, which is suspended to a point B, not in the centre.

Let $M N = 2 a$, C the centre of the bar, $C D = z$, $B p = x$, $m p = y$, $B D$

weight, P; and this is the point at which the bar would break if the weight were increased sufficiently to produce rupture.

3. If the weight be suspended from the centre, Q, its point of suspension will coincide with the point of the greatest deflection of the bar, and the curve will be symmetrical. But if the point of suspension be out of the centre, as at P in the figure, then it will no longer be the point of the greatest deflection. This greatest deflection, or maximum ordinate of the curve, will be found at M, between the point of suspension, P, and the centre of the curve, D, but much nearer to the latter. In fact, it can be shown that whatever be the horizontal distance of the point of suspension from the centre of the bar, the distance of the point M from the centre can never be greater than 0·154 of the half-length of the bar.

4. A given weight, W, suspended to the bar, will produce a greater or less amount of deflection in the entire bar, according as its point of suspension is nearer to or farther from the centre respectively, and, consequently, the greatest deflection of all when suspended from the centre itself.

5. The deflection of the point of suspension itself can be shown

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(the deflection of the point of suspension below the horizontal line) $= f$, the angle which the tangent to the curve of the bar makes at B with the horizon $= w$, the deflection which the weight 2 P would produce in the bar if suspended from the centre $= S$. Then it can be shown that for the part of the curve $B\,N$ we have

$$y = 3\,S.\frac{a+z}{a^4}\left\{\tfrac{2}{3}\overline{a-z}\,.\,zx + \tfrac{1}{2}.\overline{a-z}\,.\,x^2 - \tfrac{1}{6}.x^3\right\};$$

the equation to the other part of the curve, $B\,M$, will be found by writing z negative. We have also

$$f = \frac{S}{a^4}(a^2 - z^2)^2\ .\ \tan.w\,. = \frac{2\,S}{a^4}(a^2 - z^2)\,z.$$

The value of x, which corresponds to the greatest deflection of the curve below the horizontal line, is given by the equation

$$x = a + z - \sqrt{a^2 + \tfrac{2}{3}\,a\,z - \tfrac{1}{3}\,z^2},$$

in which x is measured backwards from B towards M. If the point of suspension be gradually shifted nearer to N, this ordinate of greatest deflection will increase its distance from the centre of the curve, which distance will be the greatest when B coincides with N, in which case $z = a$, and we obtain ·154 × a for the distance of the ordinate of greatest deflection from the centre of the curve.

to vary directly as the weight, W, multiplied by the square of the product of the segments into which the point of suspension divides the bar (supposing, which is always the case in the subject under consideration, that the deflection is small compared with the length of the bar). The most convenient expression for the deflection of the point of suspension is the following. Let P be the point of the bar from which the weight, W, is suspended, and $AN = x$, $NP = y$, be its co-ordinates; let a be half the length of the bar, or half the distance between the props; S the deflection which the weight, W, would produce in the centre of the bar, if suspended there: then, because the ordinate y is the deflection of the suspending point P, and this ordinate divides the line AB into the segments x and $2a - x$, we have, from what has been above stated,

$$S : y :: a^4 : x^2 (2a - x)^2 \therefore y = \frac{S}{a^4} (2ax - x^2)^2.$$

Having laid down these principles, which are derived from writers on the strength of materials, let us suppose the point of suspension of the given weight, W, to be shifted in succession to a series of points along the length of the bar, lying pretty close together. If a board covered with paper be fixed behind the bar, so as just to leave space for freedom of motion in the latter, and if these successive points of suspension be marked upon the paper, we shall obtain a dotted line, $APQRB$, as shown in the figure, which is the *locus* of the points of suspension; and of course, if the successive points be taken in sufficient number to lie very close together, we obtain a continuous curve for this *locus*. It is easy to see that the expression obtained above for the amount of deflection produced at the suspending point by a given weight, namely,

$$y = \frac{S}{a^4} (2ax - x^2)^2, \text{ is, in fact, the equation to this } locus.$$

It is better, perhaps, to conceive the weight to be a small heavy cylindrical body resting on the upper flat surface of the bar, and capable of rolling along it, instead of being suspended by a hook, as the former hypothesis approaches nearer to the actual problem which we have to solve, namely, the travelling of a carriage along a bridge. It will thus be perceived that the dotted curve is the path, or *trajectory*, which the centre of this

body describes in space during a very slow and gradual passage along the bar, or, rather, a shifting motion from one end to the other, point by point. This form of the trajectory only corresponds to the very slowest continued motion of the body along the bar. Always supposing the body to travel with a uniform motion from one end to the other, the slightest increase of its velocity produces a change in the form of the trajectory, which change is greater as greater velocities are taken. The exact nature and amount of this change under different circumstances will be shown below, as well as the methods by which it was determined, but the general effect is, that the curve is no longer symmetrical to the centre; the greatest depression of this curve being thrown into the second half of it, while the first half is less depressed than with the slow motion. The dotted curve $A\,P\,Q\,R\,B$, above described, is the form of the trajectory, which is the limit to all these forms, and corresponds to the very slowest motion, or, rather, to the shifting motion of the weight, in which the system is in statical equilibrium at each successive position of the load. On the other hand, the dotted curve $A\,G\,H\,K\,L$ is one of the forms which the trajectory assumes when velocity is imparted to the body. To distinguish the first form of the trajectory from the others, I shall term it the *equilibrium trajectory*. The object of the investigation which follows is to examine the form and proportion of these trajectories in general, under different relations between the elasticity, dimensions, and weight of the bar, and the magnitude and velocity of the load; first describing the experimental inquiry, and next proceeding to the theoretical principles by which the laws of the phenomena and the *modus operandi* of the forces which are called into action may be developed.

It must be carefully observed that the equilibrium trajectory is a totally different curve from the curve into which the bar is bent at every different position of the weight. In fact, the two curves only coincide at two points, namely, that at which the weight is suspended, and a point at the opposite end. These two points of intersection merge into one, and become a point of contingence at the instant the body passes the centre.* Thus the point at

* It may be useful to mention that from the equation of the equilibrium curve (z), it can be shown easily that its radius of curvature at each extremity

which the equilibrium trajectory *touches* the curve of the bar corresponds to the greatest deflection of the bar.

When we know the form of the trajectory under any of its phases, whether as the equilibrium curve or as the curve corresponding to any given velocity, we can also find the form of the bar at any moment; for the bars are so stiff and the deflections so small, that we may assume the bar at every instant of the passage of the load to be bent into the same curve which it would assume if the point of application of the load were pressed down statically to the same position.*

Thus, in fig. 2, Plate V., let $A\,E$ be the fixed points upon which the bar is supported, and let the dotted curve $A\,b_2\;c_3\;d_4\,f\,g$ be the trajectory which the body describes in its passage along the bar with considerable velocity. Draw through the points $A\,b_2\,E$ the curve $A\,b_2\;c_2\;d_2\;E$, into which the bar would be bent, if a sufficient weight were suspended at b_2, to depress the bar to that point. This curve may be supposed to be the form into which the bar is actually thrown at the instant of the body's passage over the point b_2 of the trajectory.† Similarly, when the body passes over the central line at c_3, the momentary form of the bar will be obtained by drawing through the points $A\,c_3\,E$ the proper curve $A\;b_3\;c_3\;d_3\;E$; and when the body has arrived at d_4, the

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$A, B = \frac{a^2}{8\,S}$ (measured downwards). Its central radius of curvature (measured upwards) is $\frac{a^2}{4\,S}$, or twice the former. The latter, supposing the deflection small, is half the radius of a circle drawn through the extremities of the bar and its central depressed point. The two values of x, which correspond to the two points of contrary flexure, are, $a \pm \frac{a}{\sqrt{3}}$, and the corresponding value of the ordinates is $\frac{4}{9}\,S$.

* This would not be the case if the bar were exceedingly slender, and may perhaps not be strictly true even in some of the experiments given above. I have shown below how this point may be examined, but I do not believe that any sensible error has been introduced into the result by the above assumption.

† The curve of the bar may be drawn by points from its equation, but more simply by means of a slender straight steel rod resting on two pins driven into the drawing-board at the ends of the curve of the trajectory, and depressed by hand to any desired point of the latter.

form of the bar will be $A\ b_4\ c_4\ d_4\ E$. This diagram may serve to illustrate the general nature of the action that takes place in all the experiments in question, and to show how completely different the curve of the trajectory is from the curves into which the bar is bent.

In the equilibrium curve the greatest deflection corresponds to the greatest deflection of the bar, and happens at the centre of the bar, where the two curves have a common tangent. But the above figure shows that this is not the case in the other phases of the trajectory. The point of greatest deflection of the trajectory lies a little beyond d_4. The point where the body produces the greatest central deflection of the bar will be found beyond d_4, by drawing through $A\ E$ a curve of the bar that will touch the trajectory. The entire bar will thus be evidently a little more depressed than the lowest curve shown in the figure.

The operation of the registering apparatus (see page 436) will now be more clearly understood. Five pencils were in reality attached to the bar, but, for simplicity sake, we will suppose only three to have been employed, and fixed to the bar at equal distances, from the ends and from each other respectively, at the points $B\ C\ D$, fig. 2. If these pencils were to trace their lines upon a fixed board, we should merely obtain for each a line that would give the greatest deflection that each point of the bar had attained, but no information with respect to the position of the body at which this greatest deflection was given, or with respect to the trajectory of the body.

In fig. 3, Plate V., the curves of the trajectory and bar are drawn in exact correspondence with fig. 2. The board, placed behind the bar, is supposed to receive a small constant horizontal motion, such that during the passage of the body from A to E the board shall travel through a space equal to the distance from 1 to 5 in the groups of parallel lines shown in the figure opposite to each of the points A, B, C, D, and E.

Thus, at the beginning of the motion, the point A_1 was opposite that end of the bar, and the points $B_1\ C_1\ D_1$ were similarly opposite to the respective pencils with which the bar is furnished. When the body reaches B, the motion of the board brings all the points marked 2 opposite their respective pencils, and when it has reached C, all the points marked 3 will be opposite their

respective pencils, and so on. The lines at E similarly show the points of the drawing-board that are brought opposite to that extremity of the bar by the motion. The vertical lines $B\, b_2\, b_3\, b_4$, $C\, c_2\, c_3\, c_4$, $D\, d_2\, d_3\, d_4$, shown in fig. 2, are thus, by the motion of the board, opened out into the curves designated in fig. 3 by the same letters respectively; and these curves furnish as many points through which to draw, not only the trajectory, but the curves of the bar.

When the body had arrived at A, the bar was horizontal, and its figure, therefore, passes through the points $A_1\, B_1\, C_1\, D_1\, E_1$. When the body comes to B, every line headed 2 has come opposite to the respective points of the bar, and the intersections of the pencil curves with these lines taken in order, namely, the points $b_2\, c_2\, d_2$, are points through which the bar must at that instant pass. Similarly, the points $A_3\, b_3\, c_3\, d_3\, E_3$ serve to draw the form of the bar when the body passes the centre, and $A_4\, b_4\, c_4\, d_4\, E_4$ is the curve of the bar when the body passes beneath the point D.

Points in the trajectory, on the other hand, are obtained by taking lines from the groups, each headed with a successive number; thus the lines $A_1\, B_2\, C_3\, D_4\, E_5$ will, by their intersections with the pencil curves, give the points required. For when the body was at A, A was opposite to that point of the bar, and is, therefore, a point in the trajectory. When the body reached B, the line 2 on the board was brought opposite to it, and thus b_2 is the next point in the trajectory, and so on.*

To insure the proper working of this contrivance it is necessary that it should be made with great delicacy and care. A perfectly

* It will, of course, be seen that the length of the trajectory thus obtained is greater than the length of the bar, by a quantity equal to the space 1–5, described upon the board. But this elongation is of no consequence, because it does not destroy the proportion between the abscissæ and ordinates of the curve, the velocity of the board being constant. The curves of the bar obtained in this manner are its real curves, and may serve to try whether the form of the bar is really sensibly different from its statical curvature. But the apparatus in question should only be employed when the experiments are conducted on a tolerably large scale with great loads, because the friction and inertia of its parts may seriously interfere with the motion of bar and load when the latter is small. Hence I have not introduced it into my smaller apparatus.

equable travelling motion ought to be given to the drawing-board by clockwork, or rather the pencils should be so arranged as to trace their curves upon the surface of a cylinder, which is perfectly practicable, although I have preferred describing the mechanism as applied to a travelling-board, on account of its greater simplicity. The board is objectionable, because its length, necessarily limited, compels it to be set in motion as soon as possible before the car is started, else it may arrive at the end of its course before the car has completed its journey over the bar. This increases the difficulty of giving it an equable velocity. A cylinder, on the other hand, may continue revolving as long as may be necessary.*

It will easily be seen that, however irregular the motion of the board may be, a true form of the bar will be always obtained from the group of pencil curves, by taking a series of points at the same respective distances from each other as the pencils. By means of these curves, therefore, we may, without reference to the velocity of the board, determine from each experiment not only the maximum deflection that has been given to every one of the five points in succession, but also the contemporaneous deflection of the remaining points.

Thus in fig. 3 the maximum deflection in the central pencil curve is shown to have taken place between the lines 4 and 5, that is, when the travelling load has reached a point beyond the

* The paper cylinder should be fixed below the bar with its axis parallel to it. Each pencil to be attached to the vertical arm of a right-angled bell-crank lever also mounted below the bar upon a horizontal axis at right angles to the direction of the bar, the horizontal arm of the same lever to be connected with the bar above by means of a link rod, jointed to the arm at its lower extremity and to the bar at its upper extremity. Its connection with the bar to be made by forming the link into a branch embracing the bar, each arm of which has a pointed centre-screw, which enters a small hole punched in the side of the bar (see fig. 8, Plate VI.). Thus, when the bar descends, a horizontal motion will be given to the pencil; and as the bar, the pencils, arms, and links, and the axis of the cylinder, lie in one vertical plane, the same revolving cylinder will receive all the curves. But the apparatus must be carefully constructed, so as to be as light and as free from friction as possible. The pencils should be fixed in small swing frames, and the whole mechanism be protected by a shield between itself and the bar, to avoid injury when the bar breaks.

centre between D and E; and if we take a point upon each curve at the same distance between their respective lines 4 and 5, we shall obtain the deflections at each point respectively that accompanied the maximum deflection at the centre, or, in other words, the form of the bar at that instant. Similarly we might obtain the maximum deflection at B, and the contemporaneous deflections at the other points, and so on for all.

But the form of the *trajectory* of the body can only be determined from such curves when the board moves uniformly, or at least when its motion is perfectly known, and the times of the body passing the several points of the bar registered upon it. As this was found impracticable with the Portsmouth apparatus, from the roughness of the mechanism, and a better mode had presented itself for obtaining the trajectory, the apparatus in question was confined to obtaining the maximum deflection, as above explained.

After all, however, the method of registering the trajectory by five points is evidently insufficient, and for the perfect knowledge of the effects I soon found it necessary that the entire course of the curve should be recorded. This may be effected by causing a pencil attached to the centre of the car to trace a line upon a drawing-board fixed parallel to its course. But this simple expedient can only succeed when the car moves with great steadiness,—a condition which the nature of the Portsmouth apparatus placed wholly out of the question.

The theoretical investigation of the problem is replete with difficulty, and its complete solution appears beyond the bounds of analysis. A limited solution can only be obtained by reducing the conditions to their simplest form, namely, by supposing the weight of the bar to be so small, compared with that of the load, that its mass may be wholly neglected; by considering the load as resting on the bar at one point only, and its mass to be concentrated in that point; and lastly, by supposing the deflection to be small compared with the length, which latter condition is true in practice. With these limitations not only can the form of the trajectory be obtained theoretically, but, as we shall see, other laws can be deduced which completely enable us to group the experimental phenomena and extend them to practical cases.

But for this purpose an apparatus must be so arranged as to

approach, as nearly as possible, to the simple conditions upon which the theory is based, in order the better to compare their respective results.

The simplest considerations serve to show that, provided the due proportions be maintained between the loads, velocities, and stiffness of the bar, the curves of the trajectory and bar respectively will be the same, whether small weights running on light bars be employed or heavy loads travelling upon massive bars. But in the former case the experiments may be made with an apparatus capable of construction with any required degree of delicacy and accuracy, with small friction, easily manageable and capable of being contained in an ordinary laboratory; and in the latter case the great loads and heavy bars are necessarily accompanied with unsteadiness of motion and great friction, and a general magnitude and roughness, which makes it necessary to employ several workmen and much time in each experiment, and to require the resources and space of a Government dockyard.

The radical defect of the Portsmouth apparatus, for the purpose we are now seeking, proved to be the employment of a car resting with four wheels upon two trial bars at once. In the first place the load presses with two wheels upon each bar, the bar being 9 feet long and the wheels (or rather axles) 2 feet 10 inches apart; it therefore results that when the car first enters upon the bar, the pressure of the fore wheel only acts upon the latter. When the car has advanced through a space equal to the distance between the axles, the pressure of the hind wheel also begins to act, and now the bar is subjected to the action of two loads pressing at a constant distance from each other, and this continues until the fore wheel reaches the end of the bar, which is then subjected to the pressure of the hind wheel alone. Thus a complex form of trajectory is obtained which cannot be compared with the theoretical results, and which, after all, is not much nearer to the practical effect of a four-wheel carriage upon a bridge than a load pressing on a single point would be, because the distance between the wheels is so much greater in proportion to the length of the bridge than in the real case. Again, great difficulties are introduced by the simultaneous employment of two bars. Whatever care may be taken in selecting bars, it is next to impossible to find a pair of exactly equal strength, or, if found, to

arrange the load on the carriage so that it shall press equally upon both bars and upon both hind and fore wheels. Hence an inevitable inequality in the simultaneous deflections of the bars, which, as the centre of gravity of the load is high, throws greater weight upon one side than on the other during the passage of the car. This, besides disturbing the results, tends to induce lateral oscillations that increase unduly the deflections on either side, and produce anomalies in the general effects. It was this lateral shake which prevented the trajectory from being traced by the continuous motion of a pencil. The principal excellence of the Portsmouth experiments consists in the determination of the effect of velocity upon the breaking weights on a large scale, for which purpose they will be found to give a most valuable and novel collection of facts.

For the purpose of obtaining the trajectory experimentally, I found it necessary to contrive and construct an apparatus in which the required conditions of simplicity should be complied with. The principles of this apparatus I had indeed suggested from the beginning, and was desirous of introducing into the larger machine, but it was thought advisable that the latter should be made to resemble the case of a car running on a bridge as much as possible, in order to insure the confidence of practical engineers in the results that might be obtained.

As the purpose of this small apparatus was to determine the trajectory without reference to the fracture of the bars, the material I selected was naturally steel, as being the most elastic and free from set. Thus the same bar could be used for many experiments, which greatly facilitates their comparison. Experiments upon cast iron are always embarrassed by the accumulation of set and the occasional fracture of the bars. The machine was therefore arranged to operate upon steel bars of 4 feet or less in length, and of such a stiffness as would require a weight not greater than 6 lbs. to produce a sufficient deflection.

A single trial bar was employed, and the weight pressed upon that bar at one point only. The arrangement by which these conditions were carried out consists of a carriage, which runs on four wheels, upon a kind of railway. The carriage supports a horizontal swing frame, one end of which is hinged to it; the other end has a roller, which rests on the bar, and is also capable

of being loaded at pleasure, so as to press more or less upon the bar. The trial bar, in fact, forms the continuation of an intermediate rail which lies between the two rails that support the wheels of the carriage. Thus the only purport of the carriage is to give steadiness to the weight, and confine its motion to a vertical plane. The weight presses with perfect freedom upon the bar, deflecting it during its passage, while the carriage runs steadily along the horizontal rails between which the bar is fixed. A pencil, attached to the swing frame, rises and falls proportionally to the deflection, and traces the curve of the trajectory upon a vertical drawing-board, which is fixed parallel to the trial bar, and opposite to it.

This apparatus is figured in Plate VI., and I will now proceed to describe its details.

Figs. 1 and 2 show the plan and elevation of the railway, and its inclined plane.

Figs. 3 and 4 show, on a larger scale, the central part of the railway at the place where the trial bar is fixed, and also the carriage, tracing-point, drawing-board, &c., in detail.

Figs. 5 to 8 exhibit lesser details of the mechanism.

The frame (*A A*, figs. 3 and 4) of the carriage is a simple rectangle, formed of two longitudinal bars, connected by bolts which pass through two transverse bars. The four wheels of the carriage are fixed to their axles in the manner of railway carriages, but the two axles run between pointed steel centre-screws, to reduce the friction to the least possible. These screws are seen at *D D D D*, fig. 3. The wheels have their flanges turned outwards, contrary to the usual mode. This enables the carriage to run upon a single plank of the proper breadth, and having its edges slightly rounded. The flanges are also thus kept out of the way of other portions of the mechanism in those parts of the fixed frame in which parallel bars are substituted for the plank.

The swing frame is made of thin plate iron, with cross braces, arranged so as to give it as much stiffness and lightness as possible. Its axis, *B B*, is mounted between centre-screws, *E E*, and at the other end it carries a roller, *G*, which rests upon the trial bar. Leaden weights, *H*, can be fixed in any number to this end of the swing frame, by means of a thumb-screw; and a small

stage, the end of which is seen in fig. 4, is provided to support them.

In fig. 4, the trial bar, *I K*, is shown, and the carriage is represented in the act of passing over it. The wheels of the carriage run upon the side rails of the fixed frame, or tramway. The swing frame, however, is sustained at the front end by the carriage, and at the hinder or heavy end it rests upon the trial bar, by means of the roller, and depresses it during its passage.

The weights being fixed between the roller and the axis of the swing frame, produce less pressure on the bar than their actual weight. This pressure, however, can be accurately measured by a spring dynamometer, applied to the axis of the roller.

The axis of the swing frame is placed as low as it can be, without touching the frame and trial bar in its passage. The centre of the roller, therefore, describes in its short motion an arc of a circle, which differs but little from a vertical line with respect to the frame of the carriage; for the radius of the swing frame is 20 inches, and the total vertical motion of the roller never greater than 2 inches.

The general arrangement of the tramway is shown in figs. 1 and 2. A plank, *O P*, set at an angle of 45° with the horizon, rests at the upper end, *O*, against the wall of the room, and at the lower end, *P*, upon a triple frame, *P Q R S*. The two outer portions of this frame are exactly similar. The upper edge of each, from *P* to *Q*, is straight, and inclined in continuation of the plank; and from *R* to *S* is straight and horizontal. The edge from *Q* to *R* is an arc of a circle of 7 feet radius, which touches the inclined edge at one extremity and the horizontal edge at the other, so as to connect the inclined line with the horizontal.

The frames are set at such a distance from each other as will allow the carriage-wheels to run upon their upper edges, like a railway, with as little lateral shake as possible; and the plank is carefully made of the same breadth. Thus if the carriage be set upon the plank, and released, it will run down it, and be conducted by means of the curved portion upon the horizontal rails.

The roller of the swing frame at first simply rests upon the plank; but when it passes beyond the lower end of the plank, a support for it is supplied by the intermediate frame, *P I*, seen

in the plan, fig. 1. This frame has a similar straight edge, *P Q*, and a curve, *Q R*, to the outer frames between which it is fixed. But as the trial bar *I K* is higher than the edges of the tramway, for the convenience of better access to it, the curve *Q R* is arranged to conduct the inclined part to this higher level; and accordingly, in the elevation, fig. 2, the intermediate curve is seen rising above the lateral curves, and thus ending below with a horizontal tangent, *R I*, higher by an inch and a half than the lateral rails.

The lateral rails, as already explained, are continued as far as *S*; but the middle rail is cut off at *I*, and the brass chair, or contrivance for holding the trial bar, is fixed to the end of it.

The trial bar, *I K*, thus forms the continuation of the middle rail; and the loaded roller of the swing frame thus runs from the middle rail to the trial bar. At the far end, *K*, of the trial bar, a second chair is attached to a rail, *K N*, which receives the roller after it has passed over the trial bar. To adapt the apparatus to receive bars of different lengths, this latter rail can be shifted in position. Its extremity, *K*, terminates in a flat square piece, which is rebated beneath, so as to rest upon and lie between the upper inner edges of the side rails. A similar piece of wood, *Y*, is rebated to slide between the lower inner edges of the side rails; and a bolt and thumb-screw, passing through the whole, serves to fix the end, *K*, of the shifting rail, at any distance from the other rail, *I*, that will suit the bar in question. The shifting rail is sloped gradually downwards from *K* to *N*, so that the roller of the swing frame gradually sinks downwards in its passage until it is caught by a stop in the carriage, after which the middle rail is no longer required to sustain it. *Z* is a hook-bolt, which serves to fix the middle of the shifting rail.

When the carriage has passed off the trial bar, it is necessary to check its motion and bring it to rest. To effect this, two boards, *T V*, *T V*, are fixed in continuation of the tramway. These boards are fixed at a greater interval than the tramway, and are also slightly inclined upwards, and divergent. Their interval is adjusted so that the side bars of the carriage may rest upon them, as shown by the carriage in the figure.

When the fore wheels of the carriage have nearly reached the point *T*, the lower surfaces of its frame touch the slightly

inclined edges of the boards *T V*, between *T* and *S*. Thus the frame is gently lifted, so as to raise the wheels from the railway; and as the carriage proceeds it is converted into a sledge, of which the boards *T V* form the sledgeway. But as the friction of this sledge is by no means sufficient to stop the carriage, four springs (marked *check-springs* in figs. 4 and 5, and also shown in the small figures of the carriage at each end of the figs. 1 and 2) are screwed to its sides. These springs stand completely free so long as the carriage runs on the railway; but immediately after the carriage has become shifted to the sledgeway, the springs begin to press upon the sides of the latter, which are, as the plan shows, divergent; and the divergency is greater at the beginning, *T*, because the boards are planed to a thin edge to increase it. Thus the pressure of the springs gradually increases as the carriage proceeds along the sledgeway. They are made of sufficient strength to stop the carriage before it reaches *V*, when it is released from the top of the inclined plane.

The whole of the frame above described is fixed together by bolts, which pass through the legs, the side frames, and intermediate blocks, so as to allow the whole to be readily taken to pieces, or remounted at pleasure. At *W*, a transverse frame, consisting of a horizontal piece below, with two legs, and with a sloping brace rising to the height of the vertical rail, to which it is bolted, serves to give lateral support to the whole machine.

The side rails are divided at the leg near *I*. This reduces the size of the parts of the frame, and also allows longer rails to be substituted from *I* to *S*, when longer trial bars are required.* The machine represented in the drawings will not receive bars longer than 4 feet. For the purpose of conveniently raising the carriage, and releasing it, a pulley, *O*, is fixed to the upper end of

* The frame is farther secured to the floor by a bolt near *W*, the nut of which bears upon a short transverse piece laid upon the horizontal rails, close to the upright post. This is necessary, to enable it to sustain the plank, which plank is also prevented from sagging by a brace, as shown. But nearly the whole of the phenomena of the experiments may be sufficiently shown by a less velocity than that acquired from the top of the plank, namely, 30 feet per second. About 20 feet per second will be found amply sufficient for repeating these experiments, if desired, and a plane extending about 4 feet above *P* will therefore be enough. The construction of the inclined part of the framework may thus be simplified by making the straight

the plank. The cord which passes over this is attached to a small sledge, *a*, figs. 1 and 2, upon which is fixed a latch and detent. The latch is adapted to receive a hooked pin (*n*, figs. 3 and 4), fixed to the end of the carriage; and when the detent is in the position shown in fig. 1, the carriage is thus united to the sledge, and can be drawn up with it to any desired altitude of the plane by means of the cord, and secured there. But the string, *b*, fig. 2, passes over the pulley, *c*, fixed to the little sledge, and is tied to the detent. Pulling this string, therefore, the detent is shifted, and the latch releases the carriage, which then runs down the inclined plane, and passes over the trial bar.

Fig. 5 represents the mode in which that end of the trial bar which first receives the action of the roller is fixed.

The extremity of the intermediate rail bar of the frame is cut vertically from *a* to *c*, and has a horizontal step, *c d*.

e f is a piece of metal or chair, which is secured against the vertical face, *b a c*, by means of a screw-bolt, *g*, the nut, *h*, of which is inserted into a mortise in the rail. The screw passes through a mortise in the metal piece, and the latter is kept in a vertical position by a shallow grooved recess, sunk in the vertical face, *a b c*, of the rail. Thus the chair admits of a vertical adjustment of its position. A capstan-headed screw is tapped into its lower extremity, and the head of this screw rests upon the step, *d*, which has been already mentioned. By slightly releasing the screw *g*, and turning the capstan head to right or left, the vertical adjustment is made at pleasure.

The upper end, *e*, of the metal chair has a square notch cut in it, and a steel centre-screw on each side. The points of these screws are received into corresponding centre-punch holes at the end of the trial bar, which is thus held in a manner that admits of free vertical deflection of the bar.

It is essential that the roller, as it first comes upon the bar,

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portion down to *Q* of the plank form, and sustaining the whole on its legs, without employing the heavy plank resting against the wall. In exhibiting the experiments to an audience, it is convenient to connect the centre of the bar with an index, contrived so as to magnify its deflection four or five times; thus the increase of deflection produced by velocity is shown very clearly.

should meet with no inequality of level that would either jerk it upwards or let it drop and rebound from the bar.

To effect the smooth entrance required, the vertical adjustment just described is provided, and it will be seen in the figure that the end of the bar also projects into a sunk recess formed upon the upper face of the rail. When the vertical adjustment is properly made, the upper face of the rail and the upper surface of the bar are made to coincide in level, and as the roller is sufficiently broad to run upon the sides of the above-mentioned recess, it is thus gradually brought upon the bar, the extreme end of which is slightly lowered by the file, to facilitate this action.

This chair, being attached by the single bolt, *g*, can be readily removed from the frame, to substitute others of different forms, if required for differently shaped bars.

The far extremity, *K*, of the bar is supported by the contrivance shown in fig. 6, which represents the end, *K*, of the shifting rail. When the roller has passed completely over the bar, there is no necessity to provide for its level exit, as for its level entrance, for the work has been completed at this point. All that is wanted is to support it beneath in such a manner as will allow it to slide out a little, because when it is bent by the deflection of the weight the end of it is necessarily slightly drawn out of its recess in this farthest chair. The first chair grasps its end of the bar by centre-points, as we have seen, so as to prevent this drawing action at the beginning, where it would be mischievous, and it is so small at the other end that it is not worth while to provide a friction-roller or such contrivance. . The farthest end of the bar is therefore allowed to rest in a grooved piece of metal, *a a b*, the groove of which is made rather wider than the widest bar employed, and a pair of blunt-ended screws, *c c*, serve to keep the bar steady laterally, being screwed up so as just to touch without pinching it. We shall presently see that the action of the weight tends to make the bar fly upwards when it reaches the end of its course. To keep it in its groove, therefore, a steel stirrup, *d e*, is provided; this is adjusted so as just not to touch the top of the bar, and the bar itself is filed into such a curve on its upper side as will enable it to escape contact with this stirrup during its deflection. The diagram, fig. 7, will

explain this, in which *a b* is the bottom of the groove, *e* the section of the stirrup, *d b a* the end of the bar, the upper face of which is filed into a curve, as shown. The dotted line shows the position, greatly exaggerated, into which this end is thrown by the sliding motion which accompanies its deflection; and also shows how the curve enables it to escape the stirrup.

In figs. 4 and 5 a wedge-shaped piece, *Z*, is shown attached to the end of the shifting rail. This is for the purpose of receiving the roller of the swing frame, if the bar should break. In this case the swing frame would drop downwards until it rested upon its stage in the carriage, and its roller would meet the wedge, *Z*, and be conducted to the upper face of the shifting rail, and thus prevented from stopping the carriage suddenly or throwing it off.

The board which receives the trace of the trajectory is shown at *L M*, in figs. 1, 2, 3, 4. It is sustained upon two iron pillars, screwed below to the side rails, and above to the back of the board. These pillars are curved outwards, so as to escape the heads of the centre-screws of the carriage, which would otherwise strike them in their passage. The board is thinned away at each end, as the plan, fig. 3, shows. Thus the front pencil, which is carried by the swing frame, encounters a gently-sloping surface at its first contact with the paper, and is also gradually released as it quits the paper. To fix the paper, its extremity must be first grasped in the wooden clamp at *M*, then stretched tightly along the surface of the board, and doubled over the end, *L*; a small iron clamp may then be applied, and will be found sufficient to hold it. It is better to apply a third clamp at the middle, to prevent it from sagging. The best paper that I have tried is that which is prepared by Messrs. Harwood, of Fenchurch Street. This will receive the trace of a pointed brass wire. It can be had in any length required, and should be mounted upon calico. The softness of the calico enables the pencil to act better during its rapid motion, and also allows the paper to be stretched tighter without fear of tearing.

The swing frame has a piece, *h*, figs. 3 and 4, attached to its side, as near to the roller as the wheel will allow. The pencil is clamped in a socket at the top of a small triangular swing frame, *k k*, which revolves upon centre-points, tapped into its lower

extremities, and resting in holes punched in the sides of the piece, *h*, as shown. This piece, *h*, is horizontal in the part shown in the plan, fig. 3, but is turned vertically upwards to receive these centre-screws, as seen in fig. 4. It carries also an upright post, *i*, to which is screwed a wire fork, upon whose branches is strained a ring of vulcanized caoutchouc, *m m*, which is thus stretched into the form of a double horizontal elastic strap, against which the pencil frame is pressed. A silk string fastened to this frame is wound round a fiddle peg, turning with stiff friction in a hole at the top of the post, *i*; by turning this peg to the right or left, the string is tightened or relaxed, and the swing frame pressed more or less against the elastic strap. Thus the pressure upon the pencil can be adjusted at pleasure, its point being, of course, set further outwards in the socket if the frame be drawn more backwards, and *vice versâ*. Also the string limits the outward portion of the pencil so as to insure that it shall touch the sloped part of the drawing-board exactly at the proper point for the first contact.

When the paper and pencil are properly adjusted, the first thing to be done is to conduct the carriage as slowly as possible from one end of the trial bar to the other; the pencil will then trace the *equilibrium trajectory*, as shown by the close dotted line in the figure. This trajectory serves as a curve of comparison for the dynamical trajectory, and a straight line drawn through its level extremities enables us to measure the central statical deflection. The carriage may now be drawn up the inclined plane and released. The pencil will now be found to trace a curve somewhat similar to the interrupted dotted line in the figure; this is the *dynamical trajectory*.

With respect to all the trajectories drawn by the apparatus, it must be recollected that the pencil-point is considerably above the level of the axis of the swing frame, and that its radial distance from that axis is less than that of the roller. Both these causes tend to alter the form of the trajectory, but may be corrected as follows:—

The pencil describes a short arc of a circle, which, like that of the roller's motion, coincides so nearly with a straight line, that it may be considered as one, but as a line inclined to the vertical 13° by the difference of level between the pencil-point and the

axis. In transferring the curve, therefore, a sufficient number of ordinates must be drawn on the original papers, inclined 13°, and their length must be transferred to vertical ordinates on another paper to obtain the true curve.

If the form of the curve only be required, the abscissæ of the copy may have any convenient proportion to those of the original; and accordingly, to exhibit the forms of the trajectories more strikingly, I have in Plate VIII. reduced the abscissæ to about one-fifth of the originals, and transferred to them vertically the actual lengths of the original sloped ordinates, as the most rapid and convenient mode of at once reducing the four-feet length of the original trajectories to a commodious size, and of exhibiting the required exaggeration of the form. But if the actual trajectories are required, the length of these sloped ordinates must be increased in the proportion of the radial distance of the roller from the axis of the swing frame to that of the pencil, namely, in the actual machine, of 20 inches to 17·5 inches.

The velocity of the carriage may be nearly estimated by the altitude of the point whence its centre of gravity has been liberated on the inclined plane above the position of that centre on the horizontal rails; but as some loss of this velocity is occasioned by the friction of the wheels and their rotation, &c., some method of measuring the velocity after it has passed over the rail is required. Now, immediately after this passage we have seen that the end of the carriage is received on an inclined sledgeway, and the fore wheels suddenly lifted off their rails. This happens before the check-springs have touched the sides of the sledgeway, and therefore before they have acted to retard its motion. Hence the fore wheels revolving free of the rails, their circumferences retain a velocity equal to that with which the carriage was progressing when the wheels were lifted. By observing, therefore, the velocity of rotation of the wheels when the carriage is checked, we can estimate the velocity with which it had passed over the trial bar. To facilitate this, a worm is formed upon the axis of the fore wheels, and a toothed disk, *C*, fig. 3, loosely geared into it.* An observer, stationed at a point where the carriage stops, with a stop-watch, can easily measure

* This is omitted in the section, fig. 4, to prevent confusion.

the time occupied by the passage of 10 or 20 teeth, and hence obtain the required velocity of rotation. There is, in fact, very little loss of velocity from the retarding causes. The weight of the entire carriage and its mechanism, when the swing frame is loaded to produce a pressure of 4 lbs., is 28 lbs. I find that when the carriage is released from a height that would generate, without retarding causes, a velocity of 10 feet per second on the horizontal rails, the actual velocity ascertained by the above method is 7·7 feet. Similarly, velocities that should be 15 and 20 feet, are respectively reduced to 12 feet and 16·6 feet.

In fig. 3, two centre-screws, *F F*, will be observed in the sides of the frame, supporting the axis of an arm, which is shown in dotted lines only, and terminates in a roller, *G'*, which rests, like the roller, *G*, of the swing frame, upon the trial bar, but at a distance of one foot behind it. This arm and roller is also dotted into fig. 4. Their purpose was to obtain the equilibrium and dynamical trajectories in the case of two equal pressures acting upon the trial bar, as in the Portsmouth experiments: want of time, however, having prevented me from obtaining accurate results with this part of the apparatus, I have contented myself with inserting the arm in the drawings by way of suggestion to future observers.

Beneath the bar, in fig. 4, is a contrivance termed the *Inertial Balance*. This will be fully described in Chapter IV. Figure 8 also belongs to this part of the machine.

Trajectories drawn by the apparatus above described are given in Plate VIII.; but the consideration of them is so much involved in the question of the inertia of the bar which our theoretical investigations suppose so small as to be neglected, that I must postpone their explanation until I have given, first, the theory on the above hypothesis, and next, the explanation of the methods by which the inertia of the bar can be introduced.

CHAPTER III.

Theoretical Investigation of the Trajectory.

To simplify as much as possible the mathematical calculation, the carriage must be considered as a heavy particle, and the inertia of the bar neglected. Let x y be the co-ordinates of the moving body, x being measured horizontally from the beginning of the bar and y vertically downwards, M the mass of the body, V its velocity on entering the bar, $2a$ the length of the bar, g the force of gravity, S the central statical deflection, that is to say, the deflection that is produced in the bar by the body placed at rest upon its central point, R the reaction between the body and the bar. The deflection is small,* and therefore this reaction may be supposed to act vertically, for it must be recollected that the reaction is perpendicular to the curve of the *bar* and not to the *trajectory*, and therefore, in the case of such small deflections as we have to deal with, the horizontal component of the reaction will be insignificant. Thus the horizontal velocity V will remain constant during the passage of the body along the bar. Now we have seen (pp. 453, 454) that a given weight W, suspended to the bar at a distance x from its extremity, will produce a deflection $y = c\,W\,(2\,a\,x - x^2)^2$, c being a constant

* Practically, the deflection of a girder is so small compared with the length, that this hypothesis may be fairly assumed. Engineers inform us that a deflection from $\frac{1}{400}$ to $\frac{1}{600}$ of the length may be allowed in a girder (*vide* Report, Analysis of Evidence, art. *Deflection of Girders, &c.*); but the deflections with ordinary loads are not greater than one-fourth of these. Thus, in Mr. Hawkshaw's evidence (No. 152), we find a deflection of half an inch assigned to a girder-bridge of 89 feet span under the action of a heavy locomotive engine. This is only $\frac{1}{2136}$ of the length; and in the experiments of the Commission at Ewell and Godstone, deflections of $\frac{1}{2520}$ and $\frac{1}{1440}$ of the length were obtained from a heavy locomotive and tender. In the experiments at Portsmouth, on 9-feet bars, deflections of 5 inches, that is, of $\frac{1}{21}$ of the length, were sometimes reached; but even these may be called small in the mathematical sense.

depending on the elasticity and transverse section of the bar. But as the inertia of the bar is neglected, its elastic reaction upon the travelling weight will be equal to a weight that would, if suspended to the bar at a point where the travelling weight touches it, depress that point to the same amount below the horizontal line. Therefore, $R = W = \frac{y}{c} \frac{1}{(2\,a\,x - x^2)^2}$. The constant c may be determined by observing that if $R = M\,g$ and $x = a$, y becomes S. Whence, substituting in the above equation, we obtain $c = \frac{S}{M\,g\,.\,a^4}$.

The forces which act on the body are its gravity and the reaction of the bar. Whence we obtain the equation of motion,

$$\frac{d^2 y}{d\,t^2} = g - \frac{g\,a^4}{S} \times \frac{y}{(2\,a\,x - x^2)^2}$$

which becomes, since $V = \frac{d\,x}{d.t}$,

$$\frac{d^2 y}{d\,x^2} = \frac{g}{V^2} - \frac{g\,a^4}{V^2 S} \frac{y}{(2\,a\,x - x^2)^2}:$$

from the integration of this equation we should obtain the curve of the trajectory.

Having proceeded thus far, however, I found the discussion of this equation involved in so much difficulty, that I was compelled to request my friend G. G. Stokes, Esq., Fellow of Pembroke College,* to undertake the development of it. His kind and ready compliance with my wishes, and his well-known powers of analysis, have produced a most valuable and complete discussion of the equation in question. The mathematical methods employed for this purpose are, from their nature, probably unintelligible to the majority of practical men, for whom the present essay was written; and it was thought better, therefore, that the discussion should be thrown into the form of a paper, and presented to the Cambridge Philosophical Society, before which it was read the 21st May, 1849: † to the Transactions of that

* Now Lucasian Professor in the University of Cambridge.

† The title of the paper is as follows: 'Discussion of a Differential Equation relating to the Breaking of Railway Bridges,' by G. G. Stokes, M.A., Fellow of Pembroke College, Cambridge.—*Transactions of the Cambridge Philosophical Society*, vol. viii. page 707. 1849.

Society I must beg to refer those of my readers who may desire to follow out this most elaborate and able investigation. I shall, however, give his results, extracting from the paper such of his remarks as may be necessary to make them intelligible, and shall then proceed to compare them with the trajectorial curves of my apparatus and with practice.

It appears that the equation cannot be integrated in finite terms, except for an infinite number of particular values of a certain constant involved in it; but Mr. Stokes has investigated rapidly convergent series, whereby numerical results may be obtained. By merely altering the scale of the abscissa and ordinates, the differential equation is reduced to one containing a single constant, which he terms β. This he effects as follows:—
Put

$$x = 2\,a\,X \qquad y = 16\,S\,Y \qquad \frac{g \cdot a^4}{V^2\,S} = 4\,a^2\,\beta\,;$$

and substituting these values in the equation, it becomes

$$\frac{d^2\,Y}{d\,X^2} = \beta - \frac{\beta\,Y}{X - X^2)^2}.$$

"It is to be observed that X denotes the ratio of the distance of the body from the beginning of the bar to the length of the bar; Y denotes a quantity from which the depth of the body below the horizontal plane in which it was at first moving may be obtained by multiplying by $16\ S$; and β, on the value of which depends the form of the body's path, is a constant defined by the equation $\beta = \dfrac{g\,a^2}{4\,V^2\,S}$. A small value of β, therefore, corresponds to a high velocity, and a large value to a small velocity. It appears, from the solution of the differential equation, that the trajectory of the body is unsymmetrical with respect to the centre of the bridge, the maximum depression of the body occurring beyond the centre. The character of the motion depends materially on the numerical value of β. When β is not greater than $\frac{1}{4}$, the tangent to the trajectory becomes more and more inclined to the horizontal, beyond the maximum ordinate, till the body gets to the second extremity of the bridge, when the tangent becomes vertical. At the same time the expressions for the central deflection and for the tendency of the bridge to

break become infinite. When β is greater than $\frac{1}{4}$, the analytical expression for the ordinate of the body at last becomes negative, and afterwards changes an infinite number of times from negative to positive, and from positive to negative. The expression for the reaction becomes negative at the same time with the ordinate, so that, in fact, the *body leaps*. 'The occurrence of these infinite quantities indicates one of two things; either the deflection really becomes very large, after which of course we are no longer at liberty to neglect its square, or else the effect of the inertia of the bridge is really important. Since the deflection does not really become very great, as appears from experiment, we are led to conclude that the effect of the inertia is not insignificant; and, in fact, I have shown that the value of the expression for the *vis viva* neglected at last becomes infinite. Hence, however light be the bridge, the mode of approximation adopted ceases to be legitimate before the body reaches the second extremity of the bridge, although it may be sufficiently accurate for the greater part of the body's course."

We shall presently see that in practice β is never less than $\frac{1}{4}$, and that the above conclusion can be perfectly reconciled with the experimental results when the inertia of the bar is taken into account. For the investigation of the series by which our author was enabled to calculate the numerical results, I must refer to his paper, from which I have extracted the two following Tables (V. and VI.), which contain a sufficient number of ordinates to enable the trajectory to be laid down by points, in the forms corresponding to nine values of β. Those which belong to intermediate values of β can be easily interpolated. The curves themselves are carefully laid down in Plate VII., fig. 4.

TABLE V.

x.	$\frac{y}{S}$				z.				T.			
	$\beta =$				$\beta =$				$\beta =$			
	$\frac{3}{56}$	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{3}{4}$	$\frac{3}{56}$	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{3}{4}$	$\frac{3}{56}$	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{3}{4}$
·00	·000	·000	·000	·000	·065	·111	·200	·385	·000	·000	·000	·000
·02	·000	·001	·001	·002	·067	·115	·208	·398	·005	·009	·016	·031
·04	·002	·003	·005	·010	·070	·120	·216	·412	·011	·018	·033	·063
·06	·004	·006	·011	·022	·073	·125	·224	·426	·017	·028	·051	·096
·08	·007	·011	·020	·038	·076	·130	·233	·441	·022	·038	·069	·130
·10	·010	·018	·032	·059	·080	·136	·243	·457	·029	·049	·087	·165
·12	·015	·025	·045	·085	·083	·142	·253	·474	·035	·060	·107	·200
·14	·020	·034	·061	·114	·087	·148	·264	·493	·042	·071	·127	·237
·16	·026	·045	·080	·148	·091	·155	·276	·512	·049	·083	·148	·275
·18	·033	·057	·100	·185	·096	·162	·288	·532	·056	·096	·170	·314
·20	·041	·070	·124	·227	·100	·170	·302	·553	·064	·109	·193	·354
·22	·050	·084	·149	·272	·105	·179	·316	·576	·072	·123	·217	·396
·24	·059	·100	·176	·320	·111	·188	·331	·601	·081	·137	·242	·438
·26	·069	·117	·206	·371	·117	·198	·348	·627	·090	·152	·267	·482
·28	·080	·135	·239	·418	·122	·208	·367	·642	·099	·168	·296	·519
·30	·092	·155	·272	·477	·130	·220	·386	·676	·109	·185	·324	·568
·32	·104	·176	·308	·534	·138	·232	·406	·705	·120	·202	·354	·614
·34	·118	·198	·346	·605	·146	·246	·429	·751	·131	·221	·386	·674
·36	·132	·222	·386	·665	·155	·261	·454	·783	·143	·241	·419	·721
·38	·150	·240	·427	·736	·165	·277	·480	·829	·155	·261	·453	·781
·40	·162	·272	·470	·802	·176	·295	·509	·870	·169	·283	·489	·835
·42	·178	·298	·513	·869	·188	·314	·541	·916	·182	·306	·527	·892
·44	·195	·326	·560	·939	·201	·336	·576	·966	·198	·331	·568	·951
·46	·213	·347	·607	1·01	·216	·360	·615	1·02	·214	·358	·611	1·01
·48	·231	·385	·655	1·10	·232	·386	·657	1·08	·231	·385	·656	1·08

TABLE V.—(*continued.*)

x.	$\frac{y}{S}$				z.				T.			
	$\beta =$				$\beta =$				$\beta =$			
	$\frac{5}{36}$	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{5}{4}$	$\frac{5}{36}$	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{5}{4}$	$\frac{5}{36}$	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{5}{4}$
·50	·250	·416	·705	1·14	·250	·416	·705	1·14	·250	·416	·705	1·14
·52	·270	·448	·755	1·21	·271	·449	·758	1·22	·270	·449	·757	1·21
·54	·290	·481	·807	1·28	·294	·487	·817	1·29	·292	·484	·812	1·28
·56	·311	·514	·859	1·34	·320	·529	·884	1·38	·316	·522	·871	1·36
·58	·333	·548	·911	1·40	·350	·578	·959	1·47	·342	·563	·935	1·44
·60	·355	·584	·964	1·46	·385	·633	1·05	1·58	·370	·608	1·00	1·52
·62	·378	·619	1·02	1·51	·425	·697	1·14	1·70	·401	·657	1·08	1·60
·64	·401	·654	1·07	1·55	·472	·771	1·26	1·82	·435	·710	1·16	1·68
·66	·425	·692	1·12	1·59	·527	·858	1·39	1·98	·473	·771	1·25	1·78
·68	·449	·728	1·17	1·62	·592	·961	1·54	2·13	·516	·837	1·34	1·86
·70	·473	·765	1·22	1·64	·671	1·08	1·72	2·32	·563	·910	1·45	1·95
·72	·498	·801	1·26	1·65	·765	1·23	1·94	2·54	·617	·994	1·56	2·05
·74	·523	·830	1·30	1·66	·883	1·40	2·20	2·80	·680	1·08	1·69	2·15
·76	·548	·874	1·34	1·64	1·03	1·64	2·52	3·08	·751	1·20	1·84	2·25
·78	·573	·908	1·38	1·61	1·22	1·93	2·92	3·42	·835	1·32	2·00	2·35
·80	·598	·933	1·40	1·56	1·46	2·30	3·43	3·81	·935	1·46	2·19	2·44
·82	·623	·972	1·42	1·49	1·79	2·79	4·08	4·26	1·05	1·65	2·41	2·52
·84	·647	1·00	1·43	1·39	2·24	3·46	4·96	4·79	1·20	1·86	2·67	2·58
·86	·669	1·00	1·43	1·25	2·88	4·41	6·17	5·41	1·39	2·13	2·97	2·61
·88	·691	1·04	1·41	1·09	3·87	5·84	7·91	6·10	1·63	2·47	3·34	2·58
·90	·708	1·05	1·37	·883	5·47	8·12	10·6	6·81	1·97	2·92	3·80	2·45
·92	·723	1·05	1·30	·630	8·34	12·1	15·0	7·27	2·45	3·57	4·41	2·14
·94	·730	1·00	1·18	·318	14·3	20·3	23·2	6·44	3·25	4·58	5·23	1·45
·96	·699	1·00	·987	−·014	29·6	43·5	41·8	− ·600	4·55	6·69	6·42	− ·09
·98	·690	·857	·652	−·404	112·	139·	106·	−65·8	8·80	10·9	8·32	−5·16
1·00	0	0	0	0	∞	∞	+∞	+∞	∞	∞	+∞	+∞

TABLE VI.

x.	$\frac{y}{S}$ $\beta=3$	$\beta=5$	$\beta=8$	$\beta=12$	$\beta=20$	z. $\beta=3$	$\beta=5$	$\beta=8$	$\beta=12$	$\beta=20$	T. $\beta=3$	$\beta=5$	$\beta=8$	$\beta=12$	$\beta=20$
·00	0	0	0	0	0	·600	·714	·800	·857	·909	0	0	0	0	0
·05	·023	·027	·030	·032	·034	·640	·755	·835	·886	·931	·122	·143	·159	·168	·177
·10	·089	·103	·113	·119	·123	·689	·798	·872	·915	·950	·248	·287	·314	·330	·342
·15	·195	·220	·237	·246	·252	·751	·846	·910	·945	·970	·383	·431	·464	·482	·495
·20	·327	·367	·389	·399	·405	·799	·897	·950	·975	·989	·511	·574	·608	·624	·633
·25	·486	·535	·558	·565	·572	·863	·951	·991	1·004	1·016	·647	·714	·743	·753	·762
·30	·661	·713	·722	·728	·721	·936	1·010	1·023	1·032	1·023	·786	·849	·859	·867	·859
·35	·843	·888	·889	·877	·859	1·018	1·073	1·074	1·059	1·038	·926	·976	·977	·963	·944
·40	1·023	1·049	1·026	·997	·966	1·110	1·138	1·114	1·081	1·049	1·066	1·092	1·069	1·038	1·007
·45	1·190	1·183	1·127	1·078	1·035	1·214	1·207	1·150	1·099	1·056	1·202	1·195	1·138	1·089	1·046
·50	1·331	1·274	1·180	1·111	1·060	1·331	1·274	1·180	1·111	1·060	1·331	1·274	1·180	1·111	1·060
·55	1·431	1·314	1·179	1·092	1·037	1·461	1·341	1·203	1·114	1·058	1·446	1·327	1·191	1·103	1·047
·60	1·486	1·281	1·108	1·018	·968	1·602	1·390	1·202	1·105	1·051	1·538	1·334	1·154	1·060	1·009
·65	1·446	1·173	·954	·895	·860	1·748	1·417	1·179	1·081	1·038	1·590	1·289	1·072	·983	·945
·70	1·334	·983	·781	·733	·720	1·891	1·393	1·107	1·039	1·021	1·588	1·170	·930	·873	·858
·75	1·111	·716	·564	·554	·570	1·974	1·273	1·003	·984	1·013	1·481	·955	·752	·738	·760
·80	·772	·396	·341	·382	·405	1·885	·968	·832	·932	·989	1·206	·620	·532	·596	·633
·85	·335	·090	·172	·241	·254	1·286	·344	·660	·925	·976	·656	·176	·336	·472	·498
·90	−·126	−·080	·104	·131	·123	−·970	−·616	·802	1·013	·947	−·349	−·222	·289	·365	·341
·95	−·297	+·045	·068	·026	·034	−8·227	+1·248	1·884	·720	·943	−1·563	+1·237	·358	·137	·179

In Table V. the length of the bar is divided into 50 parts; but in Table VI. 20 divisions were thought sufficient. Each Table, however, consists of three parts. In the first are contained the values of the ordinates of the curve, S being considered as unity.* In the second part of the Table, which is headed z, we have the numerical values, which express the ratio of the depression of the moving body at any point to its statical depression, that is to say, to its place in the equilibrium trajectory. In the third part, headed T, are the numbers which express the tendency of the bar to break at each point, which were thus obtained.

If a weight, W, be placed on a point of the bar whose distance from the first extremity is x, then, by the known principles of

* The equilibrium trajectory may be laid down by the help of the subjoined Table. The length of the bar is divided into 50 parts, and as the curve is symmetrical on each side of the centre, it is only necessary to give the ordinates for the first half: the central ordinate may be assumed of any convenient magnitude, and divided into 1000 parts.

TABLE VII.—EQUILIBRIUM TRAJECTORY.

x	y	x	y	x	y
1	5	10	410	19	909
2	24	11	476	20	920
3	51	12	532	21	941
4	86	13	589	22	970
5	129	14	655	23	986
6	178	15	707	24	995
7	231	16	753	25	1000
8	285	17	808	—	—
9	344	18	851	—	—

Table VI. contains the results for five values of β, namely, 3, 5, 8, 12, and 20, upon which Mr. Stokes makes the following remarks:—

"The form of these trajectories is shown in fig. 4, Plate VII. As β increases, the first point of intersection of the trajectory with the equilibrium trajectory moves towards A. Since $z = 1$ at this point, we get from the part of the Table headed z, for the abscissæ of the point of intersection (by taking proportional parts) ·34, ·29, ·26, ·24, and ·22, corresponding to the respective values 3, 5, 8, 12, and 20, of β. Beyond this point of intersection the trajectory passes below the equilibrium trajectory, and remains below it during the greater part of the remaining course. As β increases, the trajectory becomes more and more nearly symmetrical with respect to C: when $\beta = 20$ the deviation from symmetry may be considered insensible, except close to

statics,* the strain upon this point, or tendency of the bar to break, is measured by W multiplied by the product of the two parts into which the bar is divided by the point upon which the weight rests, or by $W \times (2a - x)\,x$. But, in the problem under consideration, the dynamical action of the travelling load, combined with the elastic reaction of the bar, deflects the point of the bar upon which it is momentarily placed to a distance, y, below the horizontal line. Since, therefore, the inertia of the bar is neglected, the effect to break the bar is the same as if a weight were suspended to this point sufficiently great to depress it statically to the same distance, y. Such a weight is equal to the reaction of the bar, and is therefore proportional to $\frac{y}{(2ax - x^2)^2}$. Substituting this value of W in the above expression, we obtain the tendency of the bar to break under the action of the travelling load proportional to $\frac{y}{2ax - x^2}$. Call this tendency T, and let T be so measured that it may be equal to unity when the moving body is placed at rest on the centre of the bar; in which case $y = S$ and $x = a$.

Hence $T : 1 :: \frac{y}{(2ax - x^2)} : \frac{S}{a^2}$ and $T = \frac{a^2}{S} \cdot \frac{y}{2ax - x^2}$.

In this manner, the numbers in the third part of the Tables were obtained. It must be remembered that, in this part of the investigation, the inertia of the bar or bridge is necessarily

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the extremities $A\,B$, where, however, the depression itself is insensible. The greatest depression of the body, as appears from the column which gives y, takes place a little beyond the centre; the point of greatest depression approaches indefinitely to the centre, as β increases. This greatest depression of *the body* must be carefully distinguished from the greatest depression of *the bridge*, which is decidedly larger, and occurs in a different place, and at a different time (see p. 457). The numbers in the columns headed T show that T is a maximum for a value of x, greater than that which renders y a maximum, as in fact immediately follows from a consideration of the mode in which y is derived from T. The first maximum value of T is about 1·59 for $\beta = 3$, 1·33 for $\beta = 5$, 1·19 for $\beta = 8$, 1·11 for $\beta = 12$, and 1·06 for $\beta = 20$."—*Camb. Trans.* p. 723.

* *Vide* Barlow on 'the Strength of Materials,' or any statical writer on this subject.

neglected, and it will be seen below that this inertia greatly affects some of these results.

Having now stated the results of Mr. Stokes's discussion of the equation to the trajectory, I shall endeavour to apply them to the interpretation of the experiments. This discussion has shown that the curve of the trajectory assumes different phases, each of which is characterized by a certain value of the constant $\beta = \frac{g a^2}{4 V^2 S}$. Their forms are shown in fig. 4, Plate VII. When β is large, the curve departs very little from symmetry, or from the form of the equilibrium trajectory. But, as β becomes smaller, the first half of the curve rises more and more above the equilibrium curve; the second half sinks, on the contrary, below it at first; but when the value of β is less than about $\frac{5}{4}$, the loop of the trajectory begins to rise again. On the other hand, however, as β diminishes, this loop, or lowest point of the curve, steadily increases its distance from the central position which it holds in the equilibrium trajectory.

Every one of these phases or forms of the curve may have its ordinates upon any scale of proportion with respect to the length of the whole. This scale is governed by the proportion of a to S. Accordingly, in the drawings of the curves, the proportional magnitude of the ordinates is assumed much larger than in actual practice, or, indeed, than would be consistent with the hypothesis that the deflections are small compared with the length of the bar.*

But before we can apply these results in illustration of the experiments, we must ascertain the numerical values which β holds in practical cases. In the expression for β, $g = 32{\cdot}2$ feet, a is the half-length of the bridge in feet, V the horizontal constant velocity of the body in feet per second, and S the central statical deflection, also in feet.

It will be more convenient if the value of β be expressed in

* A numerical example may explain the above remarks. In the expression for β $\left(\text{namely } \beta = 24{\cdot}15 \frac{l^2}{V^2 S}\right)$ let us substitute the values given in the two following cases. (1.) A bridge 30 feet long, over which a load that would produce a statical deflection of ·22 inch, is travelling at the rate of 90 feet per second. (2.) A bar 9 feet long, on which a load that would produce a statical deflection of 2 inches is travelling at the rate of 9 feet per second.

terms of the length (l) of the bridge, instead of the half-length, and also, if the deflection be expressed in inches, the other quantities, l and V, being expressed in feet. If we make the necessary substitutions for this purpose in the formula, we obtain $\beta = 24{\cdot}15 \frac{l^2}{V^2 S}$.

In the 9-feet bars of the Portsmouth experiments, $\beta = \frac{1956{\cdot}15}{V^2 S}$.

It is clear that, as the velocity and statical deflection vary, every experiment has a different value of β. But as certain selected values of the velocity were employed, we can exhibit corresponding values of β, as in the following Table, in which also a few values of S are taken, between which it is easy to estimate the value of β for any particular case.

TABLE VIII.

S, in Inches.	Velocity in Feet.			
	15	29	36	43
·3	29·0	7·74	5·02	3·54
·6	14·5	3·87	2·51	1·77
1	8·69	2·32	1·51	1·06
1·5	5·79	1·55	1·00	..
2	4·35	1·16	..	..
3	2·89	..	..	..

The values of S in each column are not extended beyond those which were employed in the actual experiments, as shown by the Tables (pp. 443, 488), and it thus appears that β was never less than unity, or greater than 30, in the three first series of these experiments.

To obtain less values of β, we must diminish the length of the

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We shall obtain the same value of β for each of these examples, namely, 12, very nearly. The trajectory of each of these will be the same, and also the same as that given for $\beta = 12$ in Plate VII.; in this respect, that the *proportional* increase of the statical deflection at similar points of the length is the same in all three. But the relative scale of the abscissæ and ordinates will be different in every one; for in the bridge, the central statical deflection is to the length as 30 feet to ·22 inch, that is, as 1636 to 1; in the bar the deflection is to the length as 9 feet to 2 inches, or as 54 to 1; and in the figures on the Plate as 10 to 1.

bar, or employ greater velocities and larger statical deflections; that is to say, greater weights. But greater velocities are not to be obtained with the inclined plane, which was already carried as high as practical limits allowed; and larger proportional deflections would remove the case beyond the limit of the theory upon which β was calculated, and, indeed, beyond the limits of the ordinary assumption of small deflections upon which the equations are founded in all problems in which elastic curves are concerned; so that the diminution of the length is the only practicable mode of trying experiments upon small values of β. However, the values of β in actual bridges are so much larger than any we have been experimenting upon, that they belong for the most part to totally different phases of the curve,* and therefore experiments on small values are only required to test the theory.

Thus, in Godstone Bridge, the length was 30 feet. $S = 0 \cdot 19$ in.; $\beta = \frac{114395}{V^2}$; whence for velocities of 22 feet, 40 feet, 73 feet, 90 feet, we obtain $\beta = 236,\ 71 \cdot 5,\ 21 \cdot 4,\ 14 \cdot$ respectively; of which the last belongs to a velocity, practicable indeed, but the effects of which we were not able to test.

In the Dee Bridge, $l = 98$. S varies from $\frac{7}{8}$ in. to $1\frac{1}{16}$ in.;† if we assume it equal to 1 inch, we obtain $\beta = \frac{231937}{V^2}$. In this case velocities of 20 feet, 40 feet, 70 feet, 90 feet, give values of $\beta = 580,\ 145,\ 47$, and 28 respectively.

In a bridge of 89 feet length, on the Goole line, the deflection was half an inch (*vide* Mr. Hawkshaw's evidence, Report, No.

* The principal reason of the totally different range of the values of β in the experiments, and in real bridges, respectively, is to be found in the great difference between their lengths, for as β varies (*cæteris paribus*) directly as the square of the length, and inversely as the statical deflection, it is clear that a 9-feet bar and a 30-feet bridge will at once produce a totally different set of values of β. Added to which, it is found convenient to employ a statical deflection of 1 inch or more for the sake of sufficiently developing the effects, while in real bridges the statical deflection is not greater than a quarter of an inch.

† These values of S are taken from the Report to the Commissioners of Railways, 15th June, 1847, p. 7, and consequently belong to its construction before it was strengthened.

152, &c.); this, with velocities of 25 and 90 feet, will give $\beta = 612$ and 47 respectively.

In the Ewell Bridge, $l = 48$ feet, $S = 0{\cdot}215$ in., $\beta = \frac{258789}{V^2}$, whence velocities of 25 feet and 90 feet give $\beta = 414$ and 32 respectively.

In the case of real bridges, it thus appears that β is rarely so small as 14, and may reach 600, or higher numbers, whereas, in the Portsmouth experiments, the values of β ranged between 30 and 1. In the experiments on shorter bars at Portsmouth, and in my experiments at Cambridge, still lower values of β were employed, as will presently appear. In fact, our principal experiments belong to a series of values of β that begin where those that appertain to real bridges end.*

But the better to compare the experimental results with practical cases, it will in the next place be convenient to consider the proportional increase of the central deflection of the bar that belongs to each value of β.

It has been shown in the Plate that the maximum central deflection happens when the body has reached that point of its trajectory at which the curve of the trajectory touches the corresponding curve of the bar. Every given phase of the trajectory, and therefore its appropriate value of β, has also a certain maximum central deflection in the bar, the ratio of which to the statical deflection ($= S$) can be calculated or otherwise obtained. It is not very easy to calculate it, and its value may be obtained, with sufficient accuracy for our purpose, by the drawing-board, from the curves which have been laid down from the preceding Tables, and note at foot of page 456.

However, Mr. Stokes has shown that, when β is greater than about 8, the motion of the body becomes sensibly symmetrical with

* In weak bridges still smaller values of β may be reached with high velocities. We may take, for example, the girders of the Canal Bridge near Long Eaton, which Mr. W. H. Barlow has described as exemplifying a case in which the dimensions were insufficient, and the girders removed accordingly. (Report, Minutes of Evidence, 733, and App. No. 5.) The span of the girders was 26 feet, and the statical deflection 0·3 in. This, with velocities of 70 and 90 feet, would give $\beta = 11$ and 7 respectively, and consequently increments of the statical deflection = ·12 and ·2, neglecting the inertia of the bridge, which would more than double these increments.

respect to the centre of the bridge;* and, in fact, the projections of his curves in Plate VII. show that the trajectory becoming thus nearly symmetrical, the maximum central deflection of the bar is so nearly the same as the central ordinate of the trajectory that one may be taken for the other in all cases where β is greater than 8; and of course, therefore, in real bridges, where, as we have seen, β is rarely below 14.

Now, when β is large, Mr. Stokes has given the following series,† to calculate the value of the ratio of the central deflection of the bar to S, namely (if D = central deflection of the bar):

$$\frac{D}{S} = 1 + \frac{1}{\beta} + \frac{5}{2\beta^2} + \frac{13}{\beta^3} +, \&c.$$

When β is equal to, or greater than 100, the first two terms of the series will be found true to the third place of decimals; therefore, substituting the value of β, we obtain $D = S + \frac{4 V^2 S^2}{g a^2}$. Hence, for a given load, the increment of the deflection due to velocity varies nearly as the square of the velocity directly, and the square of the length of the bridge inversely.

TABLE IX.—CORRESPONDING VALUES OF β AND $\frac{D}{S}$.

β	$\frac{D}{S}$	β	$\frac{D}{S}$	β	$\frac{D}{S}$
0·3	7·0	3·5	1·43	50	1·020
0·4	5·6	4·0	1·38	60	1·017
0·5	4·0	4·5	1·34	70	1·015
0·6	3·9	5	1·30	80	1·013
0·7	3·4	6	1·23	90	1·011
0·8	3·0	7	1·20	100	1·010
0·9	2·7	8	1·18	200	1·005
1·0	2·46	9	1·16	300	1·003
1·2	2·13	10	1·14	400	1·0025
1·4	1·92	12	1·12	500	1·0020
1·6	1·79	14	1·10	600	1·0017
1·8	1·72	16	1·09	700	1·0014
2·0	1·65	18	1·07	800	1·0012
2·3	1·59	20	1·06	900	1·0011
2·5	1·55	30	1·04	1000	1·0010
3·0	1·49	40	1·03	..	..

* Camb. Phil. Trans. p. 720.

† "In practical cases this series may be reduced to $1 + \frac{1}{\beta}$. The latter term is the same as would be got by taking into account the centrifugal force,

In Table IX. I have given with sufficient accuracy for our purpose, the numerical values of the ratio of the dynamical central deflection of the bar to the statical deflection, which correspond to different values of β. We see that the statical deflection is tripled when $\beta = 0{\cdot}8$, and doubled when $\beta = 1{\cdot}3$. When β becomes greater, the increment of the deflection diminishes rapidly; so that, for $\beta = 14$, it is only a tenth of the statical value, and one hundredth when $\beta = 100$. This Table explains the much greater development of the central deflection and other phenomena in the bulk of the Portsmouth experiments than in actual bridges; for by comparing Table VIII. with the three Tables relating to those experiments, at pp. 443–7, it will be seen that the great and startling increments of the deflection produced by the velocity of the load belong to small values of β (which never occur in practice), obtained by high velocities combined with the greatest loads. The values of β between 29 and 14, in these experiments, belong only to a few cases of the 15 feet velocity combined with the small deflections due to the least weights employed. And even these latter values of β are only reached in real bridges with velocities of 50 and 60 miles an hour. But the increase of deflection in these cases, as well in the Portsmouth experiments as in the above Table IX., is so small as to be of little practical importance. From Table IX., and from the values of β determined in page 484, it would appear that in real bridges, where β ranges from 600 to 14, the dynamical increment of the central statical deflection would be from ·0017 to ·1 only, whereas in the experiments, in which β ranges from 30 to 1, the same increment would acquire values from ·04 to 1·46 of the central statical deflection. It must always be remembered, however, that in our theory, the inertia of the bar or bridge has been supposed so small with respect to that of the load that it may be neglected, and consequently, as I will proceed to show, the theory, in this stage, although it serves very well to explain the general action of the forces in producing

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and substituting in the small term involving that force the radius of curvature of the equilibrium trajectory for the radius of curvature of the actual trajectory. The problem has been already considered in this manner by others by whom it has been attacked."—*Camb. Trans.* p. 724.

the effects in question, fails to account for the whole of the results obtained by experiment.

For the purpose of comparing the above-calculated values of the central deflection of the bars with the Portsmouth experiments, I will select those experiments in which the actual statical deflections were measured; for, as I have already explained, in the examination of the three first series I was compelled to calculate, upon somewhat uncertain data, the statical deflections for the purpose of obtaining the increase due to the motion of the load. But in the sixth and seventh series, the load was allowed to remain the same in each experiment, and successively increasing velocities were given to it, the statical deflection having been previously determined, and thus a cause of possible error was removed. In the seventh series, moreover, the load was made to press upon one point only of the bar, so as to remove one source of discrepancy between the theory and experiment (see page 441).

TABLE X.—PORTSMOUTH EXPERIMENTS, SIXTH AND SEVENTH SERIES.

Bars of Wrought Iron 9 ft. long, 1 in. broad, 3 in. deep.							
No. of Experiment.	Velocity, in feet per second.	Statical Deflection.	Dynamical Deflection.	Ratio of observed Deflection.	Calculated Ratio.	β	Calculated Dynamical Deflection.
Sixth Series.	15	·29	·38	1·31	1·05	27	·30
	29	·29	·50	1·72	1·19	7·24	·34
	36	·29	·62	2·14	1·34	4·7	·39
	..	·34	·53	1·56			·45
	43	·29	·46	1·59	1·46	3·3	·42
	..	·34	·47	1·38			·50
Bars of Cast Iron 4 ft. 6 in. long, 4 in. broad, 0·75 in. deep.							
Seventh Series. 1	15	·25	·48	1·92	1·17	8·7	·29
	29	..	·70	2·8	1·61	2·3	·40
	40	..	·84	3·36	2·18	1·2	·53
Bars of Cast Iron 4 ft. 6 in. long, 4 in. broad, 0·5 in. deep.							
2	15	·42	·60	1·43	1·29	5·2	·54
	29	..	1·58	3·76	1·92	1·4	·81

TABLE X.—(*continued.*)

No. of Experiment.	Velocity in feet per second.	Statical Deflection.	Dynamical Deflection.	Ratio of observed Deflection.	Calculated Ratio.	β	Calculated Dynamical Deflection.
Bars of Wrought Iron 4 ft. 6 in. long, 4 in. broad, 0·5 in. deep.							
3	15	·26	·39	1·5	1·17	8·2	·30
	29	..	·52	2·	1·61	2·2	·42
	40	..	·61	2·35	2·13	1·2	·55
	15	·34	·59	1·72	1·23	6·3	·42
	29	..	·82	2·41	1·75	1·7	·59
	40	..	1·00	2·94	2·70	·9	·92
Bars of Wrought Iron 4 ft. 6 in. long, 4 in. broad, 0·5 in. deep.							
4	15	·50	·74	1·48	1·36	4·3	·68
	40	..	1·95	3·9	3·9	·6	1·90
Bars of Steel 2 ft. 3 in. long, 2 in. broad, 0·25 in. deep.							
5	15	·35	·60	1·72	1·85	1·5	·65
	29	..	·88	2·52	5·6	·4	1·96
	44	..	1·03	2·94	..	·2	..
	15	·70	1·02	1·46	3·0	·8	2·10
	24	..	1·32	1·88	7·	·3	4·90
	29	..	1·46	2·08	..	·2	..
	34	..	1·30	1·85	..	·1	..
	44	..	1·03	1·47	..	·1	..

In Table X., after giving the observed statical and dynamical deflections, with their respective ratios, I have added three columns, containing quantities obtained by calculating in accordance with the above theory the value of β, the ratio of the dynamical to the statical deflection, and lastly the dynamical deflection.

By comparing the experimental and calculated values of the dynamical deflection it will be seen that, with the exception of the last set, the calculated values are smaller than the real values.

The excess, from its irregularity, is evidently due in part to some sources of error inseparable from the nature of the experiments, as, for example, the *set*, which shows itself by the greater difference exhibited in the case of cast iron; for the mean value of the excess in the five experiments on cast-iron bars is three-tenths (·32) of the statical deflection, whereas in the fourteen cases where wrought iron was employed, the mean value of the excess is one-tenth (·12) of the statical deflections. In the experiments on steel bars, on the other hand, the calculated

deflections are greater than the actual deflections. But the values of β, in the latter case, are smaller than in the experiments on wrought and cast iron, being, with one exception, less than unity.

In the next chapter I shall show that the inertia of the bar will account for the greatest part of the discrepancies above stated between the theoretical and experimental deflections, for it will appear that it tends to increase the theoretical deflections when β is greater than about 2, and to diminish them when less. In actual bridges the jolts from the joints of the rails, and the imperfect curvature or cambering of the bridge, also tends to disturb and augment the effect, and therefore we need not be surprised to find that the increase of deflection observed in the experiments of the Commission at Ewell and Godstone Bridges was greater than the theory would have assigned, as the following Table shows:

EWELL BRIDGE.				GODSTONE BRIDGE.			
Velocity in feet per second.	β.	$\frac{D}{S}$ Computed.	$\frac{D}{S}$ Observed.	Velocity in feet per second.	β.	$\frac{D}{S}$ Computed.	$\frac{D}{S}$ Observed.
25	414	1·002	1	22	236	1·004	1·23
30	287	1·004	1·07	40	72	1·015	1·15
54	88	1·01	1·14	73	22	1·06	1·31
75	46	1·02	1·09	90	14	1·10	..
90	32	1·04	..				

In the Ewell Bridge the difference is not more than the omission of the inertia of the bridge would account for; but in the Godstone Bridge the excess is much greater than in the Ewell Bridge. The Godstone Bridge was the first upon which the experiments in question were tried, and the scaffold and registering apparatus was by no means so complete and steady as that which was used for the Ewell Bridge (figured in Plate IV.). The actual quantity to be measured (about a quarter of an inch) was so small that the least unsteadiness in the apparatus would affect its correct registration. This cause may possibly account for some part of the difference between the two experiments.

In the next place I shall proceed to show how the effect of the inertia of the bridge or bar may be examined.

CHAPTER IV.

On the Effect of the Inertia of the Bridge.

In the mathematical theory of the previous chapter it has been assumed that the mass of the bridge is so small with respect to that of the load, that its inertia may be wholly neglected. But when the trajectories obtained by the apparatus just described (figured in Plate VI.) are compared with those derived by theory under the above hypothesis, considerable differences are observed which appear due to the neglect of the inertia of the bar or bridge. For example, in Plate VII., fig. 5, I have given a series of trajectories which I obtained from my apparatus.

The bar was of steel 3 feet in length between its bearing points; its section was square and about 0·22 inch in width and depth; its weight was 8 ounces avoirdupois, and the pressure on the roller was 5 lbs., which was very nearly the actual weight. Hence, the weight of the load was about ten times that of the bar; the central statical deflection (or S) $= 0{\cdot}764$ inch. In the figure the proportion of the ordinates to the abscissæ is greatly exaggerated (*vide* p. 471).

The values of β, which belong to the four trajectories in the figure, are respectively nearly 5, 2, 1, and ·4, as marked.

It happens that, with the exception of 5, the values of β in Mr. Stokes's Tables do not exactly coincide with the above, but it is easy to compare them with the trajectories in fig. 5, by taking the nearest cases.* Thus the curve of which $\beta = 2$ will

* It would have been better to have arranged the apparatus so as to have traced curves exactly corresponding to the values of β in Mr. Stokes's diagram (fig. 4, Plate VII.), as the change of form would thus have been more strikingly shown. But with respect to this, as well as to other parts of the investigation, I must remark that the necessity for presenting the Report of the Commission to Her Majesty before the recess, limited the time for carrying on this Inquiry, and therefore I have been compelled to leave many parts of it in an incomplete state, in order to hurry on to the conclusion. Experiments of the

lie between those which belong to 3 and $\frac{5}{4}$ in fig. 4, Plate VII., that for $\beta = 1$ a little above that which belongs to $\beta = \frac{5}{4}$, and that for $\beta = \cdot 4$ above that which belongs to $\beta = \frac{1}{2}$ or $\cdot 5$. Mr. Stokes also had foreseen * that the effect of the inertia of the bar would be to reduce the enormous deflections which occur in the second half of those theoretical trajectories which appertain to the values of β below unity. This view is fully confirmed by the experimental trajectories, of which fig. 5 contains specimens. But we will proceed to a more especial examination of the effect of the inertia of the bar.

There is a very striking similarity in the general forms of the corresponding trajectories in these two diagrams. In the curve that belongs to the smallest value of β, namely $\cdot 4$, the front of the experimental curve does not terminate so bluntly as in Mr. Stokes's diagram; and in all the trajectories it will be seen that their first intersection with the equilibrium curve takes place farther from the origin in the experimental cases than in the theoretical, which might be expected from the simplest view of the effect of inertia in the bar, which will of course retard the descent of the load at the beginning of the motion, and consequently tend to throw the first part of the trajectory higher up, and thus to carry the point of its intersection with the equilibrium curve to a greater distance from the origin of the curve.

It will be useful in this place to examine the relation between the weight of the load and the weight of the bridge in the experiments at Portsmouth, and in actual cases, in order to see what proportion the mass of the bridge bears to that of the load in reality. In the three first series of the Portsmouth experiments the weights of each cast-iron bar, 9 feet in length, were 67 lbs., 94 lbs., and 195 lbs. respectively. The loads laid upon each bar in the first series varied from 560 lbs. to 922 lbs.; in the second series from 560 lbs. to 1748 lbs.; and in the third series from 560 lbs. to 1648 lbs. Thus the weight of the load was con-

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nature of those given above, which are intended for the elucidation of the laws of certain mechanical phenomena, do not require the minute and delicate accuracy that are essential to physical experiments, in which the most precise numerical results are to be sought for.

* Camb. Phil. Trans. p. 708.

siderably greater than that of the bridge in all these cases. The exact ratios of load to bar in the above limiting examples are, respectively, in the first series 8·3 and 13·7; in the second series 5·9 and 18·5; in the third series 2·9 and 8·4. On the whole the weight of the load is from 3 to 14 times that of the bar. In my smaller experiments, steel bars weighing from 17 ounces to 8 ounces were employed, and loads varying from 5 lbs. to 3 lbs.; the weight of the load was therefore from 3 to 10 times that of the bar.

In the Godstone and Ewell Bridges, upon which the Commissioners experimented, the following ratios existed. It must first be observed, that every complete railway bridge for a double line consists of two bridges, one to carry each line of rails, and that the two, although lying close together, are in reality independent structures, so that the deflection of one under the action of a passing train does not affect the other. The total weight of half the Ewell Bridge is about 30 tons, and the weight of an engine and tender nearly 40 tons, so that the load is here $\frac{1}{3}$ heavier than the bridge. In the Godstone Bridge the weight of an engine and tender was 33 tons, and of the half-bridge 25 tons, which gives nearly the same proportion as the Ewell Bridge. These may serve as examples of bridges from 50 to 30 feet span. In the Dee Bridge, of which the span is 98 feet, the half-bridge is said to weigh 90 tons, and the engine and tender 30 tons.*

The Conway tube has a clear span of 400 feet, and its weight is 1146 tons. The Britannia tube in its greatest clear span is 460 feet, and the weight of the portion that belongs to this span, namely, of 472 feet of tube, is 1400 tons.† Taking an engine and train at above 60 tons, the bridge in these two cases is more than twenty times heavier than the load.

In the experimental apparatus the weight of the load was much greater with respect to the bars than in actual bridges, partly on account of the necessity for employing very flexible bars to render

* Report of the Commissioners of Railways on the Dee Bridge, page 5. At page 3 it is stated that two engines and tenders (or 60 tons) would be at the same time on one pair of girders; this would, however, be considered as a distributed load.

† Minutes of Evidence 1232, page 359, &c. Fairbairn's Account of the Britannia and Conway Tubular Bridges, page 184.

the changes of deflection sufficiently apparent, and partly on account of the great difference of length. If bars, bearing the same ratio of weight to the load as in bridges, were tried in the apparatus, the deflections would become so small that they would be scarcely appreciable. Hence it appeared impossible to obtain trajectories corresponding to different ratios of the masses of the load and bar, which were required to teach us the effect of inertia upon the trajectory; for as it plainly appears from the above data that the mass of the bridge is too considerable to be neglected, we have next to inquire whether the inertia of the bridge increases or diminishes the amount of central deflection of the bridge, which we have calculated on the supposition of the bridge being an elastic bar without sensible inertia.

The method by which I attempted to attain this object may be thus explained. In page 453 I have stated that if an elastic bar, resting on two fixed props, be deflected by a pressure applied at any point not in the centre, it will assume the form of a certain curve, in which the greatest deflection will not be at the place where the pressure is applied, but much nearer to the centre. In fact, as the deflection is small, this curve is so nearly the same in form, whether the pressure be applied in the centre or at any other point, that we may for our present purpose assume the same equation to belong to it in all cases.

The bar may thus be considered as a system of heavy particles, so connected that if motion be given to any one of them the whole will move from their initial position, and with velocities respectively proportional to the ordinates (y) of the curve which the bar assumes. Substitute for these heavy particles a mass collected in the centre of the bar, and therefore moving with a velocity proportional to the central ordinate (Y). Then as each particle m of the bar will resist the communication of motion with a force which is as the particle itself, and the square of its velocity jointly, it can be replaced by a particle at the centre of the bar, which is equal to $\frac{m y^2}{Y^2}$; and hence if this central mass be equal to the sum of these $\frac{m y^2}{Y^2}$, the effect of its inertia will be the same as that of the whole of the particles of the bar. Calculating this sum from the equation to the curve we find it to represent 0·486

of the mass of the bar, or one-half nearly. It thus appears that in considering the effect of the inertia of the bar, we may suppose a mass equal to one-half of its weight to be collected at the centre.

In the next place let there be a rod, $p\,q\,r$, below the bar (fig. 4, Plate VI.), balanced upon knife edges at q, and provided with a sliding weight at each end, and suppose these weights and the rod to be adjusted in equilibrium about the centre of motion; let k be the radius of gyration of the system, $M k^2$ its moment of inertia, and r the radial distance of the point p from the centre, then this system will resist the communication of motion to the point p, with a force equal to that of a mass $\frac{Mk^2}{r^2}$ collected at that point.

If the point p be connected to the centre o of the trial bar by a light link rod, this point will move with the same velocity as the centre of the trial bar, whenever motion is communicated to any point of the bar, and consequently the balance and its weights will revolve about the centre q. The effect of this arrangement, therefore, is the same as if a mass $\frac{Mk^2}{r^2}$ were collected in the centre of the bar. By altering the distance of the weights from the centre, always keeping them in equilibrium, we can increase or diminish the value of $\frac{Mk^2}{r^2}$ at pleasure, and as the system is in equilibrium we do not thereby affect the deflections of the bar. Thus we have at our disposal an artificial inertia applicable to the bar, by means of which we can, retaining the same bar and the same load, try successive experiments, and obtain successive trajectories appertaining to various proportions between the inertia of the load and that of the bar. Half the weight of the bar must of course be added to the mass $\frac{Mk^2}{r^2}$, which represents the inertia added by the 'Inertial Balance.' *

The link op was formed of flat steel, and was connected to the bar by a contrivance shown at large in fig. 8. The upper half of

* It may be necessary to remind my reader that the whole of this investigation proceeds upon the supposition that the deflections of the bar communicated by the travelling load take place simultaneously throughout its

the link was divided into two branches, and bent into the form shown in the drawing. Each branch carried a steel centre-point, and the branches could be set at any required distance by the thumb-screw and nut; their elasticity of course pressing them outwards. Two centre punch-holes were made in the sides of the bar at its middle point, and the steel points of the branches were adjusted so as to allow those points to enter the punched holes, and play therein with the least possible shake and friction. The lower end of the link is pierced and enters a slit in a small steel arm at *p*, screwed to the end of the lever of the balance. A wire pin passing through the holes drilled in the arm and link forms the lower joint; the lever of the balance is a square bar of oak, and graduated in ounces avoirdupois, so that the weights set to any given number of ounces on the scale, and of course balanced, shall represent the equivalent mass added to the half-weight of the bar; the sliding weights were $3\frac{1}{4}$ lbs. each.

From several sets of curves drawn with this apparatus, I have selected the three groups in figs. 6, 7, 8, Plate VIII. These trajectories were all obtained from a steel bar 4 feet long, of a square section 0·23 inch broad on each side. Its weight was 11 ounces avoirdupois, and the carriage was loaded with weights that gave an effective pressure of 3 lbs. All the curves in each group were drawn with the same velocity, and consequently have the same value of β. The curves in fig. 6 were drawn with a velocity of 7·7 feet per second, and $\beta = 6$. The curves in fig. 7 with a velocity of 11·9 feet, and $\beta = 2·4$. The curves in fig. 8 with a velocity of 16·6 feet, and $\beta = 1·2$.

The differences between the curves in each group are due to the different proportions of inertia introduced by the 'Inertial Balance.' To distinguish the several curves in each group from each other, different kinds of lines are employed. The equilibrium trajectory is necessarily the same in all the groups. This is distinguished by a plain thick line, and moreover has the name

length, and that consequently the bar at every instant of the passage of the load is bent into the same curve which it would assume if the point of application of the load were pressed down statically to the same position. See p. 456.

written upon it. The interrupted dots in cloudy masses indicate the course of the curve that corresponds to β in Mr. Stokes's Tables, and therefore to the case in which the inertia of the bar or bridge is so small as to be wholly neglected. The plain continuous line marked B, which lies close to it in the three groups, is the trajectory obtained from the bar before the Inertial Balance was connected to it; and therefore the ratio of the mass of the load to the bar in this case is more than 4 to 1.

The next trajectory in order is a dotted line, which was obtained by so adjusting the balance, that its effect should be to make the masses of the load and bar equal. The next, an interrupted line, similarly belongs to the case in which the mass of the bar is double that of the load; and the last, an interrupted line with longer strokes, alternating with dots, represents the case in which the mass of the bar is triple that of the load. Now it will be seen, on examining the three groups in figs. 6, 7, and 8, that the five curves do not follow throughout in the same order in all of them.

In the first part of the curves, indeed, as they start from their origin at the beginning of the bar, the order in all is the same; the increase of inertia uniformly throws the trajectory higher up, and we always find the equilibrium curve the lowest; the theoretical curve, in which the bar has no appreciable inertia, the next above, and the others rising in the order of their increased inertia.

All the dynamical curves intersect the equilibrium trajectory, and they all sag below it more or less in the second half. The increasing inertia carries the intersection of the curves with the equilibrium trajectory farther from the origin in every instance, and all the intersections lie farther from the origin in fig. 7 than in fig. 6, and still farther in fig. 8; that is to say, farther in the smaller values of β than in the larger.

But the effect upon the amount of the maximum depression of the trajectory is different in each of the three values of β. In fig. 6, the increase of inertia causes the successive trajectories to fall lower and lower, and thus to occasion a *greater central deflection* of the bar, as the mass of the bar is increased with respect to that of the load. In fig. 8, the increase of the inertia, on the contrary, diminishes the maximum deflection of the suc-

cessive trajectories, and thus occasions a *less central deflection* of the bar as the mass of the bar is increased.

To show this more clearly, I have introduced dotted lines in figs. 6 and 8, marked $T \dots T$, $B \dots B$, $1 \dots 1$, $2 \dots 2$, $3 \dots 3$, which lines will be observed each to begin from a point on the central ordinate of the curve, and to end in contact with the trajectory in order.* Each dotted line represents the part of the bar which lies between its centre and the trajectory, at the moment of greatest depression; and therefore shows the greatest central deflection of the bar that corresponds to each trajectory. It thus appears that in fig. 6 the increase of inertia in the bar carries it lower and lower, and in fig. 8 the reverse happens; and of course for smaller values of β the latter effect would be more strikingly developed; because, as we have seen, the central deflection goes on increasing as β diminishes, when the inertia of the bar is wholly neglected.

The succession of the curves shows pretty clearly, however, that if still more and more inertia were given to the bar in fig. 6, the series of trajectories would reach a maximum depression and then begin to rise; after which a further increase of inertia would diminish the central deflections, as in the curves of fig. 8. And this effect is shown in fig. 7, where the maximum deflection or sag of the trajectory which belongs to the bar alone sinks a little below that of the theoretical trajectory or curve of no inertia; and the next curve, in which the masses of bar and load are equal, sinks still a little lower. But the following curves, which belong to a double and triple ratio of these masses, rise higher and higher, and the central deflections of the bar follow in the same order.

It would seem from this, that for any given ratio of the masses of the bar and load some value of β may be found, for which a small variation in the ratio would neither increase nor diminish the central deflection of the bar; while for smaller values of β, the increase of inertia in the bar would diminish the central deflection, and for greater values of β, the reverse. It would require a long series of experiments to determine these values with accuracy,

* In these figures T denotes the theoretical trajectory, B the bar alone, in which the ratio of the mass of the bar to that of the load is $\frac{1}{2}$, and the figures 1, 2, 3, denote ratios of their respective values.

which the short time assigned for this research has made it impossible for me to attempt; but they may be roughly estimated as follows:

In fig. 7, $\beta = 2{\cdot}4$, and the trajectory which sinks the lowest in this figure is that which corresponds to the ratio of equality between the bar and load. It is evident from the manner in which the deflections of the bar succeed each other that the greatest deflection for this value of β would lie a little below that marked 1 in the figure, and probably correspond to about $\frac{B}{L} = {\cdot}7$ (where B is the mass of the bar and L of the load). Hence, to bring the trajectory for $\frac{B}{L} = 1$ to its maximum bar-deflection, a little larger value of β must be taken; and probably $\beta = 3$ will be very nearly the value that corresponds to the exact position of the maximum depression belonging to this trajectory.

In fig. 6, where $\beta = 6$, the last trajectory that the proportions of my apparatus enabled me to obtain belongs to the triple ratio of the masses. It seems probable that if two or perhaps three more had been drawn to correspond to the succeeding ratios, the maximum deflection would have been reached for this value of β; and that therefore the trajectory corresponding to the ratio $\frac{B}{L} = 6$ will be very nearly the one sought for.

Now Mr. Stokes has shown, as we shall see below, that when β is moderately large, and the above ratio also large, the trajectory remains constant if β varies as the ratio, that is to say, $\frac{B}{L} = c\beta$, where c is a constant. As we have just found a case in which the maximum deflection is given by a trajectory, in which the ratio of the weights of bar to load is nearly *equal* to β, and β is moderately large, we shall not err much in taking $c = 1$, and therefore in saying that the maximum deflection for any given large value of β will happen when the mass of the bridge is nearly β times that of the load.* This is sufficient to show us

* Subsequent researches of Mr. Stokes showed that in moderately large values of β, and large values of the ratio $\frac{B}{L}$, we have for the maximum deflection $\frac{B}{L\beta} = {\cdot}823$, which differs from unity by ${\cdot}177$ only. (See note, p. 504.)

that in all practical cases the inertia of the bridge will increase the deflection which is due to the velocity of the load; for in practice the value of β is always much greater than the ratio of the weights of the bridge and load. But to return to fig. 8; in this figure the central deflection (T) of the bar produced by the theoretical trajectory very nearly coincides with (B) that which is due to the trajectory for the bar alone in which $\frac{B}{L} = \frac{1}{4}$; more closely, in fact, than the figure shows, in which the distance between these two curves is slightly exaggerated. In this group, therefore, it happens that we have nearly the value of β, for which the maximum deflection of the bar is due to the theoretical curve. This value of β is 1·2, or unity, very nearly.

The general results of the experiments with the inertial balance may be therefore stated as follows:

(1.) For all values of β less than about unity, the least sensible inertia added to the bar will diminish the central deflection due to the theoretical trajectory, namely, that in which the bar is supposed to have no inertia.

(2.) For all values of β greater than about unity, inertia gradually added to the bar will at first increase the central deflection due to the theoretical trajectory, will then bring it to a maximum, and finally will diminish it.

(3.) The ratio of the masses of the bar to the load that corresponds to this maximum effect will be very nearly unity for $\beta = 3$, and for larger values of β and of $\frac{B}{L}$ will be expressed by the equation $B = \beta L$ (or more accurately $B = \cdot 823\, \beta L$).

The differences between the theoretical trajectories of fig. 4, Plate VII., and the experimental trajectories of fig. 5, are now explained. When the inertia of the bar is neglected, it was shown that for small values of β, the deflections of the bar became excessively great, and that when β is less than $\frac{1}{4}$, the tangent at the end of the trajectory is vertical, and the central deflection of the bar and the tendency to break the bridge become infinite. Mr. Stokes had already explained these startling results, by supposing that the inertia of the bridge was the cause of the practical modifications of these consequences; but without experiment it was impossible to ascertain that the inertia would,

in cases where β was greater than unity, produce the opposite effect of increasing the deflections, or indeed to understand the exact nature of the influence which different proportions between the inertia of the bar and load would have upon the trajectories.

In the last chapter it was shown that in real bridges β is rarely so small as 14, and hence it follows from the experiments of the inertial balance that the inertia of a bridge will tend to increase the deflections due to the theoretical trajectory of no inertia, which have been exhibited in Table IX. (p. 486). And the result is perfectly in conformity with the analysis of the sixth and seventh series of the Portsmouth experiments, given in Table X. (p. 488), in which the deflections for values of β greater than unity were all greater by about one-tenth, more or less, than the theoretical deflections. A similar increase was obtained from the experiments on the Godstone and Ewell Bridges, which has been now shown to be due, in part at least, to the inertia of the bridge. It also appeared from the same seventh series (Table X.) that when β was less than unity, the experimental deflections of the bar were less than the theoretical deflections of Table IX., which is also in accordance with the results obtained from the inertial balance.

It becomes therefore a point of great interest to determine the exact increment of the deflection of a real bridge that would be due to its inertia. My experiments, besides being limited to values of β considerably below 14, and therefore smaller than those that belong to practice, were, from want of time, too few in number and deficient in precision to give accurate numerical results, although amply exact enough to show the laws of the phenomena. The following values of the deflections in fig. 6 are probably not far from the truth, although subsequent and repeated experiments would be required to correct them.

In this figure $\beta = 6$, and the ratios of the dynamical to the statical deflection $\left(\frac{D}{S}\right)$ corresponding to the different ratios of inertia $\left(\frac{B}{L}\right)$ are given in the following Table:

$\frac{B}{L}$	0	¼	1	2	3
$\frac{D}{S}$	1·23	1·3	1·52	1·67	1·78

Thus for this value of β, the theoretical deflection with no inertia is increased by about ·07 when the bar has a mass of one-fourth of the load, and by ·3 when the masses of the two are equal.

These results have been obtained from experiments made on a small scale, but by setting down the equations that relate to the problem in its general form, Mr. Stokes succeeds in showing that if we have two systems in which the ratio of L to B is the same, and we conceive the travelling weights to move over the two bridges respectively, with velocities ranging from 0 to ∞, the trajectories described in the one case and the deflections of the bridge correspond exactly to the trajectories and deflections in the other case; so that to pass from the one to the other, it will be sufficient to alter all horizontal lines on the same scale as the length of the bridge, and all vertical lines on the same scale as the central statical deflection. The velocity in the one, which corresponds to a given velocity in the other, is determined by the value of the constant β.* We are thus furnished with the important result, that if by experiment a certain form of the trajectory be obtained, the same form will belong to every case in which the ratio of the masses of the bar and load is the same as in the experiment, and also the value of β the same.

Thus, by the use of the inertial balance, we shall be able to construct with facility a *dynamical model* of a large system, which we may wish to investigate experimentally. To take a numerical example, let there be a load of 25 tons moving over a girder bridge 40 feet long and weighing 25 tons, the central statical deflection being $\frac{3}{8}$ inch, and the velocity of the load 30 miles an hour, or 44 feet per second (this will give $\beta = 24$). Suppose the trial bar in the model to be 4 feet long, and the central statical deflection due to the travelling weight to be $1\frac{1}{2}$ inch. We must, in the first place, adjust the inertial balance so as to give to the bar a distributed mass equal to the mass of the load. We must now give to the carriage such a velocity as shall render β the same in the two cases. Since β is constant when the velocity varies directly as the length of the bridge, and inversely as the square root of the central statical deflection, we must alter the velocity in the direct ratio of 40 to 4, or 10 to 1, and in the

* Camb. Phil. Trans. p. 727.

inverse ratio of $\sqrt{\frac{4}{3}}$ to $\sqrt{\frac{9}{3}}$, or 2 to 3. Hence the velocity required in the model is $44 \times \frac{1}{10} \times \frac{2}{3}$, or 2·9 feet per second.

But to determine experimentally the amounts for high values of β, an apparatus calculated to operate upon longer bars with much less velocity would be necessary; fortunately, however, it happens that the investigations of Mr. Stokes will assist us in obtaining, at least in part, the information we require.

During the progress of my experiments above related, this gentleman had been simultaneously carrying on his theoretical researches with a view of determining the effect of the inertia of the bridge, which in the previous investigation had been neglected; and although he did not succeed in obtaining the complete solution of this most intricate problem, he rendered the greatest service to the question by obtaining an approximate solution; namely, one limited by the following condition, that the value of β be large or moderately large, and that the mass of the travelling body be *small* compared with the mass of the bridge.

Small values of β never occur with real bridges, and therefore the first condition includes all practical cases. Unfortunately the mass of the travelling body in practice is very nearly equal to that of the bridge, so that the latter condition does not represent the practical cases so well. But Mr. Stokes, by giving in the first place a solution of the case in which the mass of the bridge is neglected, and in the next place one in which the mass of the load is neglected, or its effect reduced to a travelling *pressure*, has solved the problem in the two extreme cases between which the practical examples lie; and has thus enabled us, assisted by the experiments, to calculate with sufficient accuracy the amount of additional deflection which is due to the velocity of the travelling load. I shall proceed, therefore, to explain the results of this most valuable addition to Mr. Stokes's former investigation, as nearly as possible in his own words, referring, as before, for the analysis to the original Paper in the 'Cambridge Philosophical Transactions.'

The general equations (which are given in the original paper) proved too complex to be manageable, but by introducing the limiting conditions above mentioned, namely that β be large or moderately large, and that the mass of the travelling body be

small compared with the mass of the bridge, Mr. Stokes succeeded in reducing the equations to a form which admitted of a complete solution, and hence has calculated the ordinates of the trajectories in a sufficient number of cases; so as to enable us to lay down the curves, and thus to understand the nature of the motion.

It appears that in these trajectories each phase is characterized by the value of a certain constant quantity, q, which occupies in this part of the investigation a similar office to the β of the previous pages.

This quantity, q, is defined as follows: let S be the central statical deflection, M the mass of the travelling body, M^1 the mass of the bar or bridge. Then

$$q^2 = \frac{63}{31}\frac{Mg}{M^1 V^2 S} = \frac{1008}{31}\frac{M\beta}{M^1}.*$$

* From this expression, it appears that if β vary directly as $\frac{M^1}{M}$ the value of q, and therefore the form of the trajectory remains unaltered; whence, having obtained from my experiments that, when $\beta = 6$, the trajectory which corresponds to the maximum deflection of the bar is very nearly that which belongs to the ratio $\frac{M^1}{M} = 6$, I inferred that we may roughly take $\frac{M_1}{M} = \beta$ to represent the case of the maximum deflection. Probably neither the ratio of the masses nor the value of β in this case is large enough to satisfy the conditions, upon which the above expression is founded, with sufficient accuracy. Upon this, however, Mr. Stokes has kindly furnished me with the following note: "In fig. C, it appears that the maximum curve of deflection lies between 3 and 4 (that is, between those which correspond to $\frac{2q}{\pi} = 3$ and 4). I have found by interpolation,

$\frac{2q}{\pi}$	Maximum value of $\frac{D}{S}$
3	1·717
4	1·697
5	1·580

And again, by interpolation, the maximum value of $\frac{D}{S} = 1{\cdot}721$, in which case $\frac{M_1}{M\beta} = {\cdot}823$, which differs only by ·177 from the result to which you were led by experiment."

Conceive the travelling mass M removed, and suppose the bar depressed through a small space and then left to itself to oscillate. It can be shown that if P be the period of motion, or twice the time of oscillation from rest to rest, S, the central statical deflection produced by a mass equal to that of the bridge and expressed in inches, and τ the time in seconds that the body takes to travel over the bridge, we have

$$P = 2\pi \sqrt{\frac{31}{63} \frac{S_1}{g}}; \quad q = 2\pi \frac{\tau}{P}.*$$

Hence the numbers 1, 2, 3, &c., written at the head of Tables A and B, and against the curves in Plate IX., represent the number of quarter periods of oscillation of the bridge which elapse during the passage of the body over it. This consideration will materially assist us in understanding the nature of the motion. It should be remarked, too, that q is increased by diminishing either the velocity of the body or the inertia of the bridge.

In Table A, the length of the bar is supposed to be divided into 20 equal parts for abscissæ, and the values of the ordinates $\frac{y}{S}$, corresponding to each of the 20 values of x, are given in the Table for 11 values of $\frac{2q}{\pi}$. The curves of this Table are the trajectories of the moving body, similarly with the trajectories of Plates VII. and VIII. To prevent the confusion which would have arisen if all these trajectories had been laid down in one figure, as in Plate VII., they have been divided into two groups in Plate IX. Fig. B contains those which appertain to the quarter periods 1, 2, 3, 4, 5, 6, and fig. D those which belong to the quarter periods 8, 10, and 16, 12 being omitted to prevent confusion. In each of these figures the equilibrium trajectory is laid down as a standard by which to compare them with each other, and with the trajectories already given.

Table B, however, to which correspond figs. C and E in Plate IX., refers to a different kind of curve, which may be termed the deflection curve. It is headed 'Values of $\frac{D}{S}$,' D

* If we suppose τ expressed in seconds, and S_1 in inches, we must put $g = 32{\cdot}2 \times 12 = 386$, nearly, and we get $q = \frac{28 \cdot \tau}{\sqrt{S_1}}$ . . . . (69).—*Camb. Phil. Trans.* p. 732.

being, as already explained, the central deflection of the bar which corresponds to any value of y.

The ordinate in these curves, therefore, represents the central deflection of the bar (expressed in its relation to S as those of the trajectories are), when the moving body has travelled over a distance represented by the abscissa, and hence the entire curve delineates the vertical motion of the centre of the bar during the progression of the body from one end to the other of the bar. It is, in fact, the curve which would be delineated by a pencil fixed to the centre of the bar (as in the apparatus described in the first chapter of this Essay), tracing its line upon a board that travels horizontally. If this board travelled uniformly at a rate equal to that of the body, the length of this curve would be exactly the same as that of the trajectory. This, for convenience sake, has been made the case with the figures in Plate IX.; for thus each of these deflection curves in figs. C and E lies immediately below the trajectory which belongs to it in figs. B and D respectively; in such a manner that when the body is at any given point in one of these trajectories, the magnitude of the central deflection of the bar at that instant is to be found in the ordinate of the deflection curve which is vertically beneath it.

TABLE A.

x	Values of $\frac{y}{S}$ when $\frac{2\eta}{\pi}$ is equal to										
	1	2	3	4	5	6	8	10	12	16	∞
·00	·000	·000	·000	·000	·000	·000	·000	·000	·000	·000	·000
·05	·001	·001	·001	·001	·001	·001	·002	·003	·004	·006	·025
·10	·003	·004	·007	·008	·012	·017	·025	·037	·050	·075	·096
·15	·008	·013	·022	·034	·050	·067	·108	·150	·190	·244	·207
·20	·015	·031	·059	·095	·137	·184	·279	·360	·414	·420	·344
·25	·029	·056	·126	·203	·290	·378	·532	·621	·630	·504	·496
·30	·045	·117	·230	·366	·509	·640	·814	·839	·744	·560	·646
·35	·063	·191	·374	·581	·778	·934	1·054	·940	·755	·727	·780
·40	·096	·285	·550	·828	1·062	1·205	1·178	·921	·759	·969	·886
·45	·133	·394	·748	1·085	1·316	1·395	1·164	·849	·846	1·084	·954
·50	·169	·516	·947	1·310	1·492	1·460	1·036	·812	1·004	·991	·977
·55	·210	·632	1·126	1·473	1·555	1·387	·860	·850	1·114	·852	·954
·60	·244	·739	1·258	1·542	1·487	1·191	·704	·923	1·062	·830	·886
·65	·274	·816	1·325	1·502	1·300	·917	·609	·942	·848	·857	·780
·70	·292	·854	1·308	1·352	1·022	·626	·565	·839	·584	·752	·646
·75	·298	·842	1·205	1·111	·705	·369	·532	·619	·391	·488	·496
·80	·282	·770	1·020	·814	·402	·180	·462	·359	·297	·280	·344
·85	·245	·644	·774	·509	·161	·069	·337	·149	·237	·178	·207
·90	·184	·463	·498	·244	·012	·020	·182	·037	·150	·121	·096
·95	·103	·248	·224	·064	−·037	·004	·051	·003	·047	·044	·025
1·00	·000	·000	·000	·000	·000	·000	·000	·000	·000	·000	·000

TABLE B.

x	Values of $\frac{D}{S}$ when $\frac{2q}{\pi}$ is equal to										
	1	2	3	4	5	6	8	10	12	16	∞
·00	·000	·000	·000	·000	·000	·000	·000	·000	·000	·000	·000
·05	·004	·004	·005	·006	·007	·008	·014	·019	·025	·041	·156
·10	·009	·013	·022	·027	·037	·053	·081	·117	·158	·239	·307
·15	·017	·028	·048	·075	·108	·146	·234	·327	·412	·530	·449
·20	·025	·052	·099	·159	·231	·309	·469	·607	·696	·707	·580
·25	·041	·093	·177	·285	·406	·531	·746	·871	·884	·707	·696
·30	·056	·144	·282	·451	·626	·787	1·003	1·031	·915	·689	·794
·35	·070	·214	·418	·650	·871	1·045	1·180	1·052	·845	·814	·873
·40	·100	·300	·578	·870	1·115	1·265	1·238	·967	·796	1·017	·930
·45	·134	·399	·757	1·097	1·332	1·412	1·178	·859	·856	1·097	·965
·50	·169	·516	·947	1·310	1·492	1·460	1·036	·812	1·004	·991	·977
·55	·213	·640	1·139	1·491	1·574	1·403	·870	·860	1·127	·862	·965
·60	·256	·776	1·321	1·619	1·562	1·250	·739	·969	1·115	·872	·930
·65	·306	·913	1·482	1·681	1·454	1·027	·682	1·054	·948	·959	·873
·70	·359	1·050	1·609	1·663	1·257	·769	·695	1·031	·718	·924	·794
·75	·419	1·181	1·691	1·560	·990	·517	·746	·869	·549	·707	·696
·80	·475	1·296	1·717	1·371	·677	·303	·777	·604	·499	·472	·580
·85	·533	1·399	1·681	1·106	·350	·149	·733	·325	·516	·384	·449
·90	·586	1·476	1·588	·776	·037	·064	·579	·117	·477	·385	·307
·95	·646	1·525	1·402	·400	−·234	·025	·321	·021	·296	·276	·156
1·00	·699	1·540	1·158	·000	−·446	·019	·000	·001	−·001	·000	·000

"In the trajectory 1, fig. *B*, the ordinates are small, because the body passed over before there was time to produce much deflection in the bridge; at least, except towards the end of the body's course, where even a large deflection of the bridge would produce only a small deflection of the body. The corresponding deflection curve (curve 1, fig. *C*) shows that the bridge was depressed, and that its deflection was rapidly increasing when the body left it.

"When the body is made to move with velocities successively one-half and one-third of the former velocity, more time is allowed for deflecting the bridge, and the trajectories marked 2, 3, are described, in which the ordinates are far larger than in that marked 1. The deflections, too, as appears from fig. *C*, are much larger than before, or at least much larger than any deflection which was produced in the first case while the body remained on the bridge. It appears from Table B, or from fig. *C*, that the greatest deflection occurs in the case of the third curve nearly, and that it exceeds the central statical deflection by about three-fourths of the whole.

"When the velocity is considerably diminished, the bridge has time to make several oscillations while the body is going over it. These oscillations may be easily observed in figs. *C* and *E*, more especially in the latter; and their effect on the form of the trajectory, which may indeed be readily understood from fig. *C*, will be seen on referring to figs. *B* and *D*." . . . . . .*

"When q is large,† as is the case in practice, the following expression will give with sufficient accuracy the value of the central deflection D_1.

$$\frac{D_1}{S} = -\frac{25}{8 \cdot q} \sin q x.$$

So that the central deflection is liable to be alternately increased

* Camb. Phil. Trans. page 733.

† Camb. Phil. Trans. p. 732 and 733. " As every thing depends on the value of q, in the approximate investigation in which the inertia of the bridge is taken into account, it will be proper to consider farther the meaning of this constant. In the first place it is to be observed that, although M appears in the equation $q^2 = \frac{1008\, M\beta}{31\, M^1}$, q is really independent of the mass of the travelling body; for when M alone varies, β varies inversely as S, and S

and decreased by the fraction $\frac{25}{8 . q}$ of the central statical deflection. And it can also be shown that

$$\frac{25}{8}\frac{1}{q} = \cdot 55\ \sqrt{\frac{M'}{M\beta}} = \cdot 112\ \frac{\sqrt{S_1}}{\tau}.$$

It is to be remembered that, in the latter of these expressions, the units of space and time are an inch and a second respectively. Since the difference between the pressure on the bridge and the weight of the body is neglected in the investigation in which the inertia of the bridge is considered, it is evident that the result will be sensibly the same, whether the bridge in its natural position be straight, or be slightly raised towards the centre, or, as it is technically called, *cambered*. The increase of deflection in the case first investigated would be diminished by a camber.

"In this Paper the problem has been worked out, or worked out approximately, only in the two extreme cases in which the mass of the travelling body is infinitely great and infinitely small respectively, compared with the mass of the bridge. The causes of the increase of deflection in these two extreme cases are quite distinct. In the former case the increase of deflection depends entirely on the difference between the pressure on the bridge and the weight of the body, and may be regarded as depending on the centrifugal force. In the latter, the effect depends on the manner in which the force, regarded as a function of the time, is applied to the bridge. In practical cases the masses of the body and of the bridge are generally comparable with each other, and the two effects are mixed up in the actual result. Nevertheless if we find that each effect, taken separately, is insensible, or so small as to be of no practical importance, we may conclude, without much

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varies directly as M, so that q remains constant. To get rid of the apparent dependence of q on M, let S_1 be the central statical deflection produced by a mass equal to that of the bridge, and at the same time restore the general unit of length. If x continue to denote the ratio of the abscissa of the body to the length of the bridge, q will be numerical, and therefore, to restore the general unit of length, it will be sufficient to take the general expression for β, namely, $\beta = \frac{g\,a}{4\,V^2\,S}$; let moreover τ be the time the body takes to travel over the bridge, $\therefore\ 2\,a = V\tau$, and we get $q^2 = \frac{63}{31}\cdot\frac{g\,\tau^2}{S_1}$."

fear of error, that the actual effect is insignificant. Now we have seen that if we take only the most important terms, the increase of deflection is measured by the fractions $\frac{1}{\beta}$ (page 486 above) and $\frac{25}{8 \cdot q}$ of S. It is only when these fractions are both small that we are at liberty to neglect all but the most important terms; but in practical cases they are actually small. The magnitude of these fractions will enable us to judge of the amount of the actual effect.

"To take a numerical example, lying within practical limits, let the span of a girder bridge be 44 feet, and suppose a weight equal to $\frac{4}{3}$ of the weight of the bridge to cause a deflection of $\frac{1}{5}$ inch. These are nearly the circumstances of the Ewell Bridge, mentioned in the Report of the Commissioners.

"In this case $S_1 = \frac{3}{4} \times \cdot 2 = \cdot 15$; and if the velocity be 44 feet in a second, or 30 miles an hour, we have $\tau = 1$, and therefore from the second of the formulæ just stated,

$$\frac{25}{8 \cdot q} = \cdot 0434 \qquad q = 72 \cdot 1 = 45 \cdot 9 \times \frac{\pi}{4}.$$

The travelling load being supposed to produce a deflection of ·2 inch, we have

$$\beta = 127 \;\therefore \frac{1}{\beta} = \cdot 0079.$$

Hence in this case the increase of the deflection due to the inertia of the bridge is between five and six times as great as that obtained by considering the bridge as infinitely light; but in neither case is the deflection important. With a velocity of 60 miles an hour, the increase of deflection ·0434 S would be doubled.

"In the case of one of the long tubes of the Britannia Bridge, β must be extremely large; but on account of the enormous mass of the tube, it might be feared that the effect of the inertia of the tube itself would be of importance. To make a supposition every way disadvantageous, regard the tube as unconnected with the rest of the structure, and suppose the weight of the whole train collected at one point. The clear span of one of the great tubes is 460 feet, and the weight of the tube 1400 tons.

"When the platform on which the tube had been built was

removed, the centre sunk 10 inches, which was very nearly what had been calculated, so that the bottom became very nearly straight, since, in anticipation of the deflection which would be produced by the weight of the tube itself, it had been originally built curved upwards. Since a uniformly distributed weight produces the same deflection as $\frac{5}{8}$ of the same weight placed at the centre, we have in this case $S_1 = \frac{8}{5} \times 10 = 16$; and supposing the train to be going at the rate of 30 miles an hour, we have $\tau = \frac{460}{44} = 10{\cdot}5$, nearly. Hence in this case $\frac{25}{8 \cdot q} = {\cdot}043$, or $\frac{1}{23}$, nearly; so that the increase of deflection due to the inertia of the bridge is unimportant."*

It appears from the above that the increase of deflection is

* In the course of the investigations undertaken by Mr. Stokes and myself, our attention was directed to an able Paper by Mr. Cox, 'On the Dynamical Deflection and Strain of Railway Girders,' which is printed in the Civil Engineers' and Architects' Journal for September, 1848. This Paper is purely theoretical, that is to say, that although the results are applied to practical cases, it is not founded upon experiments; and consequently the subject is looked at in a totally different light from that under which we have viewed it. The author has employed methods of approximation which, although they have not apparently vitiated his results, as far as real bridges are concerned, would yet cause them to fail utterly if applied to the interpretation of experiments, such as those contained in the present Essay. This must be carefully borne in mind in considering the Paper in question, which will well repay perusal. The reasons for this failure are explained in the following extract from Mr. Stokes's Paper in the Cambridge Philosophical Transactions (page 725):—"In this article the subject is treated in a very original and striking manner. There is, however, one conclusion at which Mr. Cox has arrived, which is so directly opposed to the conclusions to which I have been led, that I feel compelled to notice it. By reasoning founded on the principle of *vis viva*, Mr. Cox has arrived at the result that the moving body cannot in any case produce a deflection greater than double the central statical deflection, the elasticity of the bridge being supposed perfect. But among the sources of labouring force which can be employed in deflecting the bridge, Mr. Cox has omitted to consider the *vis viva* arising from the horizontal motion of the body. It is possible to conceive beforehand that a portion of this *vis viva* should be converted into labouring force, which is expended in deflecting the bridge; and this is, in fact, precisely what takes place. During the first part of the motion, the horizontal component of the reaction of the bridge against

measured by the two fractions $\frac{1}{\beta}$ and $\frac{25}{8.q}$ of S respectively in the two extreme cases in which the mass of the bridge or the mass of the body is neglected; and that, in practice, where these masses are very nearly equal, their effects are mixed up together in a manner that remains to be developed from the theoretical equation. It is extremely desirable, however, that we should in the mean time obtain some estimate of the practical effect of the inertia of the bridge. This Mr. Stokes suggested to me might

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the body impels the body forwards, and therefore increases the *vis viva* due to the horizontal motion; and the labouring force which produces this increase being derived from the bridge, the bridge is less deflected than it would have been had the horizontal velocity of the body been unchanged. But during the latter part of the motion the horizontal component of the reaction acts backwards, and a portion of the *vis viva* due to the horizontal motion of the body is continually converted into labouring force, which is stored up in the bridge. Now, on account of the asymmetry of the motion, the direction of the reaction is more inclined to the vertical when the body is moving over the second half of the bridge than when it is moving over the first half, and moreover the reaction itself is greater, and therefore, on both accounts, more *vis viva* depending upon the horizontal motion is destroyed in the latter portion of the body's course than is generated in the former portion: and therefore, on the whole, the bridge is more deflected than it would have been had the horizontal velocity of the body remained unchanged.

"It is true that the change of horizontal velocity is small; but nevertheless, in this mode of treating the subject, it must be taken into account; for, in applying to the problem the principle of *vis viva*, we are concerned with the square of the vertical velocity, and we must not omit any quantities which are comparable with that square. Now the square of the absolute velocity of the body is equal to the sum of the squares of the horizontal and vertical velocities, and the change in the square of the horizontal velocity depends upon the product of the horizontal velocity and the change of horizontal velocity; but this product is not small in comparison with the square of the vertical velocity."

I have great pleasure in taking this opportunity of expressing my acknowledgments to my excellent friend and fellow-labourer, Professor Stokes, for his kind and friendly co-operation with me in these investigations. I must also regret that the abstruse nature of his portion of them has prevented me from giving them at length, and thereby compelled me to do him great injustice by presenting his results only, apart from the admirable reasoning, by means of which they were obtained. It may be well to mention, however, that this course was adopted with his entire concurrence.

be roughly and empirically done by supposing the two fractions in question to represent the separate effects of the inertia of the bridge and load, and taking their sum to represent the total effect. Upon calculating the increments of the statical deflection in this manner, that were obtained experimentally by the inertial balance, (and given in the Table in page 501 above,) and comparing the results, it appears that the agreement is sufficiently close, as the following Table will show.

—	Values of $\frac{B}{L}$.			
	$\frac{1}{2}$	1	2	3
Experimental increments, $\beta = 6$ . . .	·3	·52	·67	·78
Calculated increments $\beta = 5$ . . .	·42	·55	·65	·72
Calculated increments $\beta = 6$ . . .	·34	·45	·54	·62

For larger values of β, in which the increments are smaller, we may suppose the errors to be less sensible, and therefore I have calculated the following Table for several values of β, and on the supposition that the masses of the bridge and load are equal, and, therefore, $\frac{25}{8 \cdot q} = \frac{\cdot 55}{\sqrt{\beta}}$. Rough and imperfect as this method must be, it may yet serve until further developments of the theory and more perfect experiments, both which are greatly to be desired, shall have substituted certain and logical results.

Values of β.	5	6	8	10	15	20	25	30	40	50	100	200
Increments of S when mass of bar is neglected (p. 486)	·30	·23	·18	·14	·10	·06	·05	·04	·03	·02	·01	·005
Values of $\frac{\cdot 55}{\sqrt{\beta}}$. . .	·25	·22	·19	·17	·14	·12	·11	·10	·09	·08	·05	·04
Total increment of statical deflection.	·55	·45	·37	·31	·24	·18	·16	·14	·12	·10	·06	·045

To apply this Table to any given bridge, the statical deflection due to the greatest load which is liable to pass over it must be

ascertained, and also the greatest probable velocity; from these data, and from the length of the bridge, the value of β must be calculated. (See page 483.) The increment of the statical deflection which corresponds to this value of β will be found in the lower line of the above Table.

I will conclude with a few remarks upon the purpose of the preceding pages. The experiments carried out at Portsmouth by Captain James and Lieutenant Galton had given the important and valuable result, that velocity imparted to a load increased the deflections of the bar or bridge over which it passed above those which it would have produced if set at rest upon the same bridge. The amount of this increase was also of so alarming a magnitude, that it seemed incredible that it should have escaped observation in practical cases. Accordingly, when the Commissioners visited the bridges at Ewell and Godstone, the effects there observed, although of the same character, were infinitely less in amount.

It became, therefore, necessary to investigate the laws of these phenomena; and as analysis, even in the hands of so accomplished a mathematician as Mr. Stokes, failed to give tangible results, excepting in cases limited by hypotheses that separated the problem from practical conditions, it became necessary to carry on also experiments directed to the express object of elucidating the theory and tracing its connection with practice. I have already stated that the time which remained to me for this purpose, as well as the limited funds placed in the hands of the Commission, were together insufficient to admit of either constructing the apparatus, or performing the experiments, with the minute and delicate accuracy required for the precise numerical results usually sought for in physical investigations. But my object was rather to elucidate general laws, guided by theory, than to obtain independent numerical results, and I trust that this purpose has been sufficiently answered.

It has been shown that the phenomena in question exhibit themselves in a highly developed state when the apparatus is on a small scale, but that, on the contrary, with the large dimensions of real bridges, their effects are so greatly diminished as to be comparatively of little importance, except in the cases of short

and weak bridges traversed with excessive velocities. The theoretical and experimental investigation, which is the subject of the above Essay, will, however imperfect, serve to show that such a diminution of effect, in passing from the small scale to the large, is completely accounted for.

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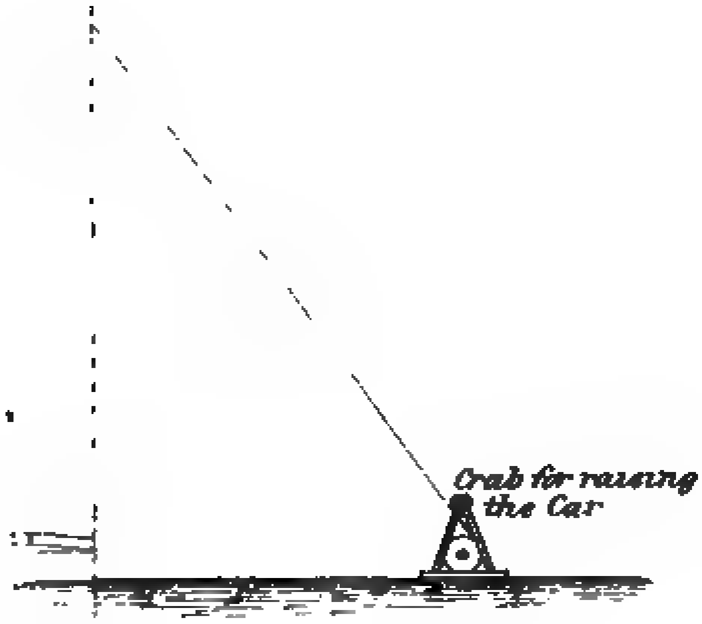
FIG. 3.

Rear Elevation



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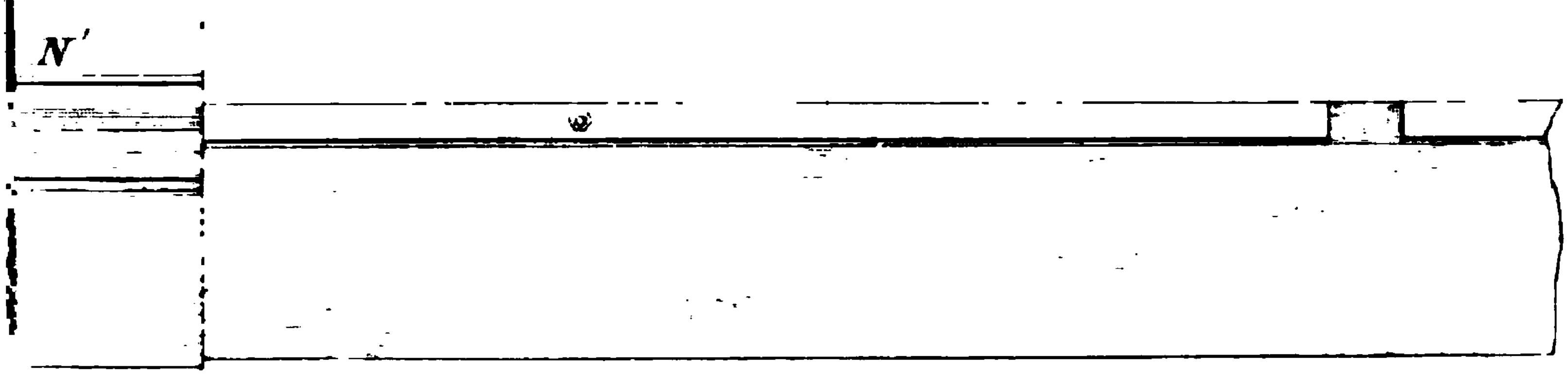

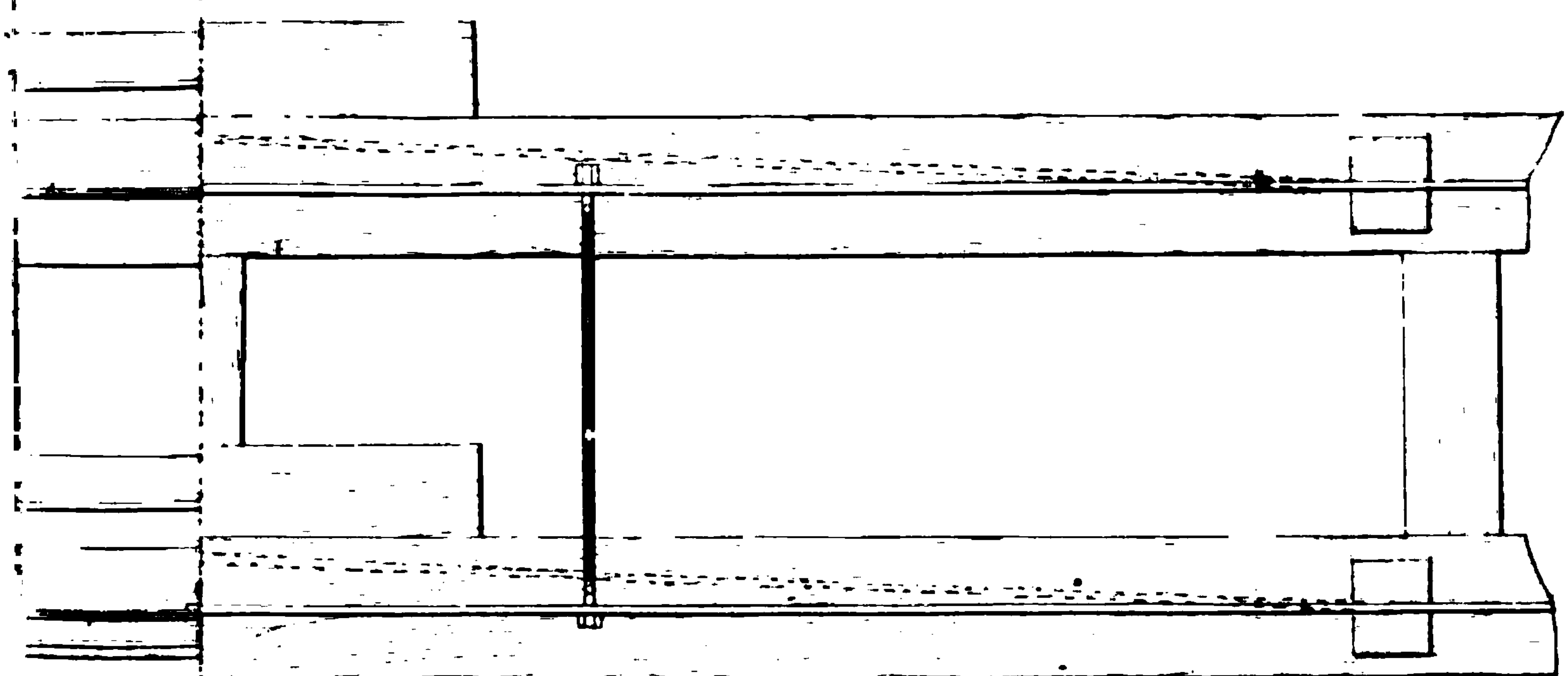

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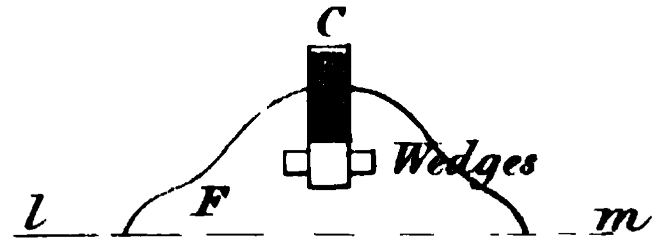
FIG. 12.



Section on l.m.

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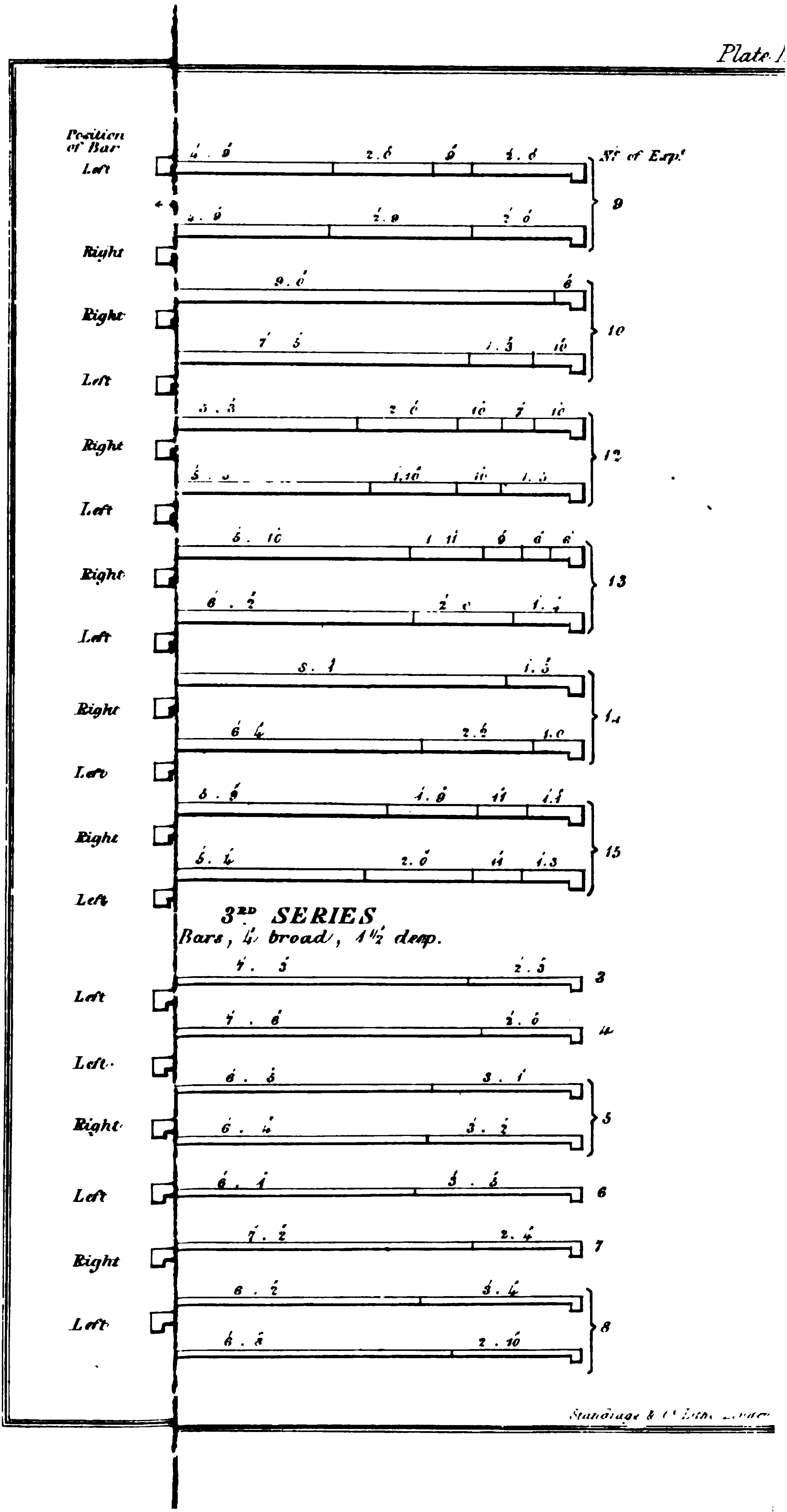
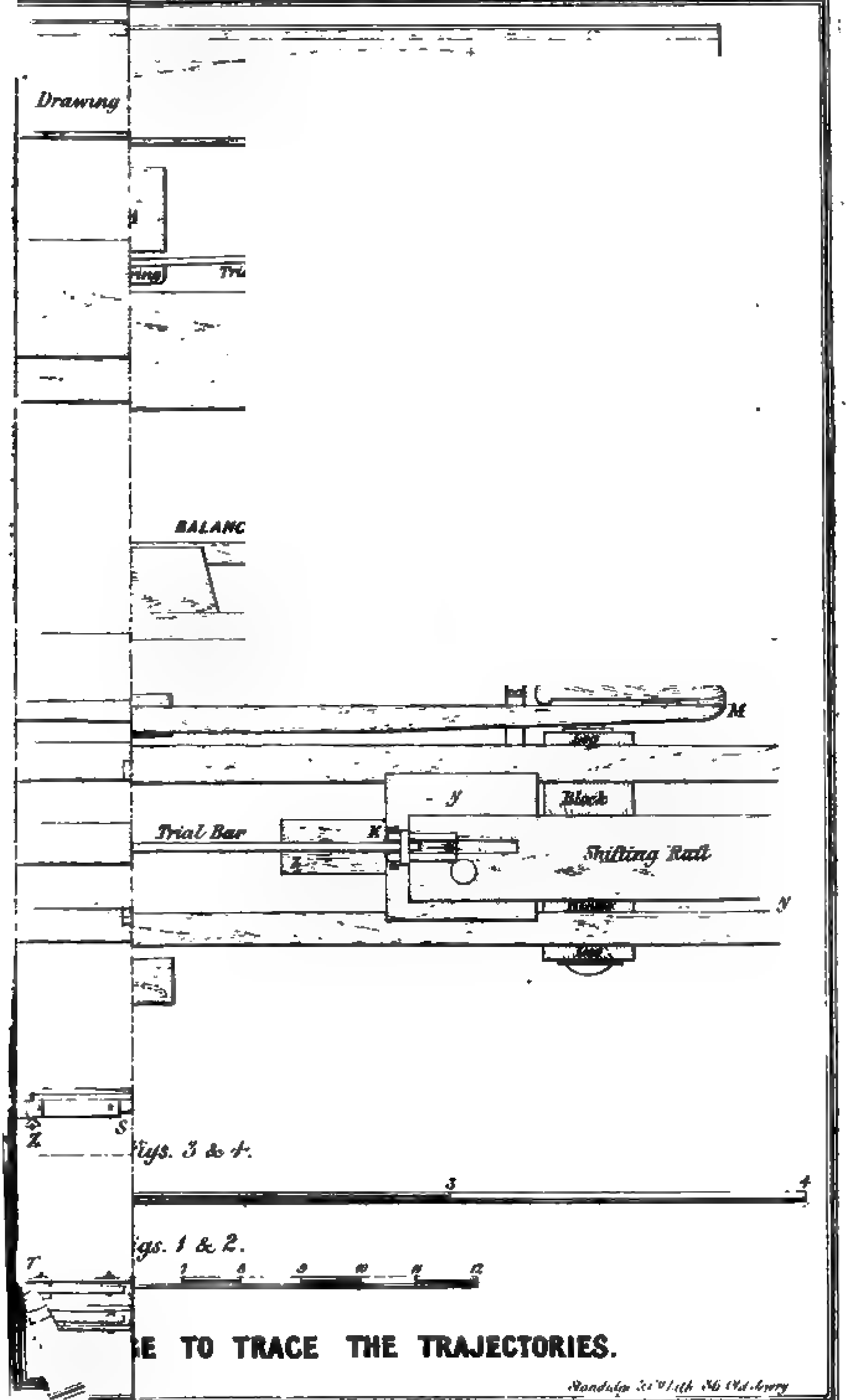

Position of Bar
No. of Expt.
Left
Right
9
10
12
13
15
3RD SERIES
Bars, 4" broad, 1½" deep.
3
4
5
6
7
8



…E TO TRACE THE TRAJECTORIES.

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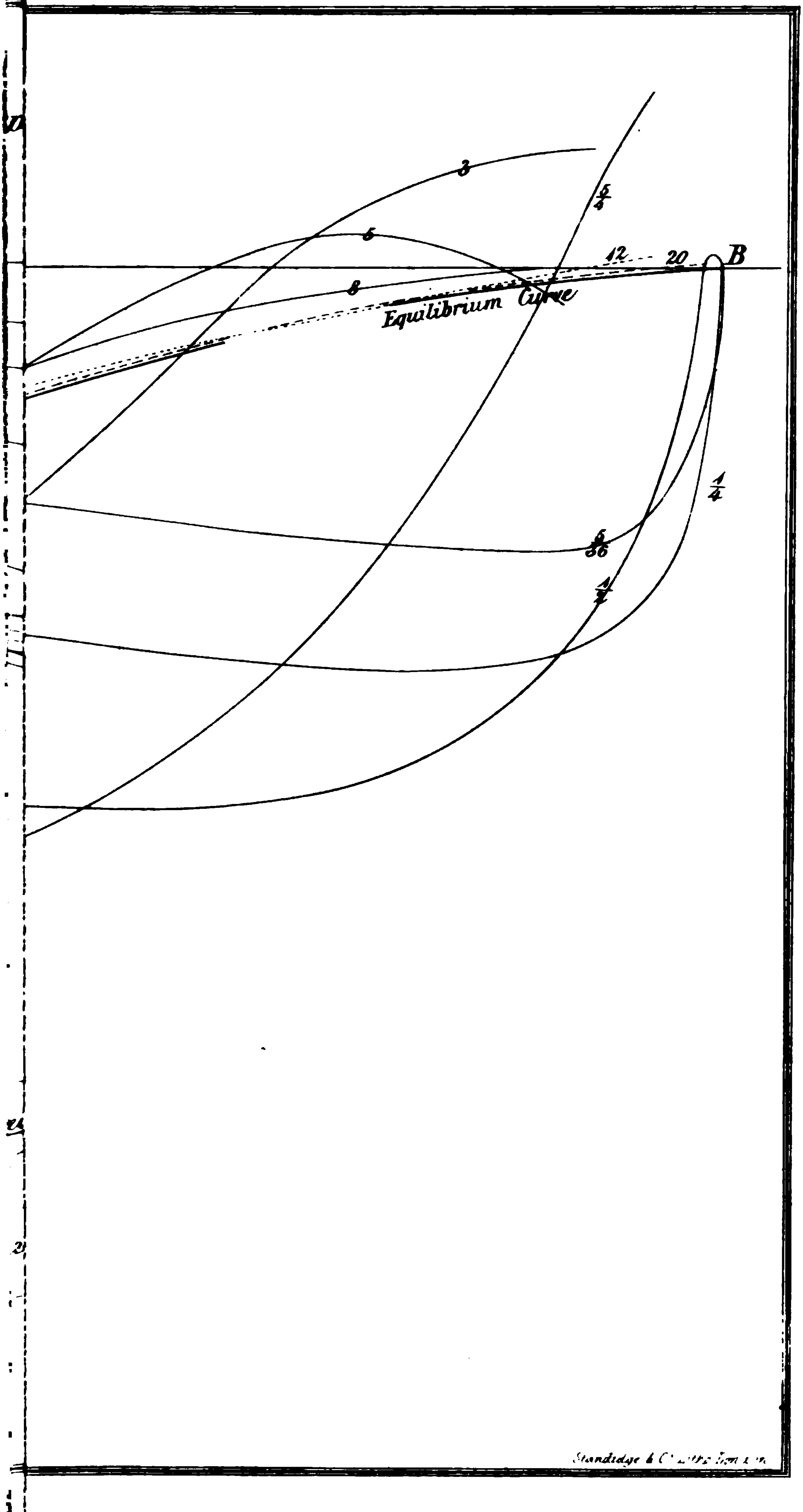
Plate VII.



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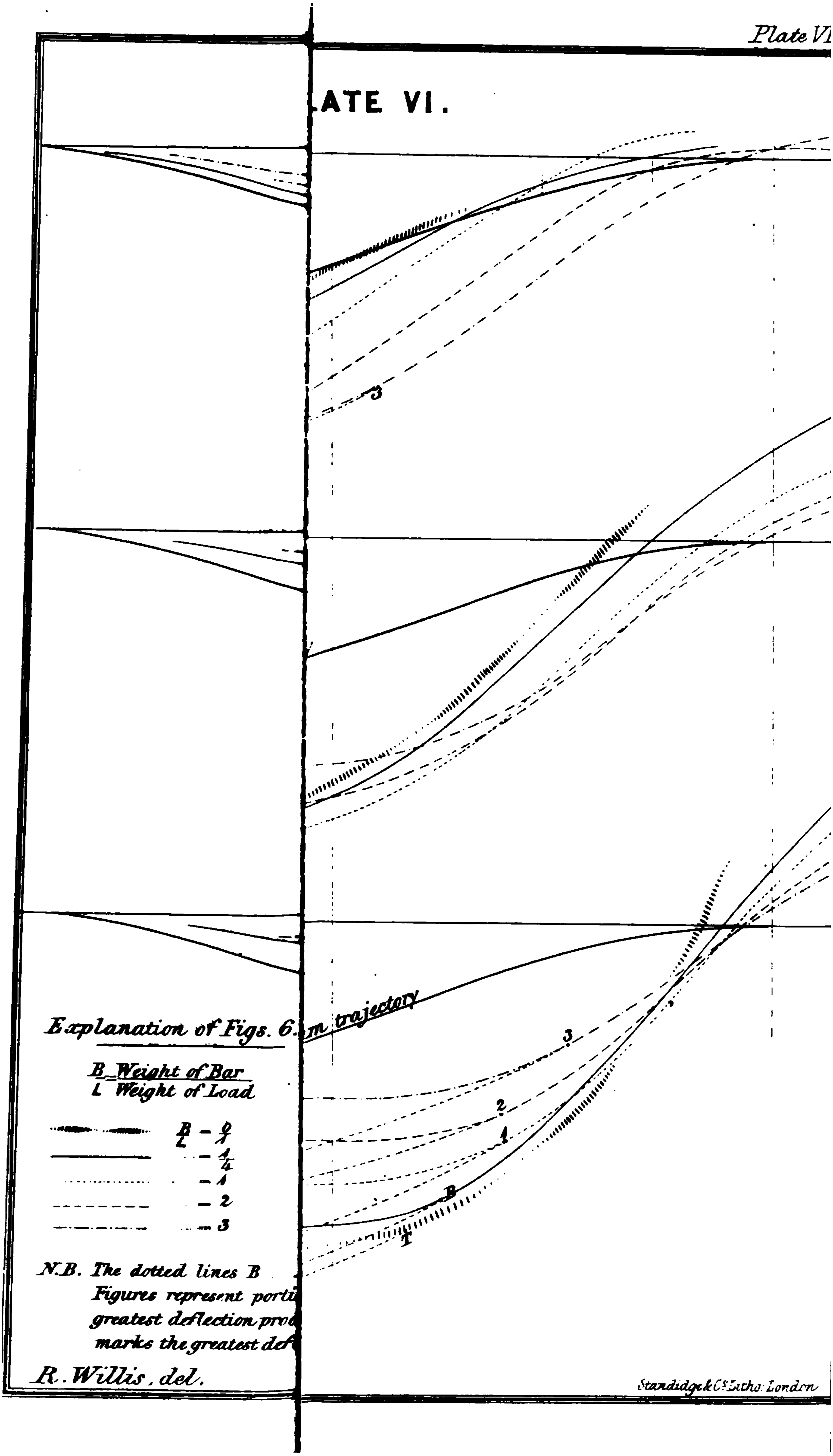
Plate VI
LATE VI.
Explanation of Figs. 6.
m trajectory
B Weight of Bar
L Weight of Load
B/L = 0/1
= 1/4
= 1
= 2
= 3
N.B. The dotted lines B
Figures represent porti
greatest deflection pro
marks the greatest def
3
2
1
B
T
R. Willis, del.
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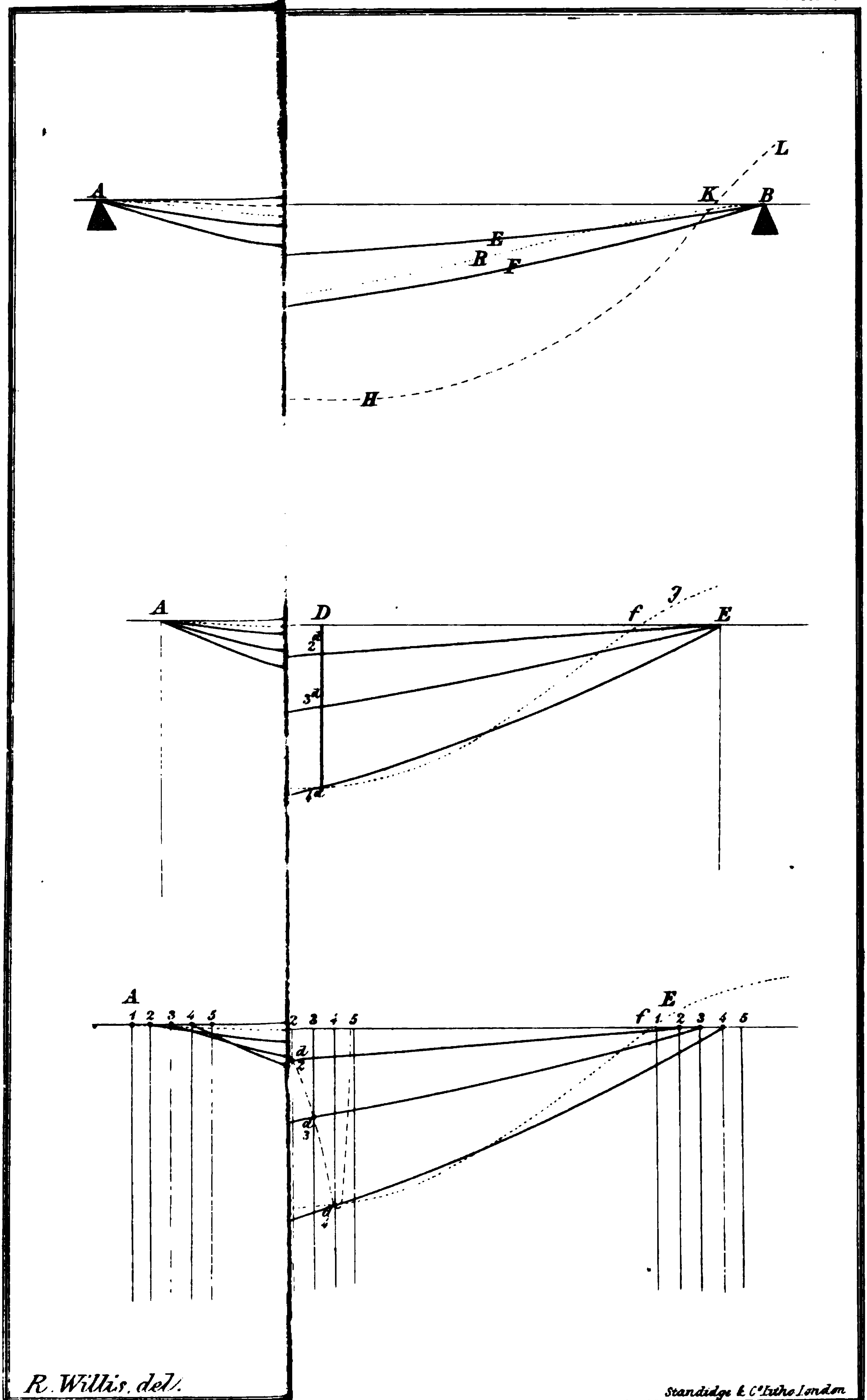
Plate V.



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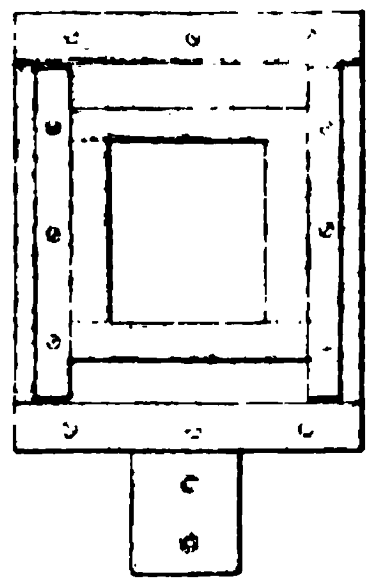
OBSERVING

E BRIDGES.

wing Board B. on an enlarged Scale.

rn.

Front Elevation.



Plan.

il and Clamp. The Pencil e was fitted very exactly
a case f, in which was a spring which caused the
il when adjusted in position to press against the
r on the Drawing Board; the adjustment was per-
ed by sliding the case f either backwards or for-
ds as was required in the cylinder g which was
ched to the Clamp and fixing it in the proper
tion by means of the screw h. The pencil was of
ss, and registered on metallic paper prepared by
Mrs Harwood.
wing Board fitted with a Zinc plate for the paper
against and adapted to slide horizontally
vertically for purposes of adjustment.
ion of Girder.
ort for Drawing Board.
e for Observer.

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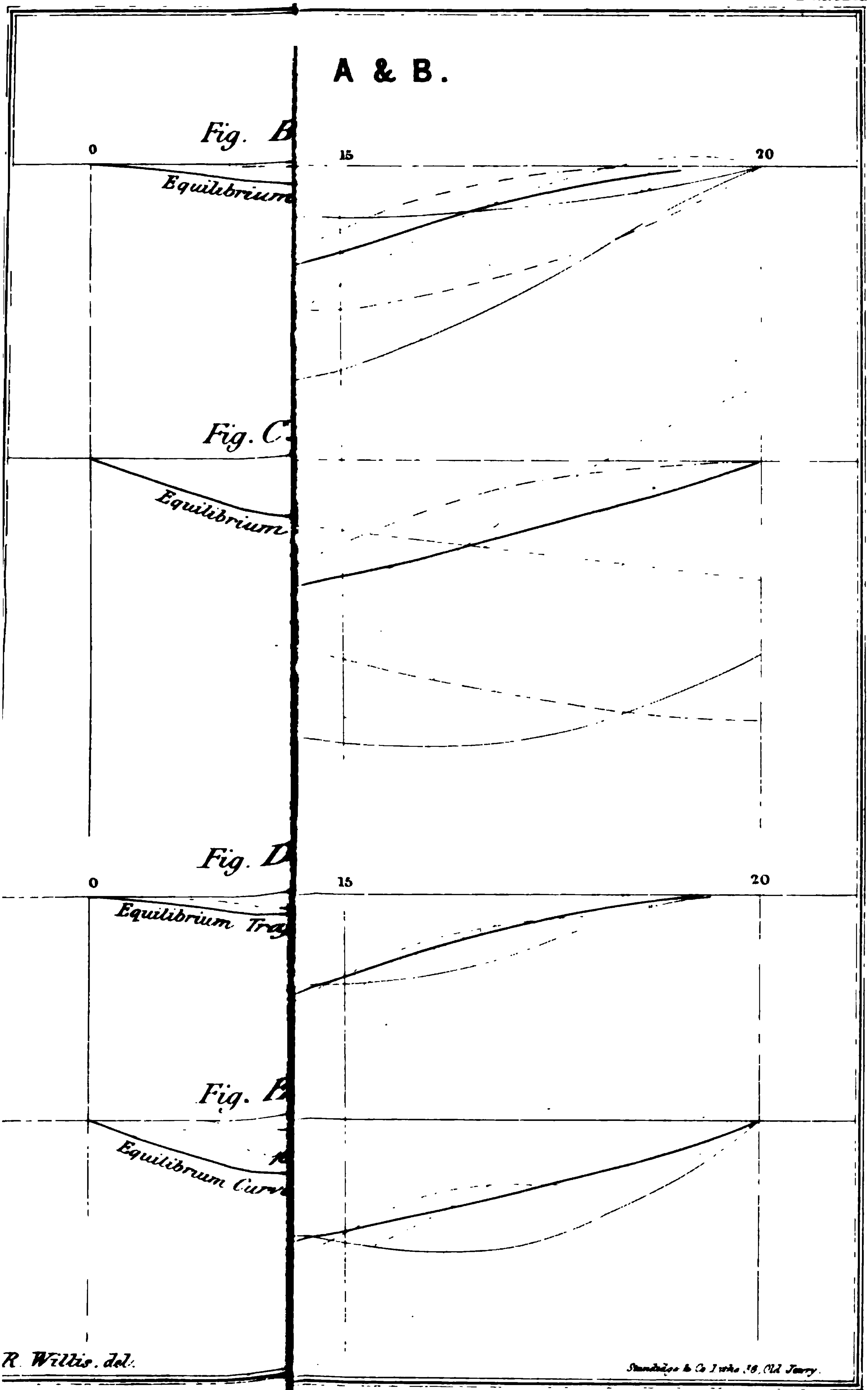
Plate IX.



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